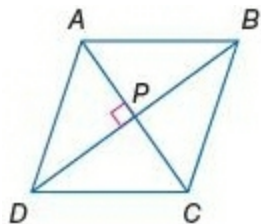


## 6-5 Special Parallelograms: Rhombi, Squares

**ALGEBRA** Quadrilateral  $ABCD$  is a rhombus. Find each value or measure.



12. If  $m\angle DPC = 3x - 15$ , find  $x$ .

**SOLUTION:**

The diagonals of a rhombus are perpendicular to each other.

$$m\angle DPC = 90$$

$$3x - 15 = 90$$

$$3x = 105$$

$$x = 35$$

**ANSWER:**

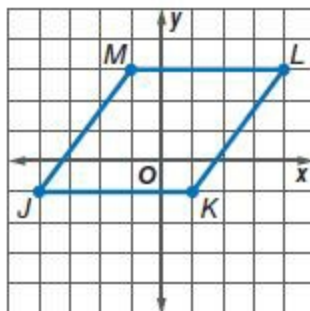
35

**COORDINATE GEOMETRY** Given each set of vertices, determine whether  $\square JKLM$  is a rhombus, a rectangle, or a square. List all that apply. Explain.

19.  $J(-4, -1)$ ,  $K(1, -1)$ ,  $L(4, 3)$ ,  $M(-1, 3)$

**SOLUTION:**

First, graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$$JL = \sqrt{(4 - (-4))^2 + (3 - (-1))^2} = \sqrt{8^2 + 4^2} = \sqrt{64 + 16} = \sqrt{80}$$

$$KM = \sqrt{(-1 - 1)^2 + (3 - (-1))^2} = \sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20}$$

The diagonals are not congruent. So, the parallelogram is not a rectangle. Check whether the two diagonals are perpendicular.

$$m_{JL} = \frac{3 - (-1)}{4 - (-4)} = \frac{1}{2}$$

$$m_{KM} = \frac{3 - (-1)}{-1 - 1} = -2$$

The diagonals are perpendicular. So, it is a rhombus.

**ANSWER:**

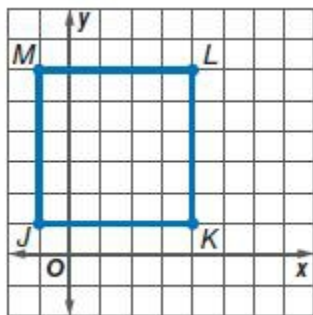
Rhombus; the diagonals are  $\perp$ .

## 6-5 Special Parallelograms: Rhombi, Squares

22.  $J(-1, 1)$ ,  $K(4, 1)$ ,  $L(4, 6)$ ,  $M(-1, 6)$

**SOLUTION:**

First, graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$$JL = \sqrt{(4 - (-1))^2 + (6 - 1)^2} = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50}$$

$$KM = \sqrt{(-1 - 4)^2 + (6 - 1)^2} = \sqrt{(-5)^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50}$$

The diagonals are congruent. So, the parallelogram is a rectangle. Check whether the two diagonals are perpendicular.

$$m_{JL} = \frac{6 - 1}{4 - (-1)} = 1$$

$$m_{KM} = \frac{6 - 1}{-1 - 4} = -1$$

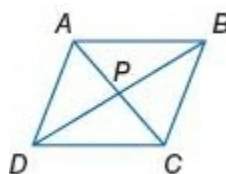
The diagonals are perpendicular. So, it is a rhombus.

Since the diagonals are both congruent and perpendicular to each other, the parallelogram is a rectangle, a rhombus, and a square.

**ANSWER:**

Square, rectangle, rhombus; all sides are  $\cong$  and  $\perp$ .

$ABCD$  is a rhombus. If  $PB = 12$ ,  $AB = 15$ , and  $m\angle ABD = 24$ , find each measure.



24.  $CP$

**SOLUTION:**

All four sides of a rhombus are congruent and the diagonals are perpendicular to each other.

So, by the Pythagorean Theorem,  $CP^2 = BC^2 - PB^2$ .

$BC = AB$ . Substitute  $AB$  for  $BC$ .

$$CP^2 = 15^2 - 12^2 = 81.$$

$$CP = \sqrt{81} = 9$$

**ANSWER:**

9

25.  $m\angle BDA$

**SOLUTION:**

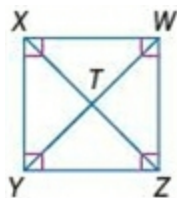
All four sides of a rhombus are congruent. So,  $\triangle ABD$  is an isosceles triangle. Then,  
 $m\angle BDA = m\angle ABD = 24$ .

**ANSWER:**

24

## 6-5 Special Parallelograms: Rhombi, Squares

$WXYZ$  is a square. If  $WT = 3$ , find each measure.



28.  $XY$

**SOLUTION:**

The diagonals of a square are congruent and bisect each other at right angles.

So,  $YT = XT = WT = 3$ .

By the Pythagorean Theorem,  $XY^2 = YT^2 + XT^2$ .

$$XY^2 = 3^2 + 3^2 = 18$$

$$XY = \sqrt{18}$$

$$XY = 3\sqrt{2}$$

**ANSWER:**

$$3\sqrt{2}$$

30.  $m\angle WYX$

**SOLUTION:**

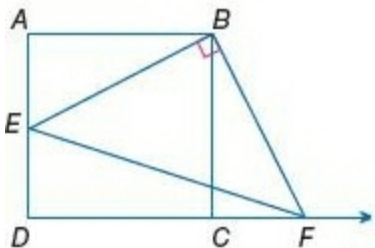
In a square, each diagonal bisects a pair of opposite angles.

So,  $m\angle WYX = \frac{1}{2}(m\angle XYZ) = 45$ .

**ANSWER:**

45

48. **CHALLENGE** The area of square  $ABCD$  is 36 square units and the area of  $\triangle EBF$  is 20 square units. If  $\overline{EB} \perp \overline{BF}$  and  $\overline{AE} = 2$ , find the length of  $\overline{CF}$ .



**SOLUTION:**

Since the area of the square is 36 square units, the

length of each side of the square is 6 units. And, all four angles of a square are right angles. So, by the Pythagorean Theorem,

$$EB^2 = AE^2 + AB^2 = 2^2 + 6^2 = 40$$

$$EB = \sqrt{40} = 2\sqrt{10}$$

The area of  $\triangle EBF$  is 20 square units. So,

$$\frac{1}{2}(EB)(BF) = 20.$$

$$\frac{1}{2}(2\sqrt{10})(BF) = 20$$

$$\sqrt{10}BF = 20$$

$$BF = \frac{20}{\sqrt{10}}$$

$$BF = \frac{20}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$BF = \frac{20\sqrt{10}}{10}$$

$$BF = 2\sqrt{10}$$

$$\overline{EB} \cong \overline{BF}.$$

Also, we have

$$\overline{BA} \cong \overline{BC} \text{ and } m\angle BAE = m\angle BCF = 90.$$

So, by the HL postulate,  $\triangle BAE \cong \triangle BCF$ .

$$AE = CF \text{ by CPCTC}$$

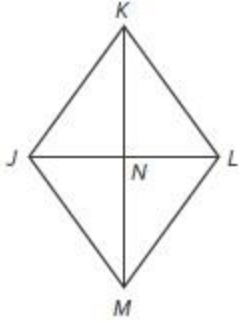
Therefore,  $CF = 2$ .

**ANSWER:**

2

## 6-5 Special Parallelograms: Rhombi, Squares

51. Julia is designing a pair of earrings. The figure shows one of the earrings. Julia knows that quadrilateral  $JKLM$  is a parallelogram and that  $m\angle KLN = 54$ .



What should the measure of  $\angle LKN$  be in order for the earring to be a rhombus?

- A 36
- B 54
- C 90
- D 108

**SOLUTION:**

The sum of the measures of the angles of a triangle is  $180^\circ$ , so write an equation and solve.

Let  $x$  represent  $m\angle LKN$ .

$$\begin{aligned} 54 + x + 90 &= 180 \\ x &= 36 \end{aligned}$$

So, the correct answer is choice A.

**ANSWER:**

A