Finance Guided Notes: Consumer Mathematics

Section F.1: Simple Interest

Interest *is the extra money that you earn from a bank account or that you pay when you take out a loan.*

Warm-up:

a) Express 0.07 as a percent.

7%

b) Express each percent as a decimal:

a. 19 %

.19

b. 3.2 %

.032

Simple Interest

$$\boxed{I = Prt}$$

I = Interest

P = Principle

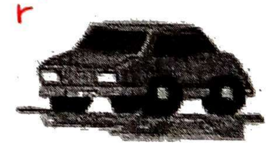
r = rate

t = time

Note: The **rate *r***, is expressed as a decimal when calculating simple interest.

Example 1: A student took out a simple interest loan for $1800 for two years at a rate of 8% to purchase a new car. Find the interest of the loan.

P t r



$I = (1800)(2)(0.08)$

$I = \$288$

Example 2: Fred made an investment for 5 years at a rate of 6% and ended up earning $120 in interest. How much was the investment for?

P=? t r I

$120 = P\cdot(0.06)(5)$

$\frac{120}{.3} = \frac{P\cdot(.3)}{.3}$

$P = \$400$

Compounding Interest

$$A = P(1 + \frac{r}{n})^{nt}$$

A = Amount your account grows to

n = number of times it's compounded in a year

For now, let's just explore interest which is <u>compounded annually</u>. In this case, n = 1

Rewrite the formula above, using n = 1:

$A = P(1 + \frac{r}{1})^t$ or $A = P(1+r)^t$

Example 3: You deposit $2000 in a savings account at Hometown Bank, which has a rate of 6%, compounded annually.

P r

a. Find *A*, the amount of money in the account after 3 years.

t

$A = 2000(1 + \frac{.06}{1})^{3 \cdot 1}$

$A = \$2382.03$

b. Find the **interest**. *(Remember that part of the amount now in your account was your own money that you deposited. The interest is the portion that the bank gave you – so* ***how much more*** *is there now compared to what you put in?)*

$$\begin{array}{r} 2382.03 \\ -\ 2000 \\ \hline \$382.03 \end{array} = \text{interest}$$

Example 4: $5000 deposited into an account with an interest rate of 7%, compounded annually.

P r

a. How much money would be in the account after 4 years?

t

$A = 5000(1 + \frac{.07}{1})^{4 \cdot 1}$

$A = \$6553.98$

b. How much interest was paid over 4 years?

$$\begin{array}{r} 6553.98 \\ -\ 5000 \\ \hline \end{array}$$

$I = \$1553.98$

F.2 Compound Interest

■**Compound Interest:** To calculate the compound interest paid **more** than once a year we use

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Note: We will assume 365 days per year and round answers to the nearest cent (__2__ decimal places.)

Example 1:

$P=7500$ $r=0.06$

You deposit $7500 in a savings account that has a rate of 6%. The interest is compounded monthly.

$n=12$

a. How much money will you have after five years?

$t=5$

$$A = 7500\left(1+\frac{.06}{12}\right)^{12\cdot 5} = \$10,116.38$$

b. Find the interest after five years.

$$\begin{array}{r} 10,116.38 \\ -\ 7500 \\ \hline \boxed{\$2616.38} \end{array}$$

Example 2: How much money would be in an account earning 4.2% interest, compounded quarterly, if $3000 is deposited and left in the account for 10 years?

$P=3000$ $t=10$ $r=0.042$ $n=4$

$$A = 3000\left(1+\frac{0.042}{4}\right)^{4\cdot 10} = 4555.8977 \quad \boxed{\$4555.90}$$

round up

Example 3: An investment is made into a fund that earns 4.2%, compounded daily. If $8,000 is initially invested, how much money will be in the account after 5 years? How much interest will have accrued?

$t=5$ $r=.042$ $n=365$ $P=8000$

n - how many times a year

monthly = 12
weekly = 52
daily = 365
yearly = 1
quarterly = 4

$$A = 8000\left(1+\frac{.042}{365}\right)^{365\cdot 5}$$

$$\boxed{\$9869.31}$$

Interest:

$$\begin{array}{r} 9869.31 \\ -\ 8000 \\ \hline \boxed{\$1869.31} \end{array}$$

■Compound Interest: Continuous Compounding

$$A = Pe^{rt}$$

Note: There is a key for "e" on your calculator – take a second to find it!

Example 4: $P = 8000$ $t = 6$ $r = 0.07$

You decide to invest $8000 for 6 years and you have a choice between two accounts. The first pays 7% per year, compounded monthly. $n = 12$

The second pays 7% per year, compounded continuously. Which is the better investment?

$$A = 8000\left(1 + \frac{.07}{12}\right)^{6 \cdot 12} \quad \text{vs.} \quad A = 8000\,e^{.07 \cdot 6}$$

$12 160.84 $12175.69

better investment

Example 5: Charlie invests $3000 in an account that earns 5% interest, compounded continuously. How much money would be in the account after 10 years?

$P = 3000$ $t = 10$ $r = 0.05$

use the e equation

$$A = 3000\,e^{(.05)(10)}$$

$$\boxed{\$4946.16}$$

F.3 Annuities

Investments - Index Funds are safe

■**Annuities**: An *annuity* is a sequence of equal payments made at equal time periods.

The *value of an annuity* is the sum of all payments plus all interest paid.

■Annuity Interest Compounded *n* times per Year

If P is the deposit made at the end of each compounding period for an annuity that pays an annual interest rate r (in decimal form) compounded n times per year, the value, A, of the annuity after t years is:

$$\boxed{A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}}$$

P=1000

Example 1: To save for retirement, you decide to deposit \$1000 into an IRA (individual retirement account) at the end of each year for the next 30 years. If you can count on an interest rate of 10% per year compounded annually,

n=1 t=30 r=.10

a. How much will you have from the IRA after 30 years?

$$A = \frac{1000\left[\left(1+\frac{.10}{1}\right)^{1\cdot 30} - 1\right]}{\left(\frac{.10}{1}\right)} = \frac{1000\left[(1+.10)^{1\cdot 30}-1\right]}{.10} = \$164{,}494.02$$

b. Find the interest earned. $30 \times 1000 = 30{,}000$

$$164{,}494.02 - 30{,}000 = \boxed{\$134{,}494.02}$$

Example 2: At age 25, to save for retirement, you decide to deposit \$200 into an IRA at the end of each month at an interest rate of 7.5% per year compounded monthly.

r=0.075 n=12 P=200

a. How much will you have from the IRA when you retire at age 65?

$t = 65 - 25 = 40$

$$A = \frac{200\left[\left(1+\frac{.075}{12}\right)^{12\cdot 40} - 1\right]}{\frac{.075}{12}} = \$604{,}764.43$$

b. Find the interest earned.

$200 \times 12 = 2400$ each year for 40 years $= \begin{array}{r} 2400 \\ \times 40 \\ \hline 96{,}000 \end{array}$

$\begin{array}{r} 604{,}764.43 \\ -96{,}000 \\ \hline \end{array}$ $\boxed{\$508{,}764.43}$

The interest is more than ~~[illegible]~~ 5 times the amount of your contributions to the IRA.

$$\frac{508764.43}{96{,}000} = 5.2996$$

$$\frac{96000 \times ?}{96000} = \frac{508764.43}{96000}$$

$P = 1000$ $n=4$

Example 3: At age 30, to save for retirement, you decide to deposit $1000 into an annuity each quarter at an interest rate of 5.5% per year compounded quarterly.

$r = 0.055$ $n=4$

a. How much will you have from the annuity when you retire at age 60? $t = 30$

$$A = \frac{1000\left[\left(1+\frac{.055}{4}\right)^{4\cdot 30} - 1\right]}{\frac{.055}{4}} = \$301{,}729.22$$

b. Find the interest earned.

1000 per quarter x 4 = 4000 per year for 30 years → 4000x30 = $120,000

$$\begin{array}{r} 301{,}729.22 \\ - \$120{,}000 \\ \hline \boxed{\$181{,}729.22} \end{array}$$

The interest is more than ___1.5___ times the amount of your contributions to the IRA.

$$\frac{181{,}729.22}{120{,}000} =$$

Example 4: At age 20, to save for retirement, you decide to deposit $3000 semi-annually into an annuity at an interest rate of 4.5% per year compounded semi-annually.

$r = .045$ $P = 3000$ $n=2$

a. How much will you have from the annuity when you retire at age 60?

$t = 60 - 20 = 40$

$$A = \frac{3000\left[\left(1+\frac{.045}{2}\right)^{2\cdot 40} - 1\right]}{\frac{.045}{2}} = \$657{,}352.71$$

b. Find the interest earned.

$3000 x 2 = $6000 per year

40 years

$240,000 deposited

$$\begin{array}{r} 657{,}352.71 \\ - 240{,}000 \\ \hline \boxed{\$417{,}352.71} \end{array}$$

1.7 times more

F.4: Installment Buying

■**Fixed Installment Loans**

❖ The ***amount financed*** is what the consumer borrows:

❖ Amount financed = cash price – down payment.

❖ The ***total installment price*** is the total Sum of all monthly payments plus the down payment:

❖ Total Installment Price = Total of all monthly payments + down payment.

❖ The finance charge is the interest on the installment loan:

Finance charge = Total installment price - Cash price.

Example 1: The cost of a used pick-up truck is $9345. We can finance the truck by paying $300 down and $194.38 per month for 60 months.

a. Determine the amount financed.

9345
− 300

$\boxed{\$9045 - \text{amount financed}}$

b. Determine the total installment price.

194.38
×60

$\$11,662.80 + 300 = \boxed{\$11,962.80}$

c. Determine the finance charge.

11,962.80
− 9345

$\boxed{\$2617.80}$

■ **Open-end Installment Loans**

- Using a credit card is an example of an open-end installment loan.
- Customers receive a statement each month.

■**Methods for Calculating Interest on Credit Cards:**

Use $I = Prt$, where r is the monthly rate and t is **one month.**

__Unpaid balance method__: The principal, P, is the balance on the first day of the billing period less payments and credits.

Example 1: Christian's credit card company starts each billing period on the first day of each month, and it uses the unpaid balance method. On the last day of January, Christian put airline tickets on his credit card, totaling $4000. His credit card charges 1.35% interest each month. Christian puts no other charges on his credit card for the rest of the year.

a) What is Christian's unpaid balance for February?

$\boxed{\$4000} + (4000)\cdot(.0135) =$

54

$4054

b) How much interest will Christian be charged during the month of February?

$4000 \times .0135$

$\boxed{\$54}$

$I = P \cdot r \cdot t$

$4000 \cdot 0.0135 \cdot 1 =$

c) What is Christian's balance on the credit card by the end of February?

$\boxed{\$4054}$

d) The credit card requires a $85 minimum payment. What is Christian's unpaid balance for the start of March, if he pays the minimum amount?

$4054 - 85 = \boxed{\$3969}$

e) How much interest will Christian be charged in March?

$3969 \times .0135 = \boxed{\$53.58}$

f) What is Christian's balance on the credit card by the end of March?

$3969 + 53.58 = \boxed{\$4022.58}$

Example 2: Christian's credit card company starts each billing period on the first day of each month, and it uses the unpaid balance method to calculate interest. His balance the last day of December was \$5000.00. The credit card company charges 18% interest per year (so 1.5 % per month.) The credit card company requires a minimum payment of \$200 per month for Christian. Fill out the table for Christian's credit card.

Kind of low - most are above 20%

Month	Amount of interest due	New balance with interest included	Ending balance with payment made.
January	5000 · 0.015 = \$75	5075.00	-200 4875
February	73.13	4948.13	4748.13
March	71.22	4819.35	4619.35
April	69.29	4688.64	4488.64
May	67.33	4555.97	4355.97
June	65.34	4421.31	4221.31
July	63.32	4284.63	4084.63
August	61.27	4145.90	3945.90
September	59.19	4005.09	3805.09
October	57.08	3862.17	3662.17
November	54.93	3717.10	3517.10
December	52.76	3569.86	3369.86

You have paid \$200 ×12 = \$2400

Total interest paid

\$ 769.86

4875 − 3369.86 = \$1505.14 paid off with your \$2400

Pay day loans - what is percentage?

F.5: The Cost of Home Ownership -- Mortgages

- A ***mortgage*** is a long-term loan for the purpose of buying a home.
- The down payment is the portion of the sale price of the home that the buyer initially pays to the seller.
- The ***amount of the mortgage*** is the difference between the price of the home and the down payment.
- Mortgages have the same **monthly payment** during the entire time of the loan.
- In addition, lending institutions can require that part of the monthly payment be deposited into an escrow account, an account used by the lender to pay real estate taxes and insurance.

Loan Payment Formula for Installment Loans

The regular payment amount, *PMT*, required to repay a loan of *P* dollars paid *n* times per years over *t* years at an annual rate *r* is given by

$$\boxed{\text{Payment} = \frac{P\left(\frac{r}{n}\right)}{\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]}}$$

Mortgages are paid monthly, so ***always use n = 12*** *for home loans.*

Example 1: The price of a home is \$195,000. The bank requires a 10% down payment since the buyer is a first-time home buyer. The cost of the home is financed with a 30-year fixed rate mortgage at 7.5%.

a. Find the required down payment. (What portion does the buyer pay right away?)

$195{,}000 \times .10 = \boxed{\$19{,}500}$

b. Find the amount of the mortgage. (How much becomes a loan?)

$195\,000 - 19{,}500 = \boxed{\$175{,}500}$

c. Use the formula above to find the monthly payment (excluding escrowed taxes and insurance).

$P = 175{,}500$
$r = .075$
$t = 30$
$n = 12$

$$\frac{175500\left(\frac{.075}{12}\right)}{\left[1-\left(1+\frac{.075}{12}\right)^{-12\cdot 30}\right]} = \$1227.12$$

d. Find the total amount paid by the owner over 30 years.

$1227.12 \times 12 \times 30 = 441{,}763.20$

per month — months — years

e. Find the total interest paid over 30 years.

$441{,}763.20 - 175{,}500 = \boxed{\$266{,}263.20}$

more than the amount you borrowed!

Example 2: The price of a home is \$465,000. The bank requires a 20% down payment at the time of closing. The home is financed with a 30-year fixed rate mortgage at 5.5% APR.

a. Find the required down payment.

$465{,}000 \times .20 = \boxed{93{,}000}$

b. Find the amount of the mortgage.

$465{,}000 - 93{,}000 = 372{,}000$

c. Find the monthly payment (excluding escrowed taxes and insurance).

$$\frac{372{,}000\left(\frac{.055}{12}\right)}{\left[1-\left(1+\frac{.055}{12}\right)^{-12\cdot 30}\right]} = \$2112.18 \text{ monthly}$$

d. Find the total amount paid by the owner over 30 years.

$2112.18 \times 12 \times 30 = \$760{,}384.80$

e. Find the total interest paid over 30 years.

$760{,}384.80 - 372{,}000 = \boxed{\$388{,}384.80}$

Example 3:
As another option, the family buying the home above decides to consider a 20-year mortgage, still at 5.5% and with a 20% down-payment. Find the monthly payment and the total interest paid over 20 years.

$$\frac{372{,}000\left(\frac{.055}{12}\right)}{\left[1-\left(1+\frac{.055}{12}\right)^{-12\cdot 20}\right]} = \$2558.94 \text{ monthly}$$

$2558.94 \times 12 \times 20 = \$614{,}145.60$

$- 372{,}000$

$\$242{,}145.60$ total interest