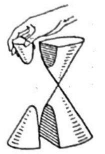
Lesson 8.4 An Exaggerated Conic!

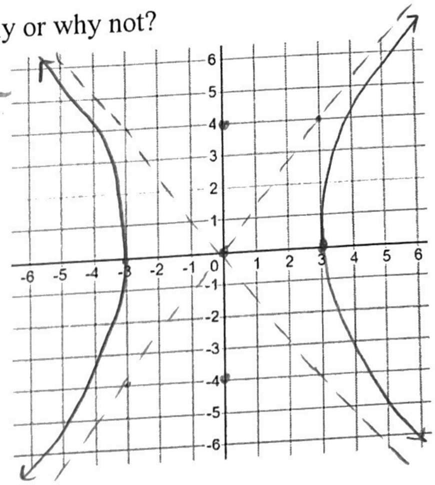


Today we are going to have the most fun you've ever had in any math class. There are a million things to learn. By the end of this lesson, you will be a professional at conic sections.

1. Today we'll look at the equation $\frac{x^2}{9} - \frac{y^2}{16} = 1$. Does this represent an ellipse? Why or why not?

NO, subtraction instead of addition

2. Graph the equation in Desmos and plot it on the coordinate plane. What do you notice? What do you wonder?



Do you see any links between the equation and the features of the graph?

3. This shape is called a hyperbola (not to be confused with hyperbole!). Let's see what happens when we make some adjustments to the equation. Use Desmos to graph each equation and then complete the table.

New Equation	What changed in the equation?	How did this affect the graph?
$\frac{x^2}{16} - \frac{y^2}{9} = 1$	denominators are switch	Distance from center to vertex is 4
$\frac{y^2}{9} - \frac{x^2}{16} = 1$	y^2 is first	Parabolas open up and down
$\frac{(x-2)^2}{9} - \frac{y^2}{16} = 1$	center changed $\leftrightarrow$	
$\frac{x^2}{9} - \frac{(y+2)^2}{16} = 1$	center changed $\leftrightarrow$	

4. The curves in a hyperbola approach two slant asymptotes. On the original equation, one of the asymptotes is $y = \frac{4}{3}x$. Plot this line on the graph in question #2. What do you think the equation of the other asymptote is?

$y = -\frac{4}{3}x$

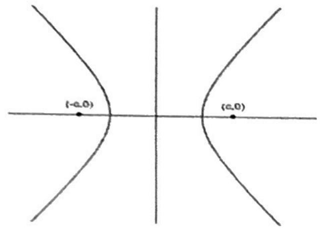
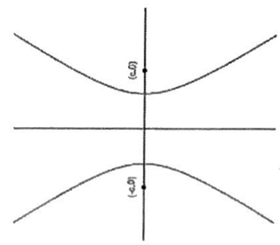
5. What do you think will be the asymptotes of $\frac{x^2}{16} - \frac{y^2}{9} = 1$? Test them in Desmos and see if you are correct.

$y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$

6. How does the slope of the asymptote equation relate to the denominators of the hyperbola equation?

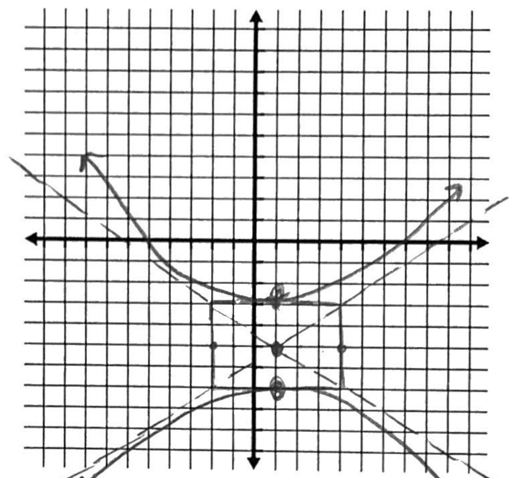
the square roots of the denominators are the rise and run.

Lesson 8.4 Hyperbolas

Horizontal Transverse Axis	Vertical Transverse Axis
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-h)^2}{a^2} - \frac{(x-k)^2}{b^2} = 1$
vertices: $(h+a, k)$ and $(h-a, k)$	vertices: $(h, k+a)$ and $(h, k+a)$
 slopes asymptotes: $m = \pm \frac{b}{a}$	 slopes asymptotes: $m = \pm \frac{a}{b}$

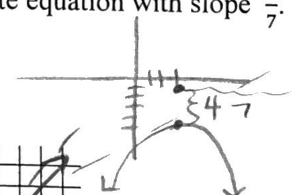
Examples:

1. Given $\frac{(y+5)^2}{4} - \frac{(x-1)^2}{9} = 1$, identify the following:
   a. Center: $(1, -5)$
   b. Asymptote Equations: $\pm 2/3$
   c. Vertices: $(1, -3)$ & $(1, -7)$
   d. Transverse Axis Length: 4
   e. Conjugate Axis Length: 6
   f. Graph

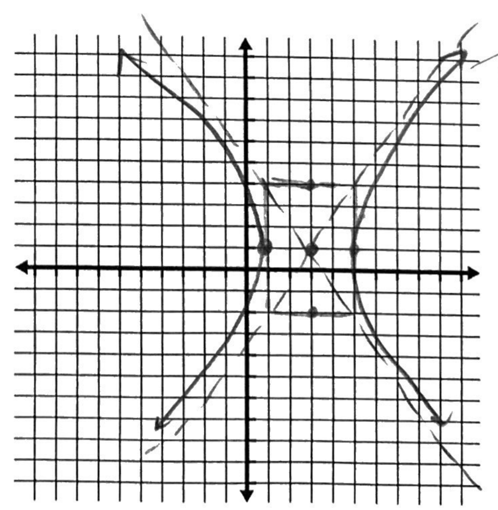


2. Write the equation of a hyperbola with a vertex at $(3, -5)$, center at $(3, -1)$ and an asymptote equation with slope $\frac{4}{7}$.

$$\frac{(y+1)^2}{16} - \frac{(x-3)^2}{49} = 1$$



3. Given $\frac{(x-3)^2}{4} - \frac{(y-1)^2}{9} = 1$, identify the following:
   a. Center: $(3, 1)$
   b. Asymptote Equations: $\pm 3/2$
   c. Vertices: $(1, 1)$ & $(5, 1)$
   d. Transverse Axis Length: 4
   e. Conjugate Axis Length: 6
   f. Graph



4. A hyperbola has asymptotes with slopes of $2/5$ and $-2/5$ and a center of $(-4, 3)$. Write two possible equations for this hyperbola.

$$\frac{(x+4)^2}{25} - \frac{(y-3)^2}{4} = 1 \quad \text{OR} \quad \frac{(y-3)^2}{4} - \frac{(x+4)^2}{25} = 1$$