2-1 Conjectures and Counterexamples

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

16. 4, 8, 12, 16, 20

ANSWER:

Each element in the pattern is four more than the previous element; 24.

18. 1, 4, 9, 16

ANSWER:

Each element is the square of increasing natural numbers; 25.

19. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

ANSWER:

Each element is one half the previous element; $\frac{1}{16}$.

21. Percent humidity: 100%, 93%, 86%, . . .

ANSWER:

Each percentage is 7% less than the previous percentage; 79%.

22. Work-out days: Sunday, Tuesday, Thursday, . . .

ANSWER:

Sample answer: each work out day is two days after the previous day; Saturday.

26. 

ANSWER:

Each figure in the pattern is the next largest regular polygon.

27. 

ANSWER:

The shading of the lower triangle in the upper right quadrant of the first figure moves clockwise through each set of triangles from one figure to the next.

28. **FITNESS** Gabriel started training with the track team two weeks ago. During the first week, he ran 0.5 mile at each practice. The next three weeks he ran 0.75 mile, 1 mile, and 1.25 miles at each practice. If he continues this pattern, how many miles will he be running at each practice during the 7th week?

ANSWER:

2 mi

Make a conjecture about each value or geometric relationship.

34. the relationship between a and b if $ab = 1$

ANSWER:

They are reciprocals.

36. the relationship between the angles of a triangle with all sides congruent

ANSWER:

The angles are all congruent.

REASONING Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

40. If n is a prime number, then $n + 1$ is not prime.

ANSWER:

False; Sample answer: If $n = 2$, then $n + 1 = 3$, a prime number.

2-1 Conjectures and Counterexamples

41. If x is an integer, then $-x$ is positive.

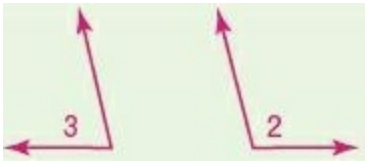
ANSWER:

False; sample answer: Suppose $x = 2$, then $-x = -2$.

42. If $\angle 2$ and $\angle 3$ are supplementary angles, then $\angle 2$ and $\angle 3$ form a linear pair.

ANSWER:

False; sample answer:



43. If you have three points A, B, and C, then A, B, C are noncollinear.

ANSWER:

False; sample answer:

44. If in $\Delta ABC, (AB)^2 + (BC)^2 = (AC)^2$, then ΔABC is a right triangle.

ANSWER:

true

45. If the area of a rectangle is 20 square meters, then the length is 10 meters and the width is 2 meters.

ANSWER:

False; sample answer: The length could be 4 m and the width could be 5 m.

51. **GOLDBACH'S CONJECTURE** Goldbach's conjecture states that every even number greater than 2 can be written as the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 3 + 5$.
a. Show that the conjecture is true for the even numbers from 10 to 20.
b. Given the conjecture *All odd numbers greater than 2 can be written as the sum of two primes,* is the conjecture *true* or *false*? Give a counterexample if the conjecture is false.

ANSWER:

a. $10 = 5 + 5$, $12 = 5 + 7$, $14 = 7 + 7$, $16 = 5 + 11$, $18 = 7 + 11$, $20 = 7 + 13$
b. False; 3 cannot be written as the sum of two primes.

60. Ray made the following conjecture: "If four points lie in a plane, then the points are collinear." Which figure is a counterexample to Ray's conjecture?

A

B

C

D

E

ANSWER:

B

63. **MULTI-STEP** Study the pattern to make conjectures about number relationships.

a. Complete the table to how the value of x^2 and $(x-1)(x+1)$.

x	x^2	$(x-1)(x+1)$
1		
2		
3		
4		
5		

b. What pattern do you observe?
c. Predict the product of 79×81.
d. Do you think that this rule will work for all real numbers? If not, provide a counter example.

ANSWER:

a.

x	x^2	$(x-1)(x+1)$
1	1	0
2	4	3
3	9	8
4	16	15
5	25	24

b. The square of a whole number is 1 greater than the product of the whole number before it and the whole number after it.
c. $80 \times 80 = 6400$. $6400 - 1 = 6399$
d. Sample answer: I think the rule will work for all real numbers.