SAMPLE ACT MATHEMATICS TEST QUESTIONS

(http://www.actstudent.org/sampletest/math/math_01.html)

An actual ACT Mathematics Test contains 60 questions to be answered in 60 minutes.

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

- 1. Illustrative figures are NOT necessarily drawn to scale.
- 2. Geometric figures lie in a plane.
- 3. The word *line* indicates a straight line.
- 4. The word *average* indicates arithmetic mean.

Set 1

1. A car averages 27 miles per gallon. If gas costs \$4.04 per gallon, which of the following is closest to how much the gas would cost for this car to travel 2,727 typical miles?

- A. \$44.44
- **B.** \$109.08
- <u>C.</u> \$118.80
- **D.** \$408.04
- **E.** \$444.40

\$4.04 per gallon

x 101 gallons

404

404

\$408.04 total

2. When x = 3 and y = 5, by how much does the value of $3x^2 - 2y$ exceed the value of $2x^2 - 3y$?

- **F.** 4
- **G.** 14
- <u>H.</u> 16
- <u>J.</u> 20
- **K.** 50

 $3x^2 - 2y$

2,727 miles

÷ 27 mpg

101 gallons

- $\frac{-2x^2+3y}{2}$
- $3^2 + 5$
- 9 + 5 = 14

3. What is the value of x when 2x + 3 = 3x - 4?

$$-\frac{1}{5}$$

$$\frac{-2x}{}$$

D.
$$\frac{1}{5}$$

$$3 = x - 4$$

$$+4 + 4$$

$$7 = x$$

2x + 3 = 3x - 4

4. What is the greatest common factor of 42, 126, and 210?

K. 42

$$42 = 2 \cdot 3 \cdot 7$$

$$126 = 2 \cdot 3 \cdot 7 \cdot 3$$

$$210 = 2 \cdot 3 \cdot 7 \quad \cdot 5$$

$$42 = 2 \cdot 3 \cdot 7$$

To get the GCF of a set of numbers, line up the prime factorization of each and multiply any prime factors that exist in every line.

5. Sales for a business were 3 million dollars more the second year than the first, and sales for the third year were double the sales for the second year. If sales for the third year were 38 million dollars, what were sales, in millions of dollars, for the first year?

$$x = year 1$$

$$y = year 2$$

$$z = year 3$$

$$y = x + 3$$

$$z = 2y$$

$$z = 38$$
 (given)

$$y = 38 \div 2 = 19$$

$$x = 19 - 3 = 16$$

6. In the figure below, ray \overrightarrow{EF} was constructed starting from rays \overrightarrow{ED} and \overrightarrow{EG} . By using a compass, D and G were marked equidistant from E on rays \overrightarrow{ED} and \overrightarrow{EG} . The compass was then used to locate a point F, distinct from E, so that F is equidistant from D and G. For all constructions defined by the above steps, the measures of $\angle DEF$ and $\angle GEF$:



G. are NOT equal.

H. sum to 30°.

J. sum to 45°.

K. sum to 60°.

Green segments are congruent.

Orange segments are congruent.

Note that: $\Delta DEF \cong \Delta GEF$ by SSS.

Therefore, $\angle DEF \cong \angle GEF$, meaning that the measures of the two angles are equal.

7. Abandoned mines frequently fill with water. Before an abandoned mine can be reopened, the water must be pumped out. The size of pump required depends on the depth of the mine. If pumping out a mine that is D feet deep requires a pump that pumps a minimum of $\frac{D^2}{25} + 4D - 250$ gallons per minute, pumping out a mine that is 150 feet deep would require a pump that pumps a minimum of how many gallons per minute?

$$x = \frac{150 \cdot 150}{25} + 4 \cdot 150 - 250$$

$$x = 6 \cdot 150 + 4 \cdot 150 - 250$$

$$x = 1.500 - 250 = 1.250$$

8. The length, in inches, of a box is 3 inches less than twice its width, in inches. Which of the following gives the length, I inches, in terms of the width, w inches, of the box?

F.
$$l = \frac{1}{2}w + 3$$

G.
$$I = w + 3$$

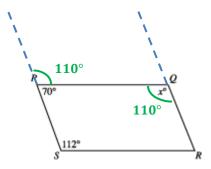
H.
$$I = w - 3$$

J.
$$l = 2w + 3$$

hence.
$$l = 2w - 3$$

$$(K.)$$
 $I = 2w - 3$

- **9.** In quadrilateral *PQRS* below, sides *PS* and *QR* are parallel for what value of *x* ?
 - A. 158
 - **B.** 132
 - <u>C.</u> 120
 - <u>D.</u> 110
 - <u>E.</u> 70



- $m \angle TPQ = 180^{\circ} 70^{\circ} = 110^{\circ}$ Then, $\angle TPQ$ and $\angle Q$ are alternating interior angles. $So, m \angle Q = 110^{\circ}$
- **10.** How many irrational numbers are there between 1 and 6?
 - <u>F.</u> 1
 - **G.** 3
 - <u>H.</u> 4
 - J. 10

- There are infinitely many rational numbers and infinitely many irrational numbers between any
- two Real numbers.
- K. Infinitely many
- **11.** A typical high school student consumes 67.5 pounds of sugar per year. As part of a new nutrition plan, each member of a track team plans to lower the sugar he or she consumes by at least 20% for the coming year. Assuming each track member had consumed sugar at the level of a typical high school student and will adhere to this plan for the coming year, what is the maximum number of pounds of sugar to be consumed by each track team member in the coming year?
 - <u>A.</u> 14
 - **B.** 44

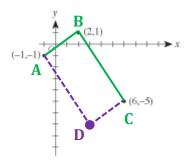
$$x = 67.5 \cdot 80\%$$

<u>C.</u> 48

$$x = 67.5 \cdot \frac{4}{5} = \frac{67.5 \cdot 4}{5} = \frac{270}{5} = 54$$

E. 66

12. In the standard (x, y) coordinate plane below, 3 of the vertices of a rectangle are shown. Which of the following is the 4th vertex of the rectangle?



- **F.** (3,-7)
- **G.** (4,–8)
- <u>H.</u> (5,−1)
- <u>J.</u> (8,–3)
- <u>K.</u> (9,–3)

Step 1: Draw two sides of the rectangle.

Step 2: The missing point appears to be below and to the left of point C. So, determine the vector that adds to B to get A.

$$B = (2, 1)$$

subtract:
$$A = (-1, -1)$$

result:
$$\mathbf{v} = < 3$$
, 2 >

Step 3: Subtract the vector, **v**, from C to get the missing point, D.

$$C = (6, -5)$$

subtract:
$$\mathbf{v} = \langle 3, 2 \rangle$$

result:
$$D = (3, -7)$$

Optional Step 4: Plot D to see if the result looks like a rectangle.

Set 2

1. The lead of a screw is the distance that the screw advances in a straight line when the screw is turned 1 complete turn. If a screw is $2\frac{1}{2}$ inches long and has a lead of $\frac{1}{8}$ inch, how many complete turns would get it all the way into a piece of wood?

- <u>A.</u> 5
- **B.** 10
- <u>C.</u> 15
- **D.** 20
- <u>E.</u> 25

2. If xy = 144, x + y = 30, and x > y, what is the value of x - y?

- <u>F.</u> 4
- Note that: x + y = 30
- **G.** 6

Fastest method is to try values that add to 30:

 $x = \frac{2\frac{1}{2}}{\frac{1}{8}} = \frac{\frac{5}{2}}{\frac{1}{8}} = \frac{5}{2} \cdot \frac{8}{1} = \frac{40}{2} = 20$

(H.)18

 $2 \cdot 28 = 56$

<u>J.</u> 22

 $4 \cdot 26 = 104$

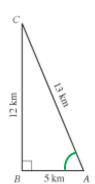
<u>K.</u> 24

 $6 \cdot 24 = 144$

Then, don't forget to subtract: 24 - 6 = 18

3. Which of the following is the sine of $\angle A$ in the right triangle below?

- A. $\frac{5}{13}$
- B 12
- $\frac{12}{13}$
- D = \frac{12}{5}
- $\frac{13}{5}$



Remember: SOH – CAH – TOA

Then,
$$\sin A = \frac{opposite}{hypotenuse} = \frac{12}{13}$$

4. Ding's Diner advertised this daily lunch special: "Choose 1 item from each column—only \$4.95!" Thus, each daily lunch special consists of a salad, a soup, a sandwich, and a drink.

How many different daily lunch specials are possible?

<u>F.</u>

G. 14

<u>H.</u> 30

J.) 120

K. 180

Salads	Soups	Sandwiches	Drinks
3	2	5	4
cole slaw	onion	meat loaf	milk
lettuce	tomato	chicken	cola
potato		hamburger	coffee
		ham	tea
		tenderloin	

$$3 \cdot 2 \cdot 5 \cdot 4 = 120$$

5. The volume, V, of the right circular cone with radius r and height h, shown below, can be found using the formula $V=\frac{1}{3}\pi r^2 h$. A cone-shaped paper cup has a volume of 142 cubic centimeters and a height of 8.5 centimeters. What is the radius, to the nearest centimeter, of the paper cup?

<u>A.</u> 2

Trick: **Approximate**

step

D. 12

E. 16

values at each

$$V = \frac{1}{3} \pi r^2 h$$

$$h \sim 9$$

$$\frac{1}{2} \pi \sim 1$$

Step 2: Substitute values:

$$142 = r^2 \cdot 9$$

$$\frac{142}{9} = r^2$$

$$r^2 \sim 16$$

6. A boat departs Port Isabelle, Texas, traveling to an oil rig. The oil rig is located 9 miles east and 12 miles north of the boat's departure point. About how many miles is the oil rig from the departure point?

3 F.

G. √63

15

21

K. 225



Recall: Pythagorean Triples (or use Pythagorean Theorem)

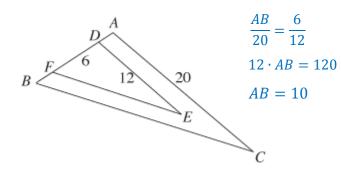
$$3 - 4 - 5$$

$$6 - 8 - 10$$

$$9 - 12 - 15$$

So,
$$x = 15$$

- 7. In the figure below, ∠ABC ≅∠DFE, ∠BAC ≅∠FDE, D and F are on AB, AD ≅FB, and distances in centimeters are as shown. What is the length of AD, in centimeters?
- <u>A.</u> 5
- **B.** 4
- <u>C.</u> 3
- D.) 2
- <u>E.</u> 1



- Then, let: x = AD = FB
 - AB = AD + DF + FB
 - 10 = x + 6 + x
 - 4 = 2x
 - 2 = x
- 8. Which of the following is a factor of the polynomial: $2x^2 3x 5$?
- <u>F.</u> x − 1
- <u>G.</u> 2x 3
- $2x^2 3x 5$
- H. 2x 5
- $=2x^2-5x+2x-5$
- = x(2x 5) + (2x 5)J. 2x + 5
- <u>K.</u> 3x + 5= (x + 1) (2x - 5)

- $-5\cdot 2 = -10$
- -5 + 2 = -3
- 9. What is x, the second term in the geometric series $\frac{1}{4} + x + \frac{1}{36} + \frac{1}{108} + \dots$? (Note: In a geometric series the ratio of any term to the following term is constant.)

The numerators are all 1, so let's look at the ratios of the denominators:

$$\frac{108}{36} = 3$$

So, the ratio of succeeding denominators is 3.

Then, the denominator of x

is:
$$4 \cdot 3 = 12$$

And, finally:

$$x = \frac{1}{12}$$

10. What is the slope of any line parallel to the line 9x + 4y = 7?

$$9x + 4y = 7$$
Parallel lines have the same slope.
$$-9x - 9x$$

$$-9x - 9x$$

$$4y = -9x + 7$$

H.
$$\frac{9}{7}$$
 In slope-intercept form, the coefficient of x is the slope.

coefficient of
$$x$$
 is to be considered as $x = x$.

11. A DVD player with a list price of \$100 is marked down 30%. If John gets an employee discount of 20% off the sale price, how much does John pay for the DVD player?

9x + 4y = 7

B. \$77.60
$$x = $100 \cdot 70\% \cdot 80\%$$

$$(C.)$56.00 x = $70 \cdot 80\%$$

D. \$50.00
$$x = $56$$

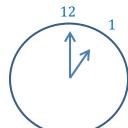
12.
$$\sqrt{-(-9)^2} = ?$$
 (Note: i = $\sqrt{-1}$)

G. 9+i
$$\sqrt{-81} = 9i$$

Set 3

1. What is the degree measure of the acute angle formed by the hands of a 12-hour clock that reads exactly 1 o'clock?

- A. 15°
- **B.** 30°
- <u>C.</u> 45°
- **D.** 60°
- **E.** 72°



The arc from 12 to 1 on a clock is $\frac{1}{12}$ of a circle.

Then,
$$\frac{360^{\circ}}{12} = 30^{\circ}$$

2. What is the probability that a number selected at random from the set {2, 3, 7, 12, 15, 22, 72, 108} will be divisible by both 2 and 3?

- F. $\frac{1}{4}$

Numbers divisible by 3 have digits that add to a number divisible by 3.

H. 5 Now, let's test each of the numbers.

So, 3 out of the 8 numbers are divisible by both 2 and 3.

$$P = \frac{3}{8}$$

3. A circle has a circumference of 16π feet. What is the radius of the circle, in feet?

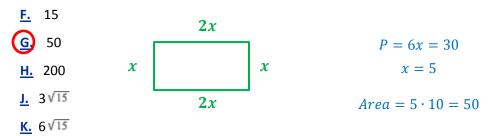
- <u>A.</u> √8
- <u>B.</u> 4
- <u>(C.)</u> 8
- <u>D.</u> 16
- <u>E.</u> 32

$$C=16\pi=2\pi r$$

$$\div 2\pi \div 2\pi$$

$$8 = r$$

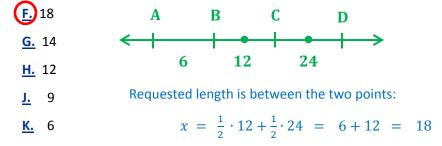
4. A rectangle with a perimeter of 30 centimeters is twice as long as it is wide. What is the area of the rectangle in square centimeters?



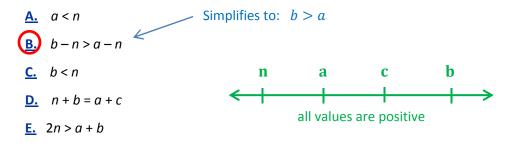
5. In the standard (x,y) coordinate plane, what are the coordinates of the midpoint of a line segment whose endpoints are (-3,0) and (7,4)?

A. (2,2)B. (2,4)C. (5,2)D. (5,4)E. (5,5) (7,4) + (-3,0) $(4,4) \div 2 = (2,2)$

6. Points *A*, *B*, *C*, and *D* are on a line such that *B* is between *A* and *C*, and *C* is between *B* and *D*. The distance from *A* to *B* is 6 units. The distance from *B* to *C* is twice the distance from *A* to *B*, and the distance from *C* to *D* is twice the distance from *B* to *C*. What is the distance, in units, from the midpoint of *BC* to the midpoint of *CD*?



7. Which of the following statements *must* be true whenever n, a, b, and c are positive integers such that n < a, c > a, and b > c?



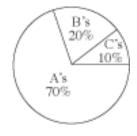
8. The distribution of Jamal's high school grades by percentage of course credits is given in the circle graph below. What is Jamal's grade point average if each A is worth 4 points; each B, 3 points; and each C, 2 points?



G. 3.4



J. 3.7



 $70\% \cdot 4 = 2.8$

 $20\% \cdot 3 = 0.6$

 $10\% \cdot 2 = 0.2$

3.6

- **K.** Cannot be determined from the given information
- **9.** What is the difference between 1.8 and $1.\overline{08}$? (Note: A bar indicates a digit pattern that is repeated.)

B. 0.71

<u>.</u> 0.71

<u>C.</u>0.7<u>19</u>

<u>D.</u> 0.72

E. 0.72

- *Note that*: 1.80 = 1.7 99 99 99 99 ... = 1.79
- Note that. 1.00 1.7 99 99 99 99 ... -
- 1.7 99 99 99 99 99 ... -1.0 80 80 80 80 80 ...
 - 0.7 19 19 19 19 19 ...
- **10.** Which of the following equations represents the linear relationship between time, t, and velocity, v, shown in the table below?

F.
$$v = 32t$$

G
$$v = 32t + 120$$

H.
$$v = 120t$$

J.
$$v = 120t + 32$$

K.
$$v = 120t + 120$$

$$y = mx + b$$

"m" is the uniform difference between terms.

"b" is the value of the function at: t = 0

11. An industrial cleaner is manufactured using only the 3 secret ingredients A, B, and C, which are mixed in the ratio of 2:3:5, respectively, by weight. How many pounds of secret ingredient B are in a 42-pound (net weight) bucket of this cleaner?

- **A.** 4.2
- **B.** 12.6
- <u>C.</u> 14.0
- <u>D.</u> 18.0
- E. 21.0

- *A*: 2*x B*: 3*x*
- C: 5x Total = 10x

- 10x = 42x = 4.2
- *B*: $3x = 3 \cdot 4.2 = 12.6$

- **12.** If n = 8 and $16 \cdot 2^m = 4^{n-8}$, then m = ?
 - **(F.)**-4
 - **G.** -2
- $16 \cdot 2^m = 4^{8-8}$

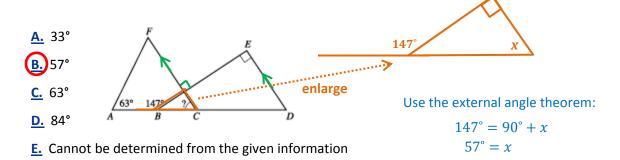
 $2^{m+4} = 1$

- **H.** 0 $2^4 \cdot 2^m = 4^0$
- <u>J.</u> 1
- <u>K.</u> 8

- Then, since: $2^0 = 1$:
 - m+4 = 0
 - m = -4

Set 4

1. In the figure below, A, B, C, and D are collinear, FC is parallel to ED, BE is perpendicular to ED, and the measures of $\angle FAB$ and $\angle EBA$ are as marked. What is the measure of $\angle FCB$?



2. Which of the following is an equation of the circle with its center at (0,0) that passes through (3,4) in the standard (x,y) coordinate plane?

F.
$$x-y=1$$

G.
$$x + y = 25$$

H.
$$x^2 + y = 25$$

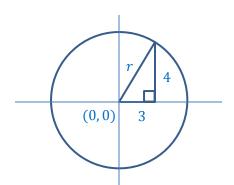
$$y^2 + y^2 = 5$$

$$(K.)$$
 $x^2 + y^2 = 25$

General equation of a circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

(h,k) is the center of the circle. r is the radius of the circle.



Recall: Pythagorean Triples (or use Pythagorean Theorem)

$$3 - 4 - 5$$

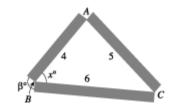
So,
$$r=5$$

Resulting equation:

$$(x-0)^2 + (y-0)^2 = 5^2$$
$$x^2 + y^2 = 25$$

Use the following information to answer questions 3–5.

Taher has decided to create a triangular flower bed border. He plans to use 3 pieces of rectangular lumber with lengths 4, 5, and 6 feet, as shown in the figure below. Points A, B, and C are located at the corners of the flower bed.



3. Taher plans to cut the 3 pieces of lumber for the flower bed border from a single piece of lumber. Each cut takes $\frac{1}{8}$ inch of wood off the length of the piece of lumber. Among the following lengths, in inches, of pieces of lumber, which is the shortest piece that he can use to cut the pieces for the flower bed border?

A. 178

A. 178

B. 179

C. 180

D. 181

E. 182

$$4 \text{ feet}$$
 5 feet
 $\frac{1}{8} \text{ inch}$

Combernia in the second of the lumber is:

 $4 \text{ feet} + \frac{1}{8} \text{ inch} + 5 \text{ feet} + \frac{1}{8} \text{ inch} + 6 \text{ feet} = 16 \text{ feet} + \frac{2}{8} \text{ inch}$
 $= 15 \cdot 12 + \frac{1}{4} \text{ inches} = 180 \frac{1}{4} \text{ inches}$

4. The measure of $\angle ABC$ in the figure is x° . Which of the following is an expression for \mathbf{b}° ?

$$G. 2x^{\circ}$$

b appears to be the measure of the external angle at B.

H.
$$(90 + x)^{\circ}$$

So. **b** +
$$x = 180$$

$$(180 - x)^{\circ}$$

Therefore,
$$\mathbf{b}^{\circ} = (180 - x)^{\circ}$$

K.
$$(180 - \frac{x}{2})^{\circ}$$

5. After arranging the flower bed, Taher decides that the flower bed would look more attractive if 1 of the angles in the triangle were a right angle. He decides to place the right angle at vertex A and to leave the lengths of AB and AC as 4 and 5 feet, respectively. To the nearest 0.1 foot, how long of a piece of lumber would he need to replace the 6-foot piece represented by BC?

Use the Pythagorean Theorem

$$4^2 + 5^2 = x^2$$

$$16 + 25 = x^2$$

$$41 = x^2$$

Then, since:

$$36 < x^2 < 49$$

We conclude that: 6 < x < 7

6. Which one of the following expressions has an even integer value for all integers a and c?

(<u>F.</u>)8a + 2ac

G. 3*a* + 3*c*

 \mathbf{H} . 2a + c

 $\frac{J}{a}$ a + 2c

K. $ac + a^2$

Only even coefficients will guarantee an even result.

7. A neighborhood recreation program serves a total of 280 children who are either 11 years old or 12 years old. The sum of the children's ages is 3,238 years. How many 11-year-old children does the recreation program serve?

A. 55

B. 122

C. 132

D. 158

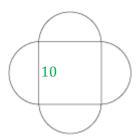
E. 208

Info Table	Number	Total Age		
11-year olds	х	11 <i>x</i>		
12-year olds	280 - x	$12 \cdot (280 - x)$		
Total	280	3,238		

Find *x* based on information in the "Total Age" column:

$$11x + 12 \cdot (280 - x) = 3,238$$
$$-x + 3,360 = 3,238$$
$$x = 122$$

8. The geometric figure shown below consists of a square and 4 semicircles. The diameters of the semicircles are the sides of the square, and each diameter is 10 centimeters long. Which of the following is the closest approximation of the total area, in square centimeters, of this geometric figure?



<u>F.</u> 100

G. 160

(H.) 260

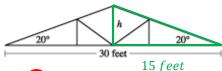
<u>J.</u> 400

K. 730

Area = Area of square +
$$4 \cdot$$
 (Area of semicircle)
= Area of square + $2 \cdot$ (Area of circle)
= $10^2 + 2 \cdot (\pi \cdot 5^2)$
= $100 + 50\pi$

$$\sim 100 + 157 = 257$$

9. Which of the following expressions is the closest approximation to the height h, in feet, of the roof truss shown below?



- A 15 tan 20°
 - **B.** 15 sin 20°
 - <u>C.</u> 30 tan 20°
 - **D.** 30 sin 20°
 - E. 15 sin 20°

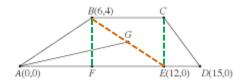
Consider only the green triangle. Remember: SOH – CAH – TOA

We have the sides opposite to and adjacent to the 20° angle. So, use the *tangent* function:

$$\tan 20^\circ = \frac{h}{15}$$

15
$$\tan 20^{\circ} = h$$

10. Quadrilateral ABCD is drawn on the standard (x,y) coordinate plane as shown below, with points E and F on AD. Point G is the center of rectangle BCEF. How many coordinate units long is AG?



- F. √10
- **G.** √13
- H. √85
- J. √97
- <u>K.</u> 11

Add the green and orange lines to the illustration as shown. Then G is the midpoint of \overline{BE} .

Plan: find the coordinates of G, then find the distance from A to G.

E:
$$+ (12,0)$$

G:
$$(18, 4) \div 2 = (9, 2)$$

Then, calculate: AG =

$$\sqrt{(9-0)^2 + (2-0)^2}$$
$$= \sqrt{81+4} = \sqrt{85}$$

11. What is the *x*-intercept of the graph of $y = x^2 - 4x + 4$?

- <u>A.</u> −2
- <u>B.</u> −1

 $0 = x^2 - 4x + 4$

<u>C.</u> 0

0 = (x-2)(x-2)

<u>D.</u> 1

x = 2

(E) :

12. For all nonzero real numbers p, t, x, and y such that $\frac{x}{y} = \frac{3p}{2t}$, which of the following expressions is equivalent to t?

$$\underline{6py}$$
 Cross multiply: $2tx = 3py$ $\underline{H.}$

$$\frac{3py}{x}$$
 Multiply both sides by $2x$:
$$\frac{2tx}{2x} = \frac{3py}{2x}$$

Combine and simplify:
$$\frac{3py}{2x} = t$$

Set 5

1. Ms. Hernandez began her math class by saying:

I'm thinking of 5 numbers such that their mean is equal to their median. If 4 of the numbers are 14, 8, 16, and 14, what is the 5th number?

What is the 5th number Ms. Hernandez is thinking of?

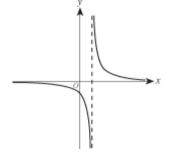
First, order the numbers given: <u>A.</u> 13

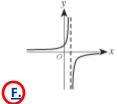
B. 14
$$8, 14, 14, 16.$$
 $\frac{8+14+14}{5}$

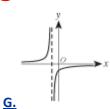
C. 15 Then notice that no matter what the
$$5^{th}$$
 number is, the median of the five numbers
$$x = 18$$

2. The graph of a certain hyperbola, y = h(x), is shown in the standard (x,y) coordinate plane below.

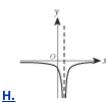
Among the following graphs, which best represents y = -h(x)?

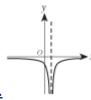


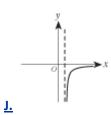


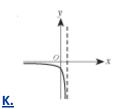


Negating a function is simply a reflection of that function over the x-axis. Answer F is a reflection over the x-axis of the function given.

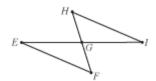








3. In the figure below, $\angle H \cong \angle F$; E, G, and I are collinear; and G is the midpoint of FH.



Plan: review the steps below and find some semblance of order.

To prove that $HI \cong FE$ given the conditions stated above, which of the following is a logical order for the 5 steps in the table below?

Statement		Reason		
1. <i>HG</i> ≅ <i>FG</i>	Must be the first step.	The midpoint of a line segment divides the segment into 2 congruent segments		
2. ∠EGF ≅ ∠IGH Must be after Step 4 (given).		Vertical angles are congruent		
3. △GHI ≅△GFE		Angle-side-angle congruence theorem		
4. ∠EGF and ∠IGH are vertical angles		Definition of vertical angles		
5. <i>HI ≅FE</i>	Must be the last step because this is the conclusion.	Corresponding parts of congruent triangles are congruent		

A. 1, 2, 3, 4, 5

B. 1, 2, 3, 5, 4

C. 1, 2, 4, 3, 5

D 1, 4, 2, 3, 5

E. 1, 5, 4, 2, 3

One of the keys to solving this problem is to note that you must establish that vertical angles exist (Step 4) before you can do anything with them (Step 2). Hence, Step 4 must precede Step 2.

The only answer given that has Step 4 before Step 2, and has Step 5 last is answer D. So, this must be the solution.

4. Each of the variables t, w, x, y, and z represents a different positive real number. Given the equations below, which of the 4 variables w, x, y, and z necessarily has the greatest value?

$$1.23w = t$$

$$1.01x = t$$

$$0.99y = t$$

0.23z = t

$$w = \frac{t}{1.23}$$

$$x = \frac{t}{1.01}$$

$$w = \frac{t}{1.23}$$
 $x = \frac{t}{1.01}$ $y = \frac{t}{0.99}$ $z = \frac{t}{0.23}$

$$z = \frac{t}{0.23}$$

<u>F.</u> *w*

The greatest value results from the division with the smallest denominator.

G. *x*





K. Cannot be determined from the given information

5. Which of the following is equivalent to $\frac{5}{k} + \frac{k+3}{k+5}$?

A.
$$\frac{k+8}{2k+5}$$
B. $\frac{k+8}{k(k+5)}$
C. $\frac{5(k+3)}{k(k+5)}$
D. $\frac{k^2+3k}{5k+25}$
E. $\frac{k^2+8k+25}{k(k+5)}$

$$\frac{k^2+8k+25}{k(k+5)}$$

$$\frac{k^2+8k+25}{k(k+5)}$$
E. $\frac{k^2+8k+25}{k(k+5)}$

6. In the 2 × 2 matrix below, b_1 and b_2 are the costs per pound of bok choy (Chinese greens) at Market 1 and Market 2, respectively; r_1 and r_2 are the costs per pound of rice flour at these 2 markets, respectively. In the following matrix product, what does q represent?

$$\begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \bullet \begin{bmatrix} b_1 & r_1 \\ b_2 & r_2 \end{bmatrix} = \begin{bmatrix} p & q \end{bmatrix}$$

- **F.** The cost of r_1 pounds of rice flour at \$0.50 per pound
- G. The cost of a half-pound of rice flour at Market 1
- H. The total cost of a half-pound of bok choy and a half-pound of rice flour at Market 1
- J. The total cost of a half-pound of bok choy and a half-pound of rice flour at Market 2
- K. The total cost of a half-pound of rice flour at Market 1 and a half-pound of rice flour at Market 2

$$q = 0.5 r_1 + 0.5 r_2$$

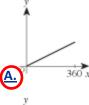
So the value of q is calculated based on the values of the cost of <u>rice flour</u> at <u>both</u> markets.

7. The 2 diagrams below show a circle of radius 1 inch with shaded sectors of angle x° , for 2 different values of x.





One of the following is the graph in the standard (x,y) coordinate plane of the area, y, of a shaded sector with angle x° , for all values of x between 0 and 360. Which is that graph?











The formula for the area of a sector of a circle, based on the illustrations above, is:

$$A = \frac{x}{360} \cdot \pi \, r^2$$

As *x* changes, Area changes proportionally, in a linear fashion. It's as if the equation were:

$$A = kx$$

Where k is a constant equal to: $\frac{\pi r^2}{360}$

The only curve that is linear is A.

8. If $h(x) = x^3 + x$ and g(x) = 2x + 3, then g(h(2)) = ?

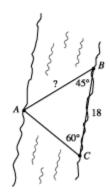
$$h(2) = 2^3 + 2 = 10$$



9. In the figure below, points A and B are on opposite banks of a small stream. Point C is on the same bank of the stream as point B and approximately 18 meters from B. The measure of $\angle CBA$ is 45°, and the measure of $\angle BCA$ is 60°.

Which of the following expressions gives the approximate distance, in meters, between point A and point B?

(Note: For $\triangle PQR$, where p, q, and r are the lengths of the sides opposite $\angle P$, $\angle Q$, and $\angle R$, respectively, $\frac{\sin \angle P}{p} = \frac{\sin \angle Q}{q} = \frac{\sin \angle R}{r}$.)



To solve this problem, we must first find the measure of angle A; then we can utilize the equalities shown.

B.
$$\frac{\sin 60^{\circ}}{18 \sin 75^{\circ}}$$

Note:
$$\frac{\sin \angle P}{p} = \frac{\sin \angle Q}{q} = \frac{\sin \angle R}{r}$$
 is called the Law of Sines.

C.
$$\frac{18 \sin 45^{\circ}}{\sin 60^{\circ}}$$

$$m \angle A = 180^{\circ} - 60^{\circ} - 45^{\circ} = 75^{\circ}$$

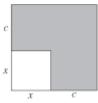
D.
$$\frac{18 \sin 60^{\circ}}{\sin 45^{\circ}}$$

$$\frac{\sin 60^{\circ}}{AB} = \frac{\sin 75^{\circ}}{18}$$

$$18 \cdot \sin 60^{\circ} = AB \cdot \sin 75^{\circ}$$

$$\frac{18 \cdot \sin 60^{\circ}}{\sin 75^{\circ}} = AB$$

10. Each side of the smaller square in the figure below is *x* inches long, and each side of the larger square is *c* inches longer than a side of the smaller square. The area of the larger square is how many square inches greater than the area of the smaller square?



To find the difference in area, calculate each area and subtract.

 $\mathbf{F.}$ c^2

Area of large square =
$$(x + c)^2$$

Area of small square =
$$x^2$$

<u>**G.**</u> *XC*

$$Difference = (x+c)^2 - x^2$$

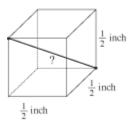
$$= x^2 + 2xc + c^2 - x^2$$

$$\underline{J}$$
. $(x+c)^2$

$$= 2xc + c^2$$

K
$$2xc + c^2$$

11. A cube with edges $\frac{1}{2}$ inch long is shown below. What is the length, in inches, of a diagonal that runs from one corner of the cube to the opposite corner?



Using the 3-dimensional distance formula, we get:

- A. $\frac{1}{4}$
- B. $\frac{3}{4}$
- <u>c.</u> $\frac{3}{2}$
- D. $\frac{\sqrt{2}}{2}$
- $\frac{\sqrt{3}}{2}$

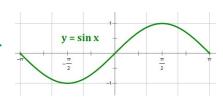
- $x = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$
- $x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

- **12.** Which of the following is equivalent to $\sin \theta \csc(-\theta)$ wherever $\sin \theta \csc(-\theta)$ is defined?
 - **(F)** -1
 - <u>G.</u> 1
 - H. –tan θ
 - \underline{J} . tan θ
 - K_{\cdot} -sin² θ

It helps to know what the sine function looks like.

It also helps to know that:

$$\csc\theta = \frac{1}{\sin\theta}$$



From the graph, we see that:

$$\sin(-\theta) = -\sin\theta$$

$$\sin\theta \csc(-\theta)$$

$$= \sin\theta \cdot \frac{1}{\sin(-\theta)}$$

$$= \sin\theta \cdot \frac{1}{-\sin(\theta)}$$

$$= \frac{\sin(\theta)}{-\sin(\theta)} = -1$$

ACT Online Sample Test Answer Key

	Set 1	Set 2	Set 3	Set 4	Set 5
1	D	D	В	В	E
2	G	Н	G	K	F
3	E	С	С	D	D
4	K	J	G	J	J
5	A	В	A	D	E
6	F	Н	F	F	K
7	D	D	В	В	A
8	K	Н	Н	Н	K
9	D	C	C	A	E
10	K	G	G	Н	K
11	D	С	В	E	E
12	F	F	F	K	F