2.1 Properties of Real Numbers and Simplifying Expressions

order doesn't matter

The grouping doesn't matter

Commutative properties		Associative properties	
$a + b = b + a$	← addition	$(a + b) + c = a + (b + c)$	← addition
$a \cdot b = b \cdot a$	← multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	← multiplication

Not true for subtraction and division

EXAMPLE 1 Using the Commutative Properties

Use a commutative property to complete each statement.

a. $-8 + 5 = 5 + (-8)$ ← put parentheses around negative numbers when adding

b. $(-2)7 = 7(-2)$

c. $7 + (-3) =$

$(-3)+7$

d. $(-5)4 =$ $4(-5)$

EXAMPLE 2 Using the Associative Properties

Use an associative property to complete each statement.

a. $-8 + (1 + 4) = (-8 + 1) + 4$

b. $[2 \bullet (-7)] \bullet 6 = 2 \bullet [(-7) \cdot 6]$

c) $-9 + (3 + 7) =$ $(-9+3)+7$

d. $5(-4 \cdot 9) =$ $[5 \cdot (-4)] \cdot 9$

Use groupings to make the problem easier

EXAMPLE 3 Distinguishing Between Properties

Is each statement an example of the associative or the commutative property?

(a) $(2 + 4) + 5 = 2 + (4 + 5)$

Associative (changes grouping)

(b) $6 \cdot (3 \cdot 10) = 6 \cdot (10 \cdot 3)$

Commutative (changes order)

(c) $(8 + 1) + 7 = 8 + (7 + 1)$

Associative & commutative!

EXAMPLE 4 Using the Commutative and Associative Properties

Find each sum or product.

a) $23 + 41 + 2 + 9 + 25$

$23+2+41+9+25$

$25 + 50 + 25$

$\boxed{100}$

b) $25(63)(4) = 25(4)(63)$

$100(63) = \boxed{6300}$

The Distributive property

$a(b+c) = a \cdot b + a \cdot c$

EXAMPLE 9 Using the Distributive Property

Use the distributive property to rewrite each expression.

(a) $5(9 + 6)$

$45 + 30 = 75$

(b) $4(x + 5 + y)$

$4x + 20 + 4y$

(c) $-\frac{1}{2}(4x + 3) = \left(-\frac{1}{2}\right) \cdot 4x + \left(-\frac{1}{2}\right) \cdot 3$

$-2x - \frac{3}{2}$

(d) $3(k - 9)$

$3k - 27$

(e) $8(3r + 11t + 5z)$

$24r + 88t + 40z$

EXAMPLE 10 Using the Distributive Property to Remove (Clear) Parentheses

Write each expression without parentheses.

When subtracting an expression in parentheses, subtract all parts

(a) $-(2y + 3)$

$-2y - 3$

(b) $-(-9w - 2)$

$-(-9w) - (-2)$

$9w + 2$

(c) $-(-x - 3y + 6z)$

$-(-x) - (-3y) - (6z)$

$x + 3y - 6z$

Simplifying Expressions

EXAMPLE 1 Simplifying Expressions

Simplify each expression.

(a) $4x + \underbrace{8 + 9}$

$4x + 17$

(b) $4(3m - 2n)$ get rid of parentheses w/ distribution

$12m - 8n$

(c) $6 + 3(4k + 5)$

$6 + 12k + 15$

$\underbrace{6 + 15} + 12k$

$\boxed{21 + 12k}$

(d) $5 - (2y - 8)$

$5 - (2y) - (-8)$

$5 - 2y + 8$

$\underbrace{5 + 8} - 2y$

$\boxed{13 - 2y}$ or $\boxed{-2y + 13}$

e) $3(3x - 4y)$

$9x - 12y$

f) $-4 - (-3y + 5)$

$-4 - (-3y) - (5)$

$-4 + 3y - 5$

$\underbrace{-4 - 5} + 3y$

$\boxed{-9 + 3y}$ or $\boxed{3y - 9}$

Combining Like Terms:  +  = ? Can't add these!

Term Everything that is multiplied together separated by + and −	**Numerical coefficient** Number in front of the term — variable ↓ 3x ↑ coefficient

EXAMPLE 2 Combining Like Terms

If the variable is the same letter raised to the same power, add the coefficients.

Combine like terms in each expression.

(a) $-9m + 5m$

$-4m$

(b) $6r + 3r + 2r$

$6+3+2=11$

$\boxed{11r}$

(c) $4x + x$

$5x$

no coefficient? It's a 1

(d) $16y^2 - 9y^2$ same variable, same power

$7y^2$

e) $32y + 10y^2$

can't combine - different power

f) $4p^2 - 3p^2$

$1p^2$ or $\boxed{p^2}$

EXAMPLE 3 Simplifying Expressions Involving Like Terms

Simplify each expression.

(a) $14y + 2(6 + 3y)$

$14y+12+6y$

$14y+6y+12$

$\boxed{20y+12}$

(b) $9k - 6 - 3(2 - 5k)$

$9k-6-6+15k$

$\underbrace{9k+15k}\ \underbrace{-6-6}$

$\boxed{24k-12}$

(c) $-(2 - r) + 10r$

$-2-(-r)+10r$

$-2\underbrace{+r+10r}$

$\boxed{-2+11r}$

d) $5(2a - 6) - 3(4a - 9)$

$10a-30-12a+27$

$\underbrace{10a-12a}\ \underbrace{-30+27}$

$\boxed{-2a-3}$

e) $5x^2 - 2y + 3(-4x^2 + 6x) + 7x - 5y$

$5x^2-2y-12x^2+18x+7x-5y$

$\underbrace{5x^2-12x^2}+\underbrace{18x+7x}\ \underbrace{-2y-5y}$

$\boxed{-7x^2+25x-7y}$

★ The term with the biggest power always goes first

f) $-2(3y^3 + 5y) - 4(2y^2 - 7y + 9)$

$-6y^3-10y-8y^2+28y-36$

$-6y^3-8y^2\underbrace{-10y+28y}-36$

$\boxed{-6y^3-8y^2+18y-36}$

g) $(3z^4 + 5z^2 - 9) + (-8z^4 - 6z + 9)$

doesn't change the things in parentheses.

$3z^4+5z^2-9+(-8z^4)-6z+9$

$\underbrace{3z^4(-8z^4)}+5z^2-6z\underbrace{-9+9}_{0}$

$\boxed{-5z^4+5z^2-6z}$

2.2 Solving Linear Equations

Identity properties

nothing added — 1 as the coefficient

$n + 0 = n$ and $1 \cdot a = a$

We want to end with this when solving an equation

Inverse properties

$a + (-a) = 0$ and $\frac{1}{a} \cdot a = 1$

These properties can be helpful in solving an equation:

To solve $x - 9 = -3$, adding 9 to the left side makes a zero and gets the variable by itself. But to keep it balanced, if 9 is added to the left side, it must also be added to the right side, and this helps to find the value of x.

$+9 \mid +9$

$x = 6$

The upside down ⊥ helps me keep the equation balanced – whatever I do to 1 side, I must do to the other.

Inverse Operations:

To undo addition, subtract from both sides.
To undo subtraction, add.

1) $x - 9 = 17$

$+9 \mid +9$

$\boxed{x = 26}$

2) $-5 = x + 2$

$-2 \mid -2$

$\boxed{-7 = x}$

Always pay attention to the variable. We want to get it alone

To undo multiplication, divide on both side of the equation.
To undo division, multiply.

3) $4x = 36$

$\frac{4x}{4} \mid \frac{36}{4}$

$\boxed{x = 9}$

4) $\frac{x}{3} = 10$

$\times 3 \mid \times 3$

$\boxed{x = 30}$

multiply to "undo" x divided by 3

Practice:

1. $x + 16 = 7$

$-16 \mid -16$

$\boxed{x = -9}$

2. $-7 = x - 22$

$+22 \mid +22$

$\boxed{15 = x}$

3. $\frac{x}{4} = 3$

$\times 4 \mid \times 4$

$\boxed{x = 12}$

4. $5x = 60$

$\div 5 \mid \div 5$

$\boxed{x = 12}$

Some eq

Some equations require more than one inverse operation. Try these two-step equations:

On 2-step equations:
1. Add or subtract first
2. Multiply or divide last

1. $2x - 7 = 15$

$+7 \quad +7$

$2x = 22$

$\div 2 \quad \div 2$

$\boxed{x = 11}$

2. $-4 = -3n + 1$

$-1 \quad -1$

$-5 = -3n$

$\div -3 \quad \div -3$

$\boxed{\frac{5}{3} = n}$ or you can write a decimal

3. $\frac{x}{3} - 8 = -2$

$+8 \quad +8$

$\frac{x}{3} = 6$

$\times 3 \quad \times 3$

$\boxed{x = 18}$

4. $\frac{1}{5}x + 9 = 3$

$-9 \quad -9$

$\frac{1}{5}x = -6$

$.5 \quad .5$

$\boxed{x = -30}$

☆ $\frac{1}{5}x$ is the same as $\frac{x}{5}$

In longer equations, use this checklist:

- Are there parentheses? Distribute!
- Are there like terms together on one side of the equal sign? Combine them!
- Is there a variable on both sides of the equation? Get rid of one of the variable terms by using inverse operations 😊.

1. $\frac{3}{5}x + 16 = \frac{8}{5}x$

$\frac{8}{5} - \frac{3}{5} = \frac{5}{5}$

$-\frac{3}{5}x \quad -\frac{3}{5}x$

$16 = 1x$

$\boxed{x = 16}$

2. $4(x - 30) = x - 6$

$4x - 120 = x - 6$

$-x \quad -x$

$3x - 120 = -6$

$+120 \quad +120$

$3x = 114$

$\frac{3x}{3} = \frac{114}{3}$

$\boxed{x = 38}$

3. $5x + 6x = 33$

$11x = 33$

$\div 11 \quad \div 11$

$\boxed{x = 3}$

4. $6 - 6x = -7x + 5$

$+7x \quad +7x$

$6 + x = 5$

$-6 \quad -6$

$\boxed{x = -1}$

5. $3x - 12 + x + 2 = 5 + 3x + 2$

$4x - 10 = 7 + 3x$

$-3x \quad -3x$

$x - 10 = 7$

$+10 \quad +10$

$\boxed{x = 17}$

2.3 Solving Complex Equations

First, let's do some shorter problems:

1) $\frac{3}{4}x = 6$

2 steps!

$\times 4 \quad \times 4$

$3x = 24$

$\div 3 \quad \div 3$

$\boxed{x = 8}$

For fractions, you can use 2 steps, or just 1

1 Step

$\frac{4}{3} \cdot \frac{3}{4}x = \frac{6}{1} \cdot \frac{4}{3}$

$x = \frac{24}{3} = \boxed{8}$

2) $-x = -17$

$\div -1 \quad \div -1$

$\boxed{x = 17}$

Remember – if the variable shows up more than once, look to see if they are together on the same side of the = or if there is a variable on both sides of the = .

3) $7x + 1 = 10x - 29$ opposite sides – use inverse

$-7x \quad -7x$

$1 = 3x - 29$

$+29 \quad +29$

$\frac{30}{3} = \frac{3x}{3}$

$\boxed{10 = x}$

4) $2 - 5(x - 3) - x = x - 6$

$\underbrace{2 - 5x + 15 - x}_{\text{same side!}} = x - 6$

$-6x + 17 = x - 6$ opposite sides

$-x \quad -x$

$-7x + 17 = -6$

$-17 \quad -17$

$-7x = -23$

$-7x = -23$

$\div -7 \quad \div -7$

$\boxed{x = \frac{23}{7}}$

5) $3x - (2x + 7) = 8x + 2$

same side $\underbrace{3x - 2x}\ -7 = 8x + 2$

opposite sides $x - 7 = 8x + 2$

$-x \quad -x$

$-7 = 7x + 2$

$-2 \quad -2$

$\frac{-9}{7} = \frac{7x}{7}$

$\boxed{x = -\frac{9}{7}}$

6) $4(4 - 3x) = 32 - 8(x + 2)$

$16 - 12x = 32 - 8x - 16$ Combine like terms

$16 - 12x = -8x + 16$

$+12x \quad +12x$

$16 = 4x + 16$

$-16 \quad -16$

$0 = 4x$

$\div 4 \quad \div 4$

$\boxed{x = 0}$

7) Consider this equation with fractions two ways and choose your favorite:

a) Use your skills adding fractions to combine like terms, etc.

$\frac{2 \cdot 2}{2 \cdot 3}x - \frac{3 \cdot 1}{3 \cdot 2}x = -\frac{1}{6}x - 2$

Common denominator will be 6

$\frac{4}{6}x - \frac{3}{6}x = -\frac{1}{6}x - 2$

$\frac{1}{6}x = -\frac{1}{6}x - 2$

$+\frac{1}{6}x \quad +\frac{1}{6}x$

$\frac{6}{2} \cdot \frac{2}{6}x = -2 \cdot \frac{6}{2}$

$\boxed{x = -6}$

b) Clear ALL the fractions by multiplying everything on both sides of the equation by 6:

the common denominator

$\frac{6}{1} \cdot \frac{2}{3}x - \frac{6}{1} \cdot \frac{1}{2}x = \frac{6}{1} \cdot -\frac{1}{6}x - 2 \cdot 6$

$\frac{12}{3}x - \frac{6}{2}x = -\frac{6}{6}x - 12$

$\underbrace{4x - 3x} = -1x - 12$

$x = -x - 12$

$+x \quad +x$

$\frac{2x}{2} = \frac{-12}{2}$

$\boxed{x = -6}$

Some equations have special solutions:

8) $5x - 15 = 5(x - 3)$

$5x - 15 = 5x - 15$ exactly the same on both sides

$-5x \quad -5x$

$0 - 15 = -15$

$-15 = -15$ True

$\boxed{\text{Infinitely Many Solutions}}$

*No matter what number I put in for x, it will be true

9) $2x + 3(x + 1) = 5x + 4$

$2x + 3x + 3 = 5x + 4$

$5x + 3 = 5x + 4$

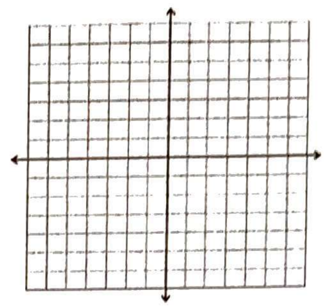
$-5x \quad -5x$

$0 + 3 = 4$

$3 = 4$? Not true!

$\boxed{\text{No Solution}}$

*No value for x with work



Solving 2-variable equations for one of the variables

Equations with both x and y can be graphed on the coordinate plane.
Often it is useful to get y by itself to make the graph.

Solve for y in each of the equations below

- Move the term with x out of the way by using inverse operations
- Divide both sides by the coefficient of y. (Be sure to divide EVERYTHING on each side!)

1) $6x + 3y = 21$

$-6x \quad -6x$

$\frac{3y}{3} = \frac{21 - 6x}{3}$ divide all terms by 3

$y = \frac{21}{3} - \frac{6}{3}x$

$\boxed{y = 7 - 2x}$

2) $-5x - 2y = 8$

$+5x \quad +5x$

order doesn't matter

$\frac{-2y}{-2} = \frac{5x + 8}{-2}$

$y = -\frac{5}{2}x - \frac{8}{2}$

$\boxed{y = -\frac{5}{2}x - 4}$

3) $-4x + 5y = 10$

$+4x \quad +4x$

$5y = 4x + 10$

$\div 5 \quad \div 5$

$\boxed{y = \frac{4}{5}x + 2}$

4) $x - 4y = 12$

$-x \quad -x$

$-4y = -x + 12$

$\div -4 \quad \div -4$

$y = +\frac{x}{4} - 3$

or

$\boxed{y = \frac{1}{4}x - 3}$

* $\frac{x}{4}$ is the same as $\frac{1}{4}x$