Lesson 8.3 Part 2: Focus on the c

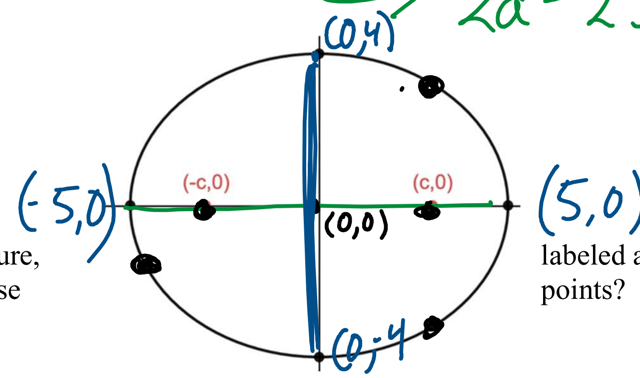
Ellipses don't just have one focus like parabolas do, they have two *foci*.
The foci of an ellipse are always located on the major axis. How do we find these foci? What properties do they have? Let's explore.

1. Consider the ellipse given by $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Is the major axis vertical or horizontal? How long is it? How do you know?

5^2 $\boxed{a=5}$ 4^2 $b=4$

$2a = 2 \cdot 5 = 10$

2. Label the center and the four vertices of the ellipse.



3. The foci of an ellipse are on the major axis inside the figure, $(c, 0)$ and $(-c, 0)$ as shown. What's so special about these labeled as points?

   a. Plot three extra points on your ellipse.

   b. Connect the first point to $(c, 0)$ and $(-c, 0)$. Measure the total distance of the two segments using one piece of string.

   c. Repeat this process for the other two points with a new piece of string.

   d. Cut one last piece of string that is the length of the major axis.

   e. What do you notice about the lengths of your four pieces of string?

Watch this video: https://www.youtube.com/watch?v=Et3OdzEGX_w

As a class, we will derive a formula to find the value of c in the focal points, or foci, $(c, 0)$ and $(-c, 0)$:

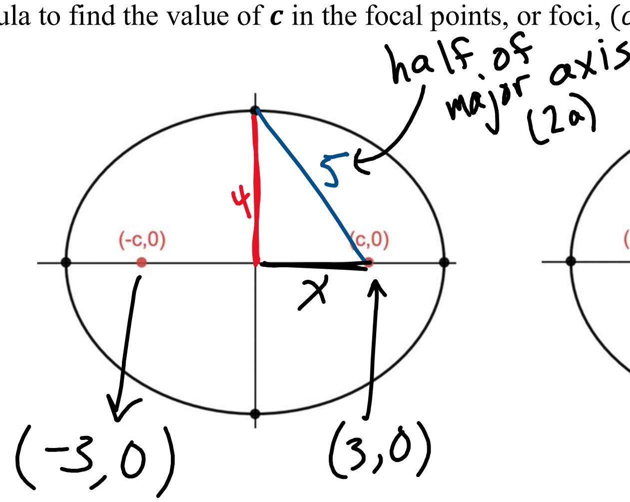
$x^2 + 4^2 = 5^2$

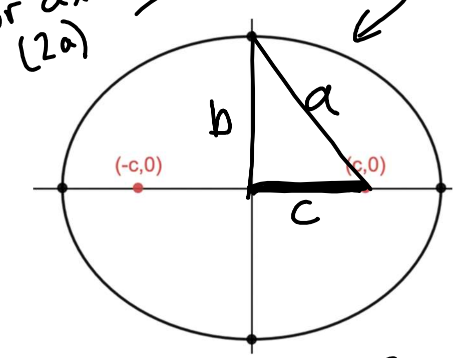
$x^2 + 16 = 25$

$\sqrt{x^2} = \sqrt{9}$

$x = 3$

focal length, c, is 3





$c^2 + b^2 = a^2$
$-b^2 \quad -b^2$

$\boxed{c^2 = a^2 - b^2}$

large small

Lesson 8.3 Part 2—Ellipses

An ellipse is the set of points for which the sum of the distances from two fixed points, the foci, is constant

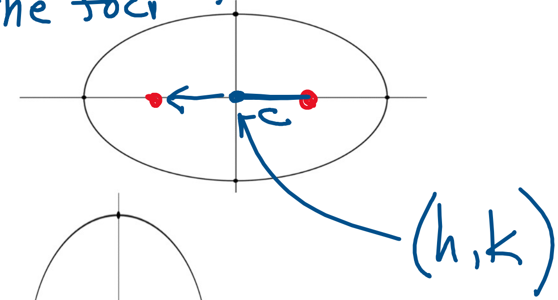
c is the distance from the center of an ellipse to each focal point.

$\boxed{c^2 = a^2 - b^2}$ ⟸ Use to find the foci

count left & right c units

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

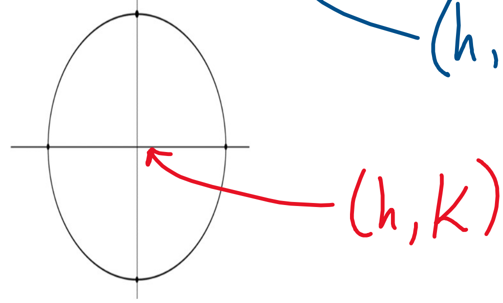
foci: ($h+c$, k) and ($h-c$, k)



count up & down c units

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

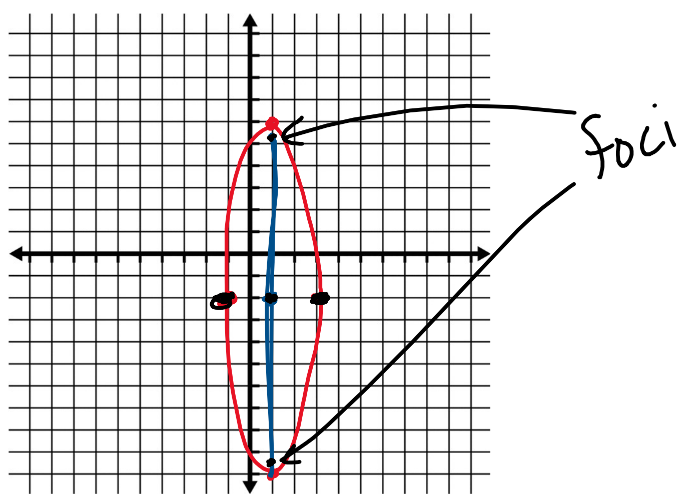
foci: (h , $k+c$) and (h , $k-c$)



Examples:

1. Graph $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{64} = 1$ and identify the following: 8^2 2^2
   a. Center: (1, -2)
   b. Vertices: (1, 6) & (1, -10)
   c. Co-Vertices: (3, -2) & (-1, -2)
   d. Foci: (1, 5.7) & (1, -9.7)

Find c
$c^2 = a^2 - b^2$
$c^2 = 64 - 4$
$\sqrt{c^2} = \sqrt{60}$
$c \approx 7.7$

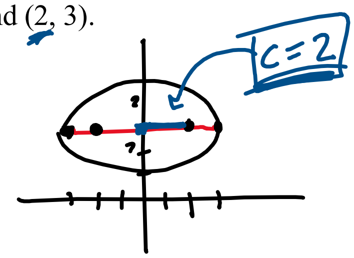


2. Write the equation of an ellipse with a major axis length of 6 and foci (-2, 3) and (2, 3).

Used $\boxed{c^2 = a^2 - b^2}$
$2^2 = 3^2 - b^2$
$4 = 9 - b^2$
$b^2 = 9 - 4 \Rightarrow b^2 = 5$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{9} + \frac{(y-3)^2}{5} = 1$$

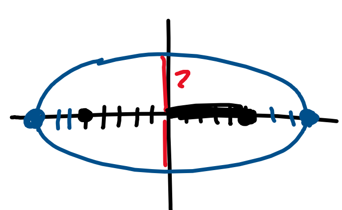


3. Write the equation of an ellipse with foci at (-5,0) and (5,0) and vertices at (-8,0) and (8,0).

$$\boxed{\frac{x^2}{64} + \frac{y^2}{39} = 1}$$

8^2

$c^2 = a^2 - b^2$
$5^2 = 8^2 - b^2$
$25 = 64 - b^2$
$b^2 = 64 - 25$
$b^2 = 39$



center: (0,0)

4. Find the foci of $4y^2 + x^2 - 6x - 8y - 3 = 0$.

→ Complete the square

→ Use $c^2 = a^2 - b^2$

$x^2 - 6x + \underline{9} + 4y^2 - 8y + \underline{4} = 3 + \underline{9} + \underline{4}$

$(x-3)^2 + 4(y^2 - 2y + \underline{1}) = 16$

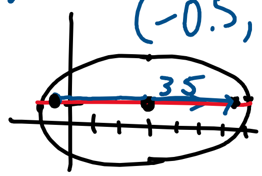
$\dfrac{(x-3)^2}{16} + \dfrac{4(y-1)^2}{16} = \dfrac{16}{16}$

$$\boxed{\frac{(x-3)^2}{16} + \frac{(y-1)^2}{4} = 1}$$

center: $(3,1)$

foci: $(6.5, 1)$

$(-0.5, 1)$



$c^2 = a^2 - b^2$

$(c^2 = \text{lg} - \text{sm})$

$c^2 = 16 - 4$

$c^2 = 12$

$\boxed{c \approx 3.5}$

5. Write the equation of an ellipse with Major axis vertical with length 20, minor length of axis 10, and center at $(2, -3)$.

6. Write the equation in standard form: $4x^2 + 25y^2 - 24x + 100y + 36 = 0$