Unit 10 Calendar **Name:** ____________________

Day	Date	Assignment (Due the next class meeting)
Tuesday Wednesday	3/29/22 (A) 3/30/22 (B)	10.1 Worksheet **Properties of Exponents & Base e**
Thursday Friday	3/31/22 (A) 4/1/22 (B)	10.2 Worksheet **Graphing Exponential Functions & Base e (Day 1)**
Monday Tuesday	4/4/22 (A) 4/5/22 (B)	10.3 Worksheet **Graphing Exponential Functions (Day 2)**
Wednesday Thursday	4/6/22 (A) 4/7/22 (B)	10.4 Worksheet **Changing the Base of an Exponential Function**
Friday Monday	4/8/22 (A) 4/11/22 (B)	10.5 Worksheet **Modeling with Exponential Functions (Growth and Decay)**
Tuesday Wednesday	4/12/22 (A) 4/13/22 (A)	10.6 Worksheet **Solving Exponential Equations**
Thursday Friday	4/14/22 (A) 4/15/22 (B)	**Unit 10 Practice Test**
Tuesday Wednesday	4/19/22 (A) 4/20/22 (B)	**Unit 10 Review**
Thursday Friday	4/21/22 (A) 4/22/22 (B)	**Unit 10 Test**

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Try www.khanacademy.org if you need help outside of school hours.
- Student who complete 100% of their homework second semester on-time will receive a pizza party and 2% bonus to their grade!
- Don't forget about the webpage: www.washoeschools.net/drhsmath

10.1 Notes: Properties of Exponents & Base e

Let *a* and *b* be real numbers and let *m* and *n* be integers

Product of Powers	$a^m \bullet a^n = a^{m+n}$	$x^2 x^3 = x^5$
Power of a Power	$(a^m)^n = a^{mn}$	$(x^2)^3 = x^6$
Power of a Product	$(ab)^m = a^m b^m$	$(xy)^2 = x^2 y^2$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$	$a^{-5} = \frac{1}{a^5}$ or $\frac{1}{a^{-5}} = a^5$
Zero Exponent	$a^0 = 1$	$(\ldots.)^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^{10}}{x^7} = x^3$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{x^2}{y^5}\right)^{10} = \frac{x^{20}}{y^{50}}$

Examples: Simplify.

1) $(x^3y^6)^3$

$x^{3\cdot3}\ y^{6\cdot3}$

x^9y^{18}

2) $(x^3)^2 \cdot (xy^2)^4$

$x^{3\cdot2} \cdot x^4y^{2\cdot4}$

$x^6 \cdot x^4y^8$

$x^{10}y^8$

3) $(x^2y^{-6})^7$

$x^{2\cdot7}\ y^{-6\cdot7}$

$x^{14}\ y^{-42}$

$\frac{x^{14}}{y^{42}}$

4) $(2a^2b^8)^0$

1

Try one of the following :

a) $(x^2y^7)^6$

$x^{12}y^{42}$

b) $(x^{-2}y)^3 \cdot y^4$

$\frac{y^3}{x^6} \cdot y^4$

$\frac{y^7}{x^6}$

c) 15^0

1

Examples: Simplify.

5) $\frac{x^5y^2}{x^{15}y^8}$

$\cancel{xxxxx}\ \cancel{yy}$

$\cancel{xxxxx}xxxxxxxxxx\cancel{yy}yyyyyy$

$\frac{1}{x^{10}y^6}$

6) $\left(\frac{a^4}{b^2}\right)^2$

$\frac{a^8}{b^4}$

7) $\left(\frac{r^{-2}}{s^3}\right)^{-3}$

$\frac{r^6}{s^{-9}} = r^6s^9$

8) $\frac{c \cdot c^4}{c^2}$

$\frac{c^5}{c^2} = c^3$

Try one of the following: * try one of the following *

a) $\frac{x^7y^{16}}{x^{15}y^{12}}$

$\frac{y^4}{x^8}$

b) $\left(\frac{q^7}{r^{-2}}\right)^4$

$q^{28}r^8$

Examples: Simplify.

9) $\dfrac{16m^4n^{-5}}{2n^{-5}m^7}$

$\dfrac{8}{m^3}$

10) $\dfrac{x^2y^{-3}}{(2x^3y^{-2})^2}$

$\dfrac{x^2y^{-3}}{4x^6y^{-4}}$

$\dfrac{x^2y^4}{4x^6y^3} = \dfrac{y}{4x^3}$

11) $\dfrac{4^2 \cdot 64^3}{4^4}$

4^3

$\dfrac{4^2 \cdot (4^3)^3}{4^4}$

$\dfrac{4^2 \cdot 4^9}{4^4}$

$\dfrac{4^{11}}{4^4} = 4^7$

Try one of the following!

a) $\dfrac{(a^2b^4)^2}{a^{-3}b}$

$\dfrac{a^4b^8}{a^{-3}b} = a^7b^7$

b) $\dfrac{24xy^6}{4x^{-2}y^4}$

$6x^3y^2$

c) $\dfrac{4^8 \cdot 2^2}{2^{20}}$

$\dfrac{2^{16} \cdot 2^2}{2^{20}}$

$\dfrac{2^{18}}{2^{20}} = \dfrac{1}{2^2}$

The Natural Base *e*:

e is a # 2.7.....
(like π is 3.14...)

Examples: Simplify the following expressions.

12) $3e^2 \cdot 6e^5$

$18e^7$

13) $\dfrac{18e^4}{9e^3}$

$2e$

14) $(-4e^{-5x})^3$

$-64e^{-15x} = \dfrac{-64}{e^{15x}}$

Try one of the following!

a) $-5e^3 \cdot 2e^6$

$-10e^9$

b) $\dfrac{24e^4}{6e^3}$

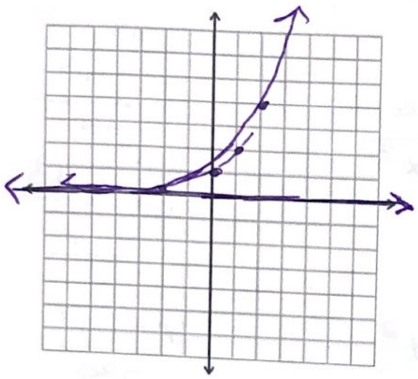
$4e$

c) $(-3e^{-4x})^2$

$9e^{-8x} = \dfrac{9}{e^{8x}}$

10.2 Notes: Graphing Exponential Functions & Base e (Day 1)

Graphing Exponential Functions:



(in set notation)
Domain: $(-\infty, \infty)$

Range: $y > 0$

Linear Parent Function: $y = 2^x$

x	y	(x, y)
−2	$2^{-2} = 1/4$	$(-2, 1/4)$
−1	$2^{-1} = 1/2$	$(-1, 1/2)$
0	$2^0 = 1$	$(0, 1)$
1	$2^1 = 2$	$(1, 2)$
2	$2^2 = 4$	$(2, 4)$

y-intercept: $(0, 1)$

Horizontal Asymptote: $y = 0$

What happens when we change b (when $b > 1$)?

Graph each of the functions on the graphing calculator. Sketch your results on the graph provided.

a. $y = 2^x$

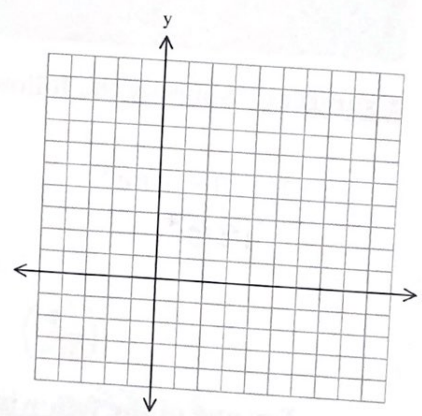
b. $y = e^x$

b. $y = 3^x$

c. $y = 4^x$

d. $y = 10^x$

when the base changes, the graph gets steeper but still goes through (0,1

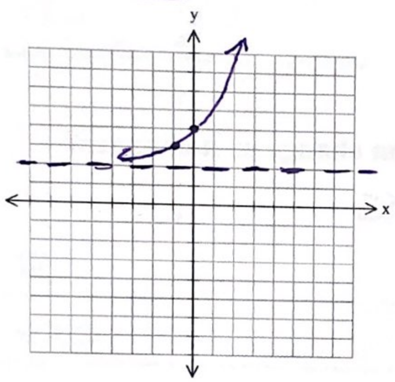


Graphing $f(x) = ab^{x-h} + k$, when $b > 1$ (Exponential Growth)

What happens when we change h & k?

Graph the following exponential equation. Explain how the graph is transformed from the parent function $f(x) = 2^x$. Also, state the domain and range for each function & describe the end behavior.

$f(x) = 2^{x+1} \boxed{+2}$ -asymptote



Transformation: ↑2 ←1

Domain: ℝ

Range: y > 2

End Behavior:
as x → ∞ y → ∞
x →

How does the graph of the exponential function change as h & k changes?

⟷ ↕

How does the graph of the exponential function change as the base b changes?

gets steeper

What happens when we change a?

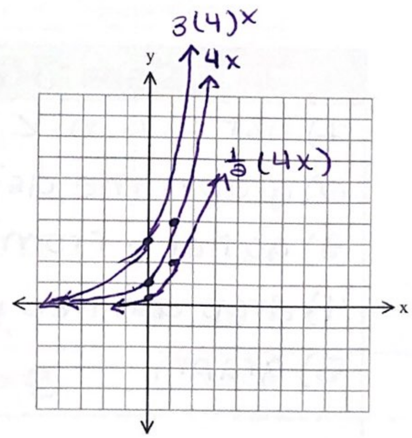
Graph each function on the graphing calculator.
Sketch your results on the graph provided.

a. $f(x) = 4^x$

b. $g(x) = \boxed{3}(4)^x$ → goes through (0,3) instead of (0,1)

c. $h(x) = \boxed{\frac{1}{2}}(4)^x$

↳ goes through (0, 1/2)



Compare the parent graph, $f(x)$, with $g(x)$ & $h(x)$. What is the domain, range, & end behavior for each graph? What do you notice about the y −intercepts?

everything is the same
execpt for y-intercept

How does the graph of the exponential function change as a changes?

stretch|compress

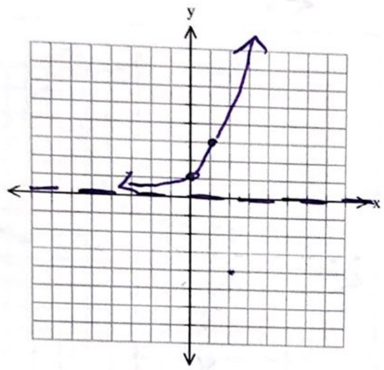
Steps to Graph Exponential Functions:

1) list a, b, h, k
2) graph the asymptote (k-value)
3) go ↑ or ↓ from y-axis & then ← or → h (opp. h) First pt.
4) drop ~~b~~ like it's not, → 1, ↑ or ↓ a · base
5) graph

Examples

Graph each exponential function. Describe the domain & range. asymptote

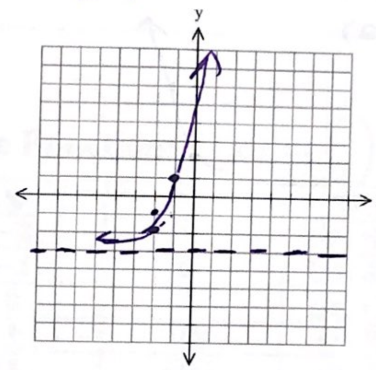
1. $y = 3^x$



Domain: $\mathbb{R}$

Range: $y>0$

2. $y = 4^{x+2}\boxed{-3}$ ~~asymptote~~

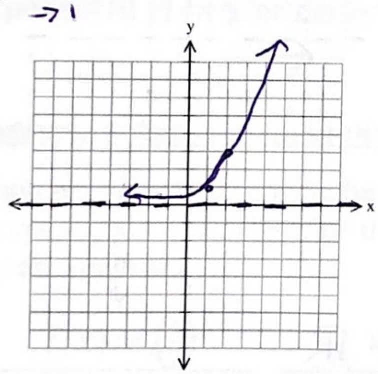


Domain: $\mathbb{R}$

Range: $y>-3$

Try one of the following:

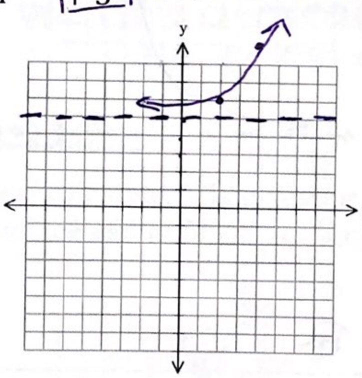
3. $y = 3^{x-1}$ →1



Domain: $\mathbb{R}$

Range: $y>0$

4. →2 $y = 4^{x-2}\boxed{+5}$ asymptote



Domain: $\mathbb{R}$

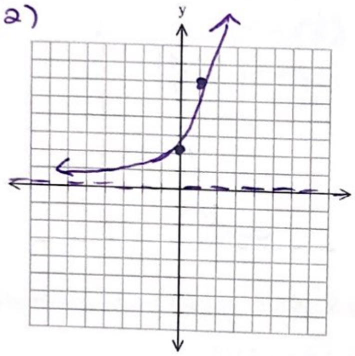
Range: $y>5$

Examples

Graph each exponential function. Describe the domain & range.

5. $y = 2 \cdot 3^x$

(0,2)

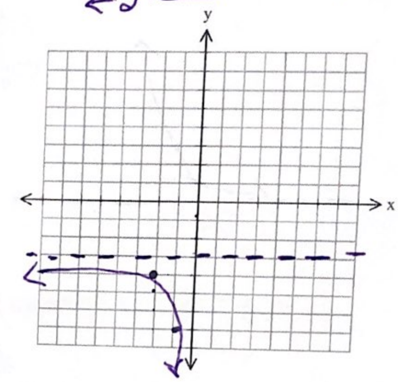


Domain: $\mathbb{R}$

Range: $y > 0$

6. $y = -1 \cdot 4^{x+2} \boxed{-3}$ asymptote

←2



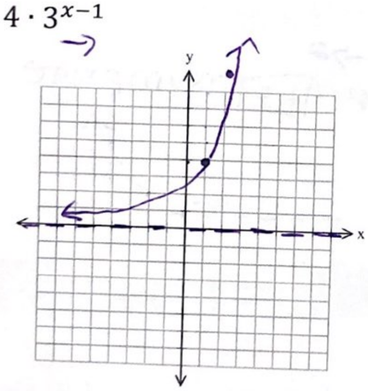
Domain: $\mathbb{R}$

Range: $y < -3$

Try one of the following:

7. $y = 4 \cdot 3^{x-1}$

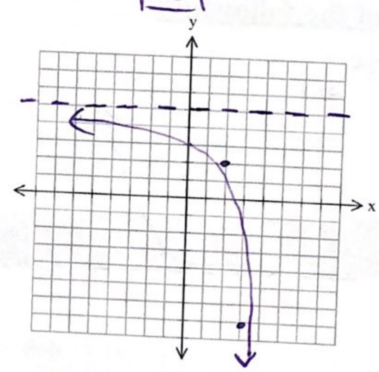
→



Domain: $\mathbb{R}$

Range: $y > 0$

8. $y = -3 \cdot 4^{x-2} \boxed{+5}$



Domain: $\mathbb{R}$

Range: $y < 5$

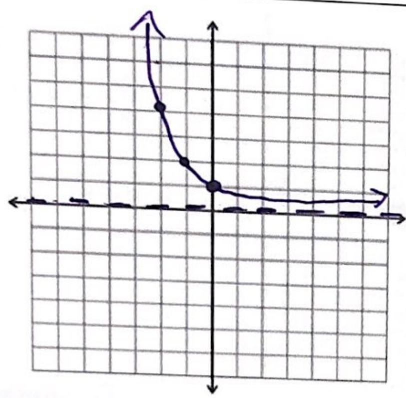
9. When evaluating the function $f(x) = 2^{x-4}$ for any real number x, what must be true about the value of $f(x)$?

A. The value of f(x) is always negative
B. The value of f(x) is always positive
C. The value of f(x) is always greater than 4
D. The value of f(x) is always less than 4

what is the range?

10.3 Notes: Graphing Exponential Functions (Day 2)

Graphing $f(x) = ab^{x-h} + k$, when $0 < b < 1$ (Exponential Decay)



(in set notation)
Domain: ℝ
Range: $y > 0$

Graph the Function: $f(x) = \left(\frac{1}{2}\right)^x$

x	y	(x, y)
−2	4	(-2, 4)
−1	2	(-1, 2)
0	1	(0, 1)
1	1/2	(1, 1/2)
2	1/4	(2, 1/4)

y-intercept: (0, 1)
Horizontal Asymptote: $y = 0$

As your go right, are the values increasing or decreasing? decreasing

Is this exponential growth or decay? Why? decay, b/c it is decreasing ($0 < b < 1$)

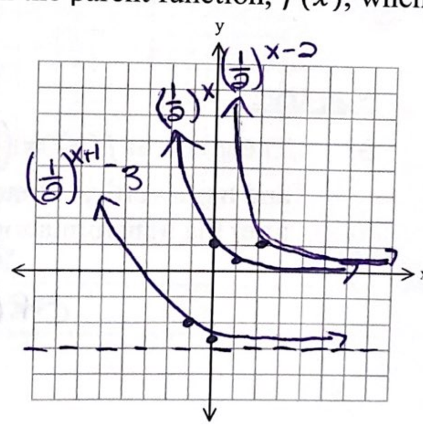
What happens when we change h & k (when $0 < b < 1$)?

Graph each of the following functions on the graphing calculator. Sketch your results on the graph provided. Describe the transformation from the parent function, $f(x)$, when you change h & k.

a. $f(x) = \left(\frac{1}{2}\right)^x$

b. $g(x) = \left(\frac{1}{2}\right)^{x-2}$

c. $h(x) = \left(\frac{1}{2}\right)^{x+1} - 3$



Vertical & Horizontal Reflections

Use the graphing calculator to graph each of the following functions.

a. $y = 2^{-x}$ $\left(\frac{1}{2}\right)^x$

b. $y = 3^{-x}$ $\left(\frac{1}{3}\right)^x$

c. $y = e^{-x}$ $\left(\frac{1}{e}\right)^x$

d. $y = \left(\frac{1}{2}\right)^x$

e. $y = \left(\frac{1}{3}\right)^x$

f. $y = e^x$

Which of these are exponential growth functions?

F

Which of these are exponential decay functions?

a, b, c, d, e

Examples:

1. The graph $f(x) = 2^x$ is translated two (2) units up, four (4) units right, & has a vertical reflection (reflected across the x-axis). Write the equation of the function after the transformation.

$f(x) = -2^{x-4} + 2$

2. The graph $f(x) = e^x$ is translated down five (5) units. Write the equation of the function after the transformation.

$f(x) = e^x - 5$

You try !

3. The graph of $f(x) = \left(\frac{1}{2}\right)^x$ is translated two (2) units to the right, three (3) units up, and has a vertical stretch by a factor of four (4). Write the equation of the function after the transformation.

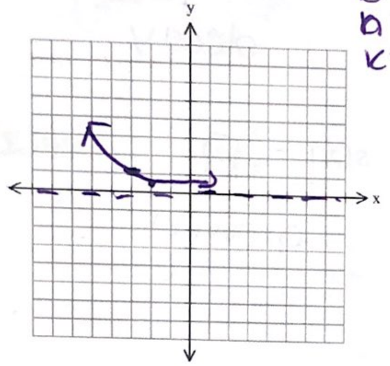
$f(x) = 4\left(\frac{1}{2}\right)^{x-2} + 3$

Examples:

Graph each exponential function. Describe the domain & range.

4. $y = \left(\frac{1}{2}\right)^{x+3}$

$a = 1$
$b = 1/2$
$h = -3$
$k = 0$

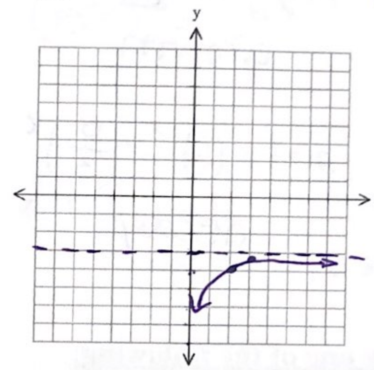


Domain: $\mathbb{R}$

Range: $y > 0$

5. $y = -\left(\frac{1}{3}\right)^{x-2} - 4$

$a = -1$
$b = 1/3$
$h = 2$
$k = -4$



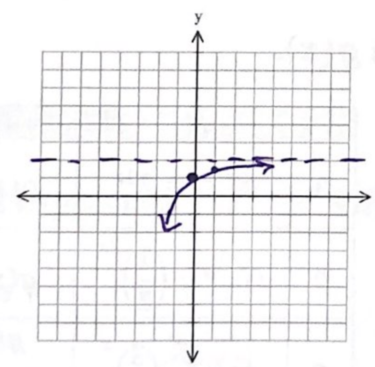
Domain: $\mathbb{R}$

Range: $y < -4$

Try on of the following!

6. $y = -\left(\frac{1}{2}\right)^{x} + 2$

$a = -1$
$b = 1/2$
$h = 0$
$k = 2$

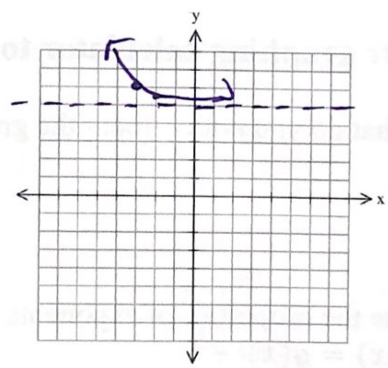


Domain: $\mathbb{R}$

Range: $y < 2$

7. $y = \left(\frac{1}{3}\right)^{x+3} + 5$

$a = 1$
$b = 1/3$
$h = -3$
$k = 5$



Domain: $\mathbb{R}$

Range: $y > 5$

Examples:

Which of the following functions are examples of exponential growth & which are examples of exponential decay? Why?

8. $f(x) = 0.25\boxed{4}^x$

growth

9. $h(x) = \boxed{0.9^x}$

decay

10. $g(x) = \left(\frac{3}{2}\right)^{-x} = \left(\frac{2}{3}\right)^x$

decay

11. $s(x) = \frac{2}{3}\boxed{e}^x$

growth

Try one of the following:

12. $k(x) = \boxed{\left(\frac{2}{3}\right)}^x$

decay

13. $p(x) = \left(\frac{2}{3}\right)^{-x} = \left(\frac{3}{2}\right)^x$

growth

10.4 Notes: Changing the Base of Exponential Functions

Use your graphing calculator to compare $f(x)$ & $g(x)$.

What do you notice about the graphs of each pair?

	$f(x)$	$g(x)$
A	$f(x) = 2^{3x}$	$g(x) = 8^x$
B	$f(x) = \left(\frac{1}{2}\right)^{2x}$	$g(x) = \frac{1^x}{4}$
C	$f(x) = \left(\frac{3}{2}\right)^x$	$g(x) = \left(\frac{2}{3}\right)^{-x}$

Use the properties of exponents to explain why $f(x) = g(x)$

$2^{3x} = (2^3)^x = 8^x$

$\left(\frac{1}{2^2}\right)^x = \left(\frac{1^2}{2^2}\right)^x = \frac{1}{4}^x$

Example:

Write each of the following exponential functions as the same function with a different base.

1. $f(x) = 2^{5x}$

$(2^5)^x = 32^x$

2. $g(x) = 25^x$

$(5^2)^x = 5^{2x}$

Try these!

3 $f(x) = 3^{3x}$

$(3^3)^x = 27^x$

4. $f(x) = 16^x$

$(4^2)^x = 4^{2x}$

Example:

5. Which of the following would NOT produce the same graph as $g(x) = 729^x$?

A. $h(x) = 3^{6x}$

B. $h(x) = 9^{3x}$

C. $h(x) = 6^{4x}$

D. $h(x) = 27^{2x}$

Rational Roots

Rational Exponents: $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Simplify:

a. $x^{4/3}$

$\sqrt[3]{x^4}$

$\downarrow$

$x\sqrt[3]{x}$

b. $x^{5/2}$

$\sqrt[2]{x^5}$

$\downarrow$

$x^2\sqrt{x}$

c. $6^{5/3}$

$\sqrt[3]{6^5}$

$\downarrow$

$6\sqrt[3]{6^2}$

$\downarrow$

$6\sqrt[3]{36}$

Think back to previous units…apply properties & rules that we have learned about to simplify the following problems as best you can with a partner.

6. $9^{\frac{1}{2}} \cdot 9^{\frac{3}{2}}$

$9^{\frac{4}{2}} = 9^2 = 81$

7. $\dfrac{3^{\frac{5}{6}}}{3^{\frac{1}{3}}}$

$\frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

$3^{1/2}$

$\sqrt{3}$

8. $\sqrt[5]{27} \cdot \sqrt[5]{9}$

$\sqrt[5]{3^3} \cdot \sqrt[5]{3^2}$

$\sqrt[5]{3^5}$

$= 3$

Examples:

Simplify the following expressions. Assume all variables are positive values.

9. $\dfrac{16^2}{2^3}$

$\dfrac{(2^4)^2}{2^3}$

$\dfrac{2^8}{2^3} = 2^5$

10. $\dfrac{3^2 \cdot 9^3}{3^4}$

$\dfrac{3^2 \cdot 3^6}{3^4} = 3^4$

11. $x^{3/4} \cdot y^{2/3} \cdot x^{3/4} \cdot \sqrt[3]{y}$ $y^{\frac{1}{3}}$

$x^{6/4} \cdot y^{3/3}$

$x^{3/2}\, y$

12. $\dfrac{a^{1/3}\sqrt{b}}{a^{4/3}b^{1/2}}$

$\dfrac{a^{1/3}\cancel{b^{1/2}}}{a^{4/3}\cancel{b^{1/2}}}$

$\dfrac{1}{a^{3/3}} = \dfrac{1}{a}$

13. $\left(\dfrac{a^4b^{2/3}c^{1/5}}{a^6b^{1/3}c^{2/5}}\right)^5$

$\dfrac{a^{20}b^{10/3}c^{5/5}}{a^{30}b^{5/3}c^{10/5}}$

$\dfrac{b^{5/3}}{a^{10}c}$

14. $\left(\dfrac{-2x^3y^{1/3}}{3x^{2/3}y^{2/3}}\right)^3$

$\dfrac{-2^3x^9y^1}{27x^{6/3}y^{6/3}}$

$\dfrac{-8x^9y}{27x^2y^2}$

$\dfrac{-8x^7}{27y}$

Try one of the following!

Simplify the following expressions. Assume all variables are positive values.

15. $\left(\frac{5^2}{5^4}\right)^{\frac{3}{2}}$

$\frac{5^3}{5^6} = \frac{1}{5^3}$

16. $\frac{64^{1/2} \cdot 4}{4^3}$

$\frac{(4^4)^{1/2} \cdot 4}{4^3}$

$\frac{4^2 \cdot 4}{4^3} = 1$

17. $\left(\frac{-3\sqrt{a} \cdot b^{3/4}}{4a^{5/2}b^{1/4}}\right)^2$

$\frac{9ab^{3/2}}{16a^5b^{1/2}} = \frac{9b}{16a^4}$

10.5 Notes: Modeling with Exponential Functions

Exponential Growth & Decay

Exponential Growth Formula: $A(t) = A_o\boxed{(1+r)}^t$ → greater than 1

Exponential Decay Formula: $A(t) = A_o\boxed{(1-r)}^t$ → less than 1

Vocabulary

- **Principle:** A_o
  same
- **Initial Amount:** A_o
- **Rate:** r

- **Compound Interest:**
  - Compounded Annually
  - Compounded Quarterly
  - Compounded Monthly
  - Compounded Weekly
  - Compounded Daily
  - Compounded Continuously

Example 1:

Janelle invests \$5000 in an account that earns interest at a rate of 2% compounded annually.

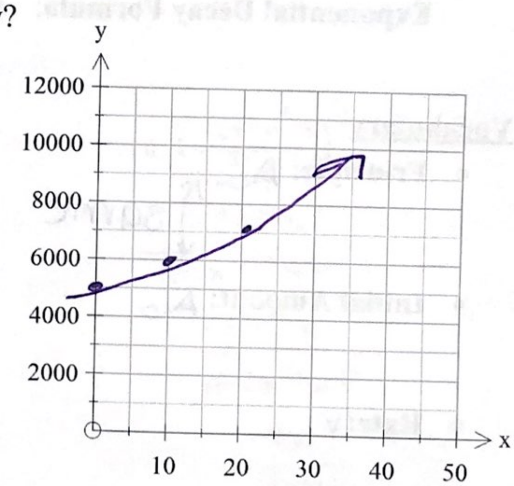
.02

a. Is this exponential growth or exponential decay?

b. Write the function that gives the balance in the account after t years.

$f(t) = 5000(1+0.02)^x$

c. Graph the function.



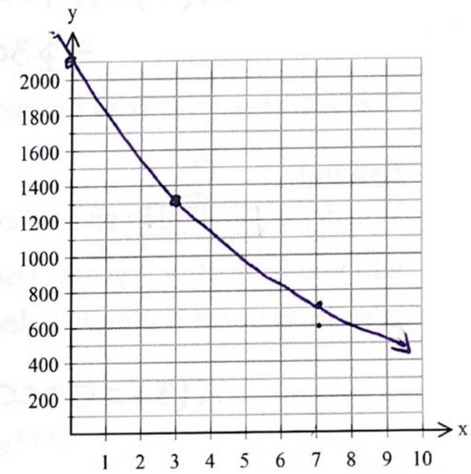
d. Find the balance after 6 years.

$5000(1.02)^6 = \$5630.81$

YOU TRY!

Example 2:

A bacteria population starts at 2,032 and $\boxed{\text{decreases}}$ at about 15% per day. Graph the function. Then predict how many bacteria there will be after 7 days.



a. Is this exponential growth or exponential decay?

b. Write a function representing the number of bacteria present each day.

$f(t)=2032(1-.15)^t$

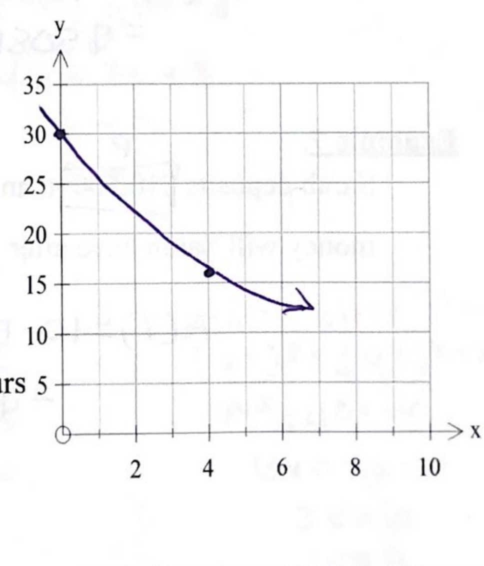
c. Graph the function.

d. Find the number of bacteria after 7 days.

$f(7)=2032(1-0.15)^7$
$=651$

Example 3:

The rate at which caffeine is $\boxed{\text{eliminated}}$ from the bloodstream of an adult is about 15% per hour. An adult drinks a caffeinated soda, and the caffeine in his/her bloodstream reaches a peak level of 30 milligrams.



a. Is this exponential growth or exponential decay?

b. Write the function that gives the remaining caffeine at t hours after the peak level.

$f(t)=30(1-0.15)^t$

c. Graph the function.

d. Find the amount of caffeine remaining after 4 hours

$f(4)=30(1-.15)^4$

Example 4: P r

Keiko invests $\boxed{\$2700}$ in an account that earns 2.5% annual interest compounded ***continuously***. How much money will she have in her account after 5 years? Use $\boldsymbol{A(t) = Pe^{rt}}$.

t

$A(5) = 2700e^{0.025(5)}$

$= \$3059.50$

Example 5: P r

You deposit $\boxed{\$5000}$ in an account that earns 3.5% compounded quarterly. How much money will you have after 3 years? Use $\boldsymbol{A(t) = P\left(1+\frac{r}{n}\right)^{nt}}$; where n is the number of times per year at an investment is compounded.

t

$A(3) = 5000\left(1+\frac{0.035}{4}\right)^{4(3)} = \$5551.02$

You try these!

Example 6: P r

Miguel invests $\boxed{\$4800}$ at 1.9% annual interest compounded continuously. How much money will he have in his account after 3 years? Use $\boldsymbol{A(t) = Pe^{rt}}$.

$A(3) = 4800e^{(0.019)(3)}$

$= \$5081.55$

Example 7: P r

Sarah deposits $\boxed{\$10,500}$ in an account that earns 6.7% compounded daily. How much money will Sarah have after 7 years? Use $\boldsymbol{A(t) = P\left(1+\frac{r}{n}\right)^{nt}}$

t

$A(7) = 10,500\left(1+\frac{0.067}{365}\right)^{365(7)}$

$= \$16782.43$

10.6 Notes: Solving Exponential Equations

Property of Equality for Exponential Equations:

Work with a partner and try to find the value of *x*. Be prepared to share your process with the class.

$$2^{x+4} = 2^{2x+3}$$

$x+4 = 2x+3$

$1 = x$

Examples: Solve for *x* and check your solutions.

1) $2^{x-1} = 32$

$x-1=5$

$x=6$

2) $e^{3x} = e^{x+12}$

$3x = x+12$

$2x=12$

$x=6$

3) $\frac{1}{64} = 4^{2x-4}$

$\frac{1}{4^3} = 4^{2x-4}$

$4^{-3} = 4^{2x-4}$

$-3 = 2x-4$

$1=2x \rightarrow x=\frac{1}{2}$

4) $9^{2x} = 27^{x+1}$

$(3^2)^{2x} = (3^3)^{x-1}$

$3^{4x} = 3^{3x+3}$

$4x = 3x+3$

$x=3$

Try one of the following!

5) $15^{2x-9} = 15^{5x+6}$

$2x-9 = 5x+6$

$-15 = 3x$

$x=-5$

6) $2^{3x+1} = \frac{1}{32}$

$2^{3x+1} = 2^{-5}$

$3x+1 = -5$

$3x = -6$

$x = -2$

7) $16^{3x} = 64^{x+2}$

$(4^2)^{3x} = (4^3)^{x+2}$

$4^{6x} = 4^{3x+6}$

$6x = 3x+6$

$3x = 6$

$x=2$

Examples: Solve each system of exponential equations for x by setting $f(x) = g(x)$. Verify your answers using a graphing calculator.

8. $\begin{cases} f(x) = 3 \\ g(x) = 27^x \end{cases}$

$3 = 27^x$
$3^1 = 3^{3x}$
$1 = 3x$
$x = 1/3$

9. $\begin{cases} f(x) = 5^{2x} \\ g(x) = 125^{x-2} \end{cases}$

$5^{2x} = (5^3)^{x-2}$
$2x = 3x - 6$
$-x = -6$
$x = 6$

You try these!

10. $\begin{cases} f(x) = e^{2x} \\ g(x) = e^{x+5} \end{cases}$

$e^{2x} = e^{x+5}$
$2x = x + 5$
$x = 5$

11. $\begin{cases} f(x) = 4^x \\ g(x) = 32^{x-3} \end{cases}$

$4^x = (2^5)^{x-3}$
$2^{2x} = 2^{5x-15}$
$2x = 5x - 15$ $x = 5$
$-3x = -15$

Example 12: Use your graphing calculator to solve the following problem

The equation $f(x) = 4.1(1.33)^x$ models the population of the United States, in millions, from 1790 to 1890. In this equation, x is the number of decades since 1790, and $f(x)$ is the population in millions. In what year did the population reach 71 million?

a. Let $f(x) = 4.1(1.33)^x$ & let $g(x) = 71$. To solve for x, find where $f(x) = g(x)$.

$\frac{71}{4.1} = \frac{4.1(1.33)^x}{4.1}$ $\quad \frac{71}{4.1} = 1.33^x$ $\quad x =$

$\frac{\ln \frac{71}{4.1}}{\ln 1.33} = \frac{x \ln 1.33}{\ln 1.33}$ $\quad 9.99$ or $x \approx 10$ decades

b. In what year did the population reach 71 million?

$1790 + 99 = 1889$ (At the end of the year)

Example 13: Write an exponential function in the form $y = ab^x$ whose graph passes through the points (2, 12.5) and (4, 312.5).

$12.5 = ab^2$
$a = \frac{12.5}{b^2}$

$312.5 = ab^4$
$312.5 = \frac{12.5}{b^2} \cdot b^4$
$\frac{312.5}{12.5} = \frac{12.5b^2}{12.5}$

$25 = b^2$
$b = 5$
$12.5 = a(5)^2$
$\frac{1}{2} = a$

$\boxed{y = \frac{1}{2} \cdot 5^x}$