Unit 10 Calendar **Name:** ___________________________

Day	Date	Assignment (Due the next class meeting)
Tuesday Wednesday	3/29/22 (A) 3/30/22 (B)	10.1 Worksheet **Properties of Exponents & Base e**
Thursday Friday	3/31/22 (A) 4/1/22 (B)	10.2 Worksheet **Graphing Exponential Functions & Base e (Day 1)**
Monday Tuesday	4/4/22 (A) 4/5/22 (B)	10.3 Worksheet **Graphing Exponential Functions (Day 2)**
Wednesday Thursday	4/6/22 (A) 4/7/22 (B)	10.4 Worksheet **Changing the Base of an Exponential Function**
Friday Monday	4/8/22 (A) 4/11/22 (B)	10.5 Worksheet **Modeling with Exponential Functions (Growth and Decay)**
Tuesday Wednesday	4/12/22 (A) 4/13/22 (A)	10.6 Worksheet **Solving Exponential Equations**
Thursday Friday	4/14/22 (A) 4/15/22 (B)	**Unit 10 Practice Test**
Tuesday Wednesday	4/19/22 (A) 4/20/22 (B)	**Unit 10 Review**
Thursday Friday	4/21/22 (A) 4/22/22 (B)	**Unit 10 Test**

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Try www.khanacademy.org if you need help outside of school hours.
- Student who complete 100% of their homework second semester on-time will receive a pizza party and 2% bonus to their grade!
- Don't forget about the webpage: www.washoeschools.net/drhsmath

10.1 Notes: Properties of Exponents & Base e

Let *a* and *b* be real numbers and let *m* and *n* be integers

Product of Powers	$a^m \bullet a^n =$	
Power of a Power	$(a^m)^n =$	
Power of a Product	$(ab)^m =$	
Negative Exponent	$a^{-m} =$	
Zero Exponent	$a^0 =$	
Quotient of Powers	$\frac{a^m}{a^n} =$	
Power of a Quotient	$\left(\frac{a}{b}\right)^m =$	

<u>Examples</u>: Simplify.

1) $(x^3y^6)^3$ 2) $(x^3)^2 \cdot (xy^2)^4$ 3) $(x^2y^{-6})^7$ 4) $(2a^2b^8)^0$

Try one of the following :

a) $(x^2y^7)^6$ b) $(x^{-2}y)^3 \cdot y^4$ c) 15^0

<u>Examples</u>: Simplify.

5) $\frac{x^5y^2}{x^{15}y^8}$ 6) $\left(\frac{a^4}{b^2}\right)^2$ 7) $\left(\frac{r^{-2}}{s^3}\right)^{-3}$ 8) $\frac{c \cdot c^4}{c^2}$

Try one of the following:

a) $\frac{x^7y^{16}}{x^{15}y^{12}}$ b) $\left(\frac{q^7}{r^{-2}}\right)^4$

Examples: Simplify.

9) $\dfrac{16m^4n^{-5}}{2n^{-5}m^7}$

10) $\dfrac{x^2y^{-3}}{(2x^3y^{-2})^2}$

11) $\dfrac{4^2 \cdot 64^3}{4^4}$

Try one of the following!

a) $\dfrac{(a^2b^4)^2}{a^{-3}b}$

b) $\dfrac{24xy^6}{4x^{-2}y^4}$

c) $\dfrac{4^8 \cdot 2^2}{2^{20}}$

The Natural Base *e*:

Examples: Simplify the following expressions.

12) $3e^2 \cdot 6e^5$

13) $\dfrac{18e^4}{9e^3}$

14) $(-4e^{-5x})^3$

Try one of the following!

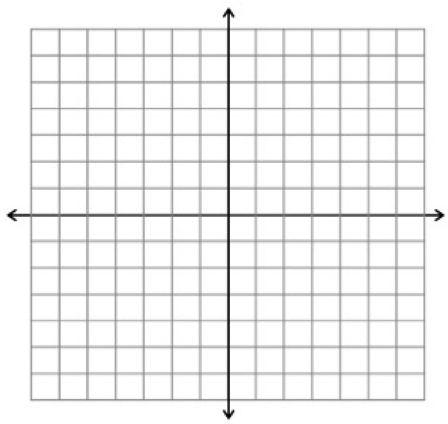
a) $-5e^3 \cdot 2e^6$

b) $\dfrac{24e^4}{6e^3}$

c) $(-3e^{-4x})^2$

10.2 Notes: Graphing Exponential Functions & Base e (Day 1)

Graphing Exponential Functions:



(in set notation)
Domain:

Range:

Linear Parent Function: $y = 2^x$

x	y	(x, y)
-2		
-1		
0		
1		
2		

y-intercept:

Horizontal Asymptote:

What happens when we change b (when $b > 1$)?

Graph each of the functions on the graphing calculator. Sketch your results on the graph provided.

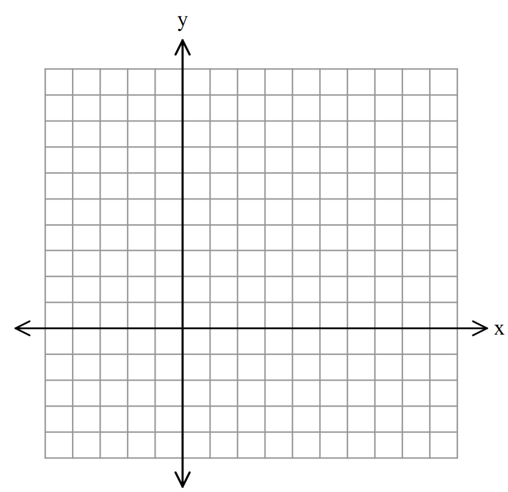
a. $y = 2^x$

b. $y = e^x$

b. $y = 3^x$

c. $y = 4^x$

d. $y = 10^x$

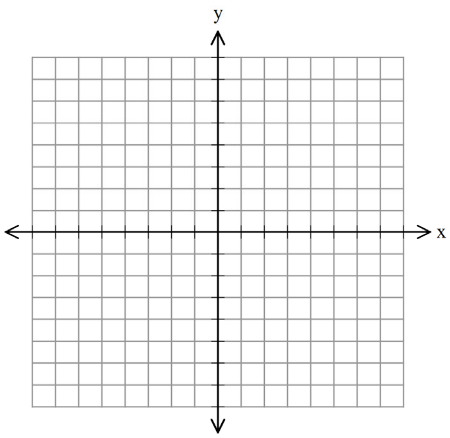


Graphing $f(x) = ab^{x-h} + k$, when $b > 1$ (Exponential Growth)

What happens when we change h & k?

Graph the following exponential equation. Explain how the graph is transformed from the parent function $f(x) = 2^x$. Also, state the domain and range for each function & describe the end behavior.

$f(x) = 2^{x+1} + 2$



Transformation:

Domain:

Range:

End Behavior:

How does the graph of the exponential function change as h & k changes?

How does the graph of the exponential function change as the base b changes?

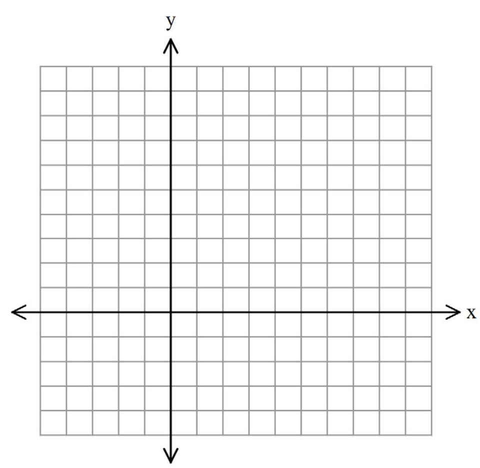
What happens when we change a?

Graph each function on the graphing calculator. Sketch your results on the graph provided.

a. $f(x) = 4^x$

b. $g(x) = 3(4)^x$

c. $h(x) = \frac{1}{2}(4)^x$



Compare the parent graph, $f(x)$, with $g(x)$ & $h(x)$. What is the domain, range, & end behavior for each graph? What do you notice about the y −intercepts?

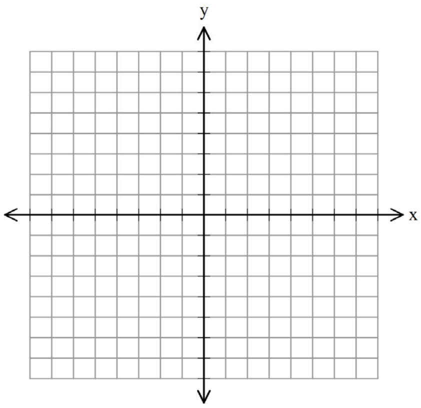
How does the graph of the exponential function change as a changes?

Steps to Graph Exponential Functions:

Examples

Graph each exponential function. Describe the domain & range.

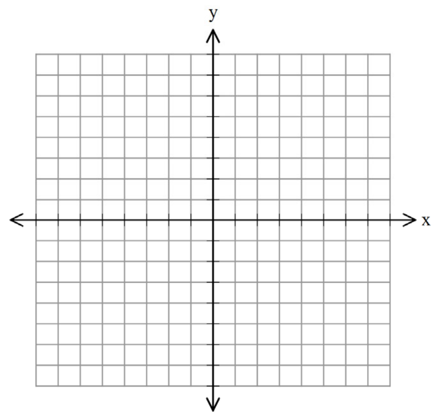
1. $y = 3^x$



Domain:

Range:

2. $y = 4^{x+2} - 3$

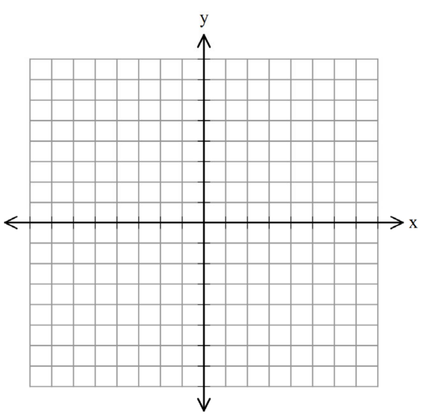


Domain:

Range:

Try one of the following:

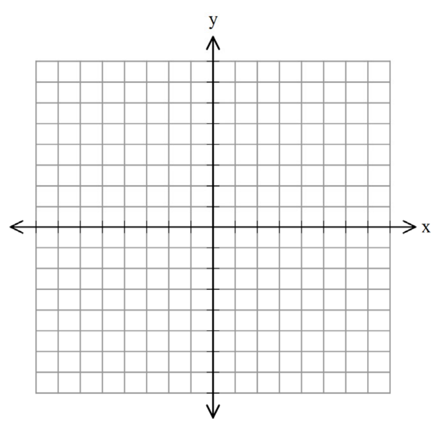
3. $y = 3^{x-1}$



Domain:

Range:

4. $y = 4^{x-2} + 5$



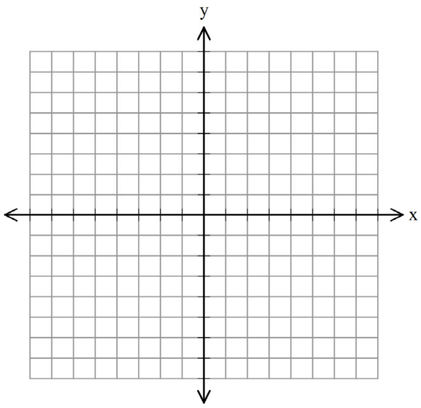
Domain:

Range:

Examples

Graph each exponential function. Describe the domain & range.

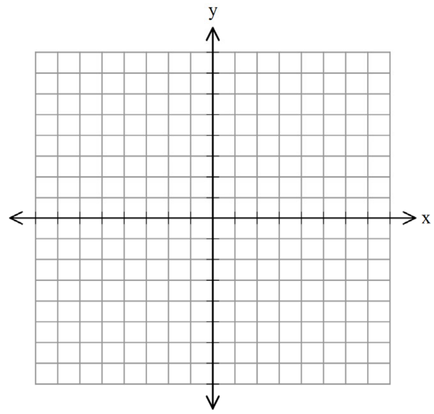
5. $y = 2 \cdot 3^{x}$



Domain:

Range:

6. $y = -1 \cdot 4^{x+2} - 3$

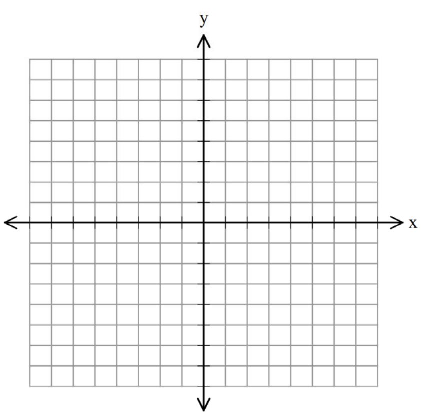


Domain:

Range:

Try one of the following:

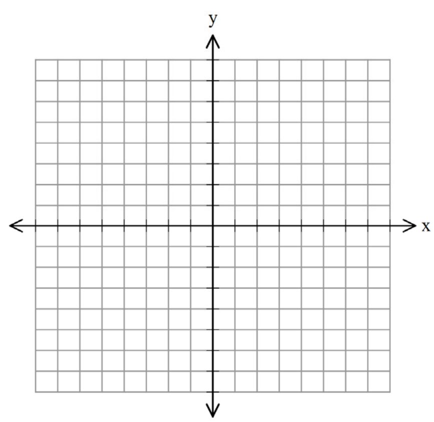
7. $y = 4 \cdot 3^{x-1}$



Domain:

Range:

8. $y = -3 \cdot 4^{x-2} + 5$



Domain:

Range:

9. When evaluating the function $f(x) = 2^{x-4}$ for any real number x, what must be true about the value of f(x) ?

A. The value of f(x) is always negative

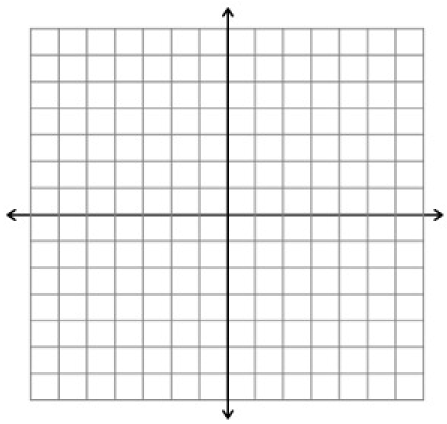
B. The value of f(x) is always positive

C. The value of f(x) is always greater than 4

D. The value of f(x) is always less than 4

10.3 Notes: Graphing Exponential Functions (Day 2)

Graphing $f(x) = ab^{x-h} + k$, when $0 < b < 1$ (Exponential Decay)



Graph the Function: $f(x) = \left(\frac{1}{2}\right)^{x}$

x	y	(x, y)
-2		
-1		
0		
1		
2		

y-intercept:

Horizontal Asymptote:

(in set notation)
Domain:

Range:

As your go right, are the values increasing or decreasing?

Is this exponential growth or decay? Why?

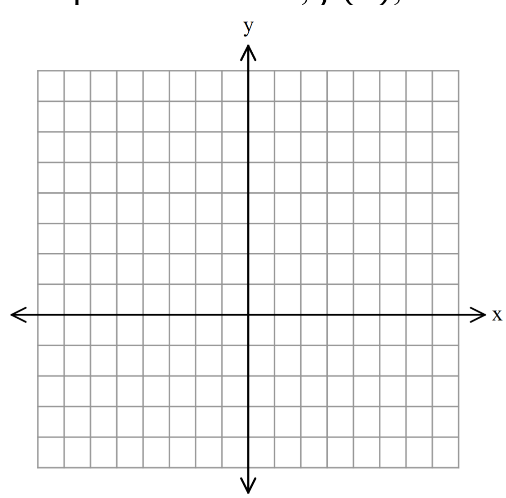
What happens when we change h & k (when $0 < b < 1$)?

Graph each of the following functions on the graphing calculator. Sketch your results on the graph provided. Describe the transformation from the parent function, $f(x)$, when you change h & k.

a. $f(x) = \left(\frac{1}{2}\right)^{x}$

b. $g(x) = \left(\frac{1}{2}\right)^{x-2}$

c. $h(x) = \left(\frac{1}{2}\right)^{x+1} - 3$



Vertical & Horizontal Reflections

Use the graphing calculator to graph each of the following functions.

a. $y = 2^{-x}$

b. $y = 3^{-x}$

c. $y = e^{-x}$

d. $y = \left(\frac{1}{2}\right)^x$

e. $y = \left(\frac{1}{3}\right)^x$

f. $y = e^x$

Which of these are exponential growth functions?

Which of these are exponential decay functions?

Examples:

1. The graph $f(x) = 2^x$ is translated two (2) units up, four (4) units right, & has a vertical reflection (reflected across the x-axis). Write the equation of the function after the transformation.

2. The graph $f(x) = e^x$ is translated down five (5) units. Write the equation of the function after the transformation.

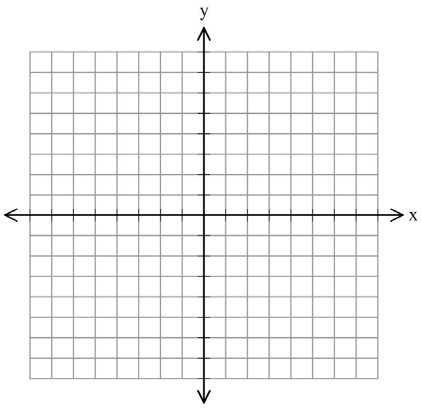
You try !

3. The graph of $f(x) = \left(\frac{1}{2}\right)^x$ is translated two (2) units to the right, three (3) units up, and has a vertical stretch by a factor of four (4). Write the equation of the function after the transformation.

Examples:

Graph each exponential function. Describe the domain & range.

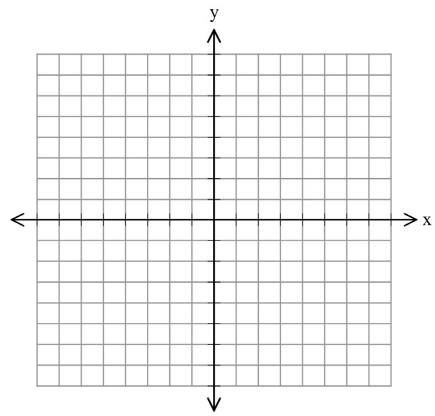
4. $y = \left(\frac{1}{2}\right)^{x+3}$



Domain:

Range:

5. $y = -\left(\frac{1}{3}\right)^{x-2} - 4$

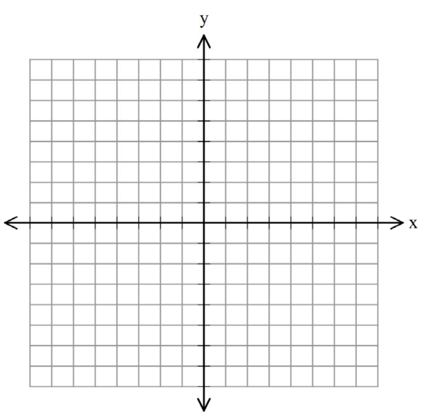


Domain:

Range:

Try on of the following!

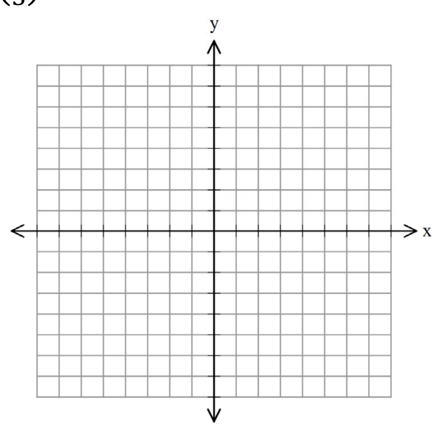
6. $y = -\left(\frac{1}{2}\right)^{x} + 2$



Domain:

Range:

7. $y = \left(\frac{1}{3}\right)^{x+3} + 5$



Domain:

Range:

Examples:

Which of the following functions are examples of exponential growth & which are examples of exponential decay? Why?

8. $f(x) = 0.25(4)^x$

9. $h(x) = 0.9^x$

10. $g(x) = \left(\frac{3}{2}\right)^{-x}$

11. $s(x) = \frac{2}{3}(e)^x$

Try one of the following:

12. $k(x) = \left(\frac{2}{3}\right)^x$

13. $p(x) = \left(\frac{2}{3}\right)^{-x}$

10.4 Notes: Changing the Base of Exponential Functions

Use your graphing calculator to compare $f(x)$ & $g(x)$.

What do you notice about the graphs of each pair?

Use the properties of exponents to explain why $f(x) = g(x)$

	$f(x)$	$g(x)$
A	$f(x) = 2^{3x}$	$g(x) = 8^x$
B	$f(x) = \left(\frac{1}{2}\right)^{2x}$	$g(x) = \frac{1}{4}^x$
C	$f(x) = \left(\frac{3}{2}\right)^x$	$g(x) = \left(\frac{2}{3}\right)^{-x}$

Example:

Write each of the following exponential functions as the same function with a different base.

1. $f(x) = 2^{5x}$

2. $g(x) = 25^{x}$

Try these!

3 $f(x) = 3^{3x}$

4. $f(x) = 16^{x}$

Example:

5. Which of the following would NOT produce the same graph as $g(x) = 729^{x}$?

A. $h(x) = 3^{6x}$

B. $h(x) = 9^{3x}$

C. $h(x) = 6^{4x}$

D. $h(x) = 27^{2x}$

Rational Roots

Rational Exponents: $\boldsymbol{a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m}$

Simplify: a. $x^{4/3}$ b. $x^{5/2}$ c. $6^{5/3}$

Think back to previous units…apply properties & rules that we have learned about to simplify the following problems as best you can with a partner.

6. $9^{\frac{1}{2}} \cdot 9^{\frac{3}{2}}$

7. $\dfrac{3^{\frac{5}{6}}}{3^{\frac{1}{3}}}$

8. $\sqrt[5]{27} \cdot \sqrt[5]{9}$

Examples:

Simplify the following expressions. Assume all variables are positive values.

9. $\dfrac{16^2}{2^3}$

10. $\dfrac{3^2 \cdot 9^3}{3^4}$

11. $x^{3/4} \cdot y^{2/3} \cdot x^{3/4} \cdot \sqrt[3]{y}$

12. $\dfrac{a^{1/3}\sqrt{b}}{a^{4/3}b^{1/2}}$

13. $\left(\dfrac{a^4 b^{2/3} c^{1/5}}{a^6 b^{1/3} c^{2/5}}\right)^5$

14. $\left(\dfrac{-2x^3 y^{1/3}}{3x^{2/3} y^{2/3}}\right)^3$

Try one of the following!

Simplify the following expressions. Assume all variables are positive values.

15. $\left(\frac{5^2}{5^4}\right)^{\frac{3}{2}}$

16. $\frac{64^{1/2} \cdot 4}{4^3}$

17. $\left(\frac{-3\sqrt{a} \cdot b^{3/4}}{4a^{5/2}b^{1/4}}\right)^2$

10.5 Notes: Modeling with Exponential Functions

Exponential Growth & Decay

Exponential Growth Formula: $\boldsymbol{A(t) = A_o(1 + r)^t}$

Exponential Decay Formula: $\boldsymbol{A(t) = A_o(1 - r)^t}$

Vocabulary

- **Principle:**
- **Initial Amount:**
- **Rate:**

- **Compound Interest:**

  - Compounded Annually
  - Compounded Quarterly
  - Compounded Monthly
  - Compounded Weekly
  - Compounded Daily
  - Compounded Continuously

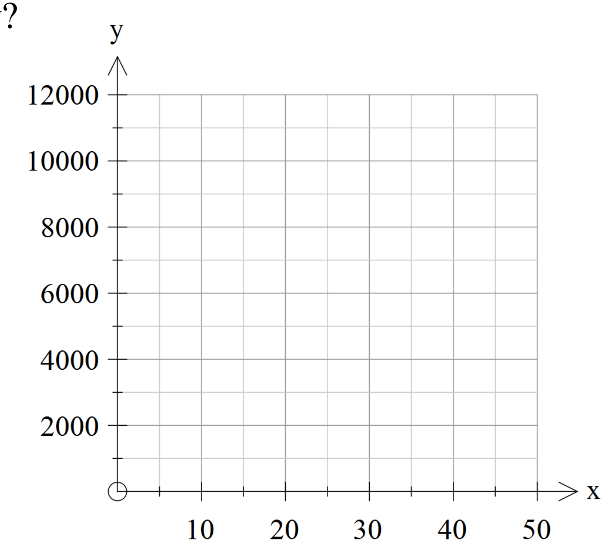
<u>Example 1</u>:

Janelle invests $5000 in an account that earns interest at a rate of 2% compounded annually.

a. Is this exponential growth or exponential decay?

b. Write the function that gives the balance in the account after t years.

c. Graph the function.

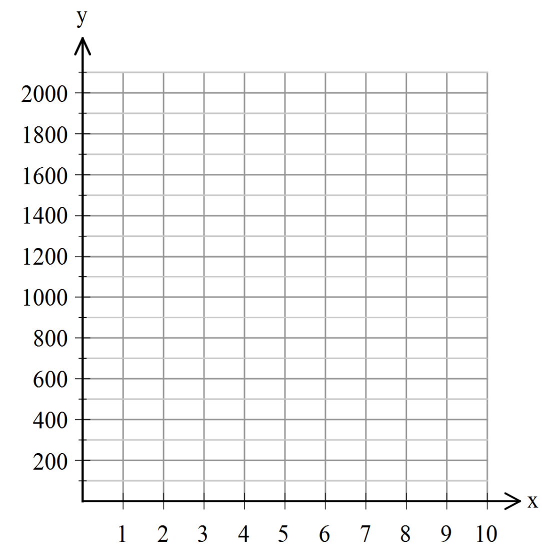


d. Find the balance after 6 years.

YOU TRY!

Example 2:

A bacteria population starts at 2,032 and decreases at about 15% per day. Graph the function. Then predict how many bacteria there will be after 7 days.



a. Is this exponential growth or exponential decay?

b. Write a function representing the number of bacteria present each day.

c. Graph the function.

d. Find the number of bacteria after 7 days.

Example 3:

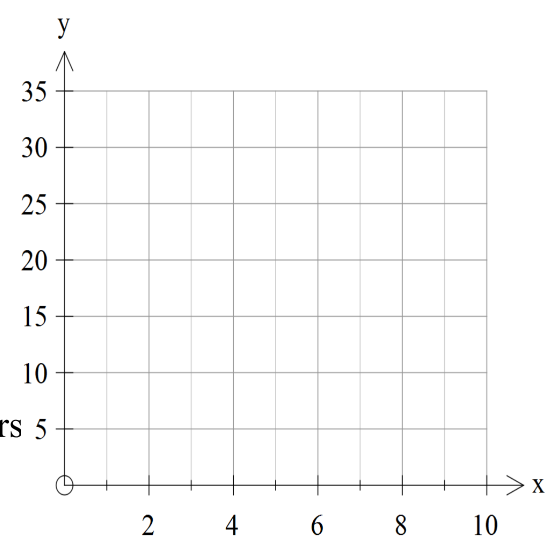
The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. An adult drinks a caffeinated soda, and the caffeine in his/her bloodstream reaches a peak level of 30 milligrams.

a. Is this exponential growth or exponential decay?

b. Write the function that gives the remaining caffeine at t hours after the peak level.

c. Graph the function.

d. Find the amount of caffeine remaining after 4 hours



Example 4:
Keiko invests \$2700 in an account that earns 2.5% annual interest compounded ***continuously***. How much money will she have in her account after 5 years? Use $\boldsymbol{A(t) = Pe^{rt}}$.

Example 5:
You deposit \$5000 in an account that earns 3.5% compounded quarterly. How much money will you have after 3 years? Use $\boldsymbol{A(t) = P\left(1 + \frac{r}{n}\right)^{nt}}$; where n is the number of times per year at an investment is compounded.

You try these!

Example 6:
Miguel invests \$4800 at 1.9% annual interest compounded continuously. How much money will he have in his account after 3 years? Use $\boldsymbol{A(t) = Pe^{rt}}$.

Example 7:
Sarah deposits \$10,500 in an account that earns 6.7% compounded daily. How much money will Sarah have after 7 years? Use $\boldsymbol{A(t) = P\left(1 + \frac{r}{n}\right)^{nt}}$

10.6 Notes: Solving Exponential Equations

Property of Equality for Exponential Equations:

Work with a partner and try to find the value of *x*. Be prepared to share your process with the class.

$$2^{x+4} = 2^{2x+3}$$

Examples: Solve for *x* and check your solutions.

1) $2^{x-1} = 32$

2) $e^{3x} = e^{x+12}$

3) $\frac{1}{64} = 4^{2x-4}$

4) $9^{2x} = 27^{x+1}$

Try one of the following!

5) $15^{2x-9} = 15^{5x+6}$

6) $2^{3x+1} = \frac{1}{32}$

7) $16^{3x} = 64^{x+2}$

Examples: Solve each system of exponential equations for x by setting $f(x) = g(x)$. Verify your answers using a graphing calculator.

8. $\begin{cases} f(x) = 3 \\ g(x) = 27^x \end{cases}$

9. $\begin{cases} f(x) = 5^{2x} \\ g(x) = 125^{x-2} \end{cases}$

You try these!

10. $\begin{cases} f(x) = e^{2x} \\ g(x) = e^{x+5} \end{cases}$

11. $\begin{cases} f(x) = 4^x \\ g(x) = 32^{x-3} \end{cases}$

Example 12: Use your graphing calculator to solve the following problem

The equation $f(x) = 4.1(1.33)^x$ models the population of the United States, in millions, from 1790 to 1890. In this equation, x is the number of decades since 1790, and $f(x)$ is the population in millions. In what year did the population reach 71 million?

a. Let $f(x) = 4.1(1.33)^x$ & let $g(x) = 71$. To solve for x, find where $f(x) = g(x)$.

b. In what year did the population reach 71 million?

Example 13: Write an exponential function in the form $y = ab^x$ whose graph passes through the points

$(2, 12.5)$ and $(4, 312.5)$.