Chapter 7 Calendar **Name:** ____________________

Day	Date	Assignment (Due the next class meeting)
Monday Tuesday	1/3/22 (A) 1/4/22 (B)	7.1 Worksheet Factoring Review
Wednesday Thursday	1/5/22 (A) 1/6/22 (B)	7.2 Worksheet Simplifying Rational Expressions
Friday Monday	1/7/22 (A) 1/10/22 (B)	7.3 Worksheet Multiplying and Dividing Rational Functions
Tuesday Wednesday	1/11/22 (A) 1/12/22 (B)	7.4 Worksheet Adding and Subtracting Rational Functions
Thursday Friday	1/13/22 (A) 1/14/22 (B)	7.5 Worksheet Solving Rational Equations
Tuesday Wednesday	1/18/22 (A) 1/19/22 (B)	Unit 7 Practice Test
Thursday Friday	1/20/22 (A) 1/21/22 (B)	**Unit 7 Test**
Monday Tuesday	1/24/22 (A) 1/25/22 (B)	6.3 Worksheet Polynomial division, remainder and factor theorems
Wednesday Thursday	1/26/22 (A) 1/27/22 (B)	6.4 Worksheet Rational Roots Theorem
Friday Monday	1/28/22 (A) 1/31/22 (B)	7.6 Worksheet Inverse Variation and the Reciprocal Function
Tuesday Wednesday	2/1/22 (A) 2/2/22 (B)	7.7 Worksheet Graphing with Rational Functions
Thursday Friday	2/3/22 (A) 2/4/22 (B)	Graphing Review
Monday Tuesday	2/7/22 (A) 2/8/22 (B)	**Unit 7 Graphing Quiz**

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Try www.khanacademy.org if you need help outside of school hours.
- Student who complete 100% of their homework second semester will receive a 2% bonus to their grade! Students with no late homework will receive a pizza party too!

7.1 Notes: Review of Factoring

1) Create a polynomial that has 3 unique factors.

$x^3 - x^2 - 6x$

$x(x^2 - x - 6)$

$x(x^2 + 2x - 3x - 6)$

$x(x-3)(x+2)$

2) Now create a binomial that factors to a pair of conjugates.

$x^2 - 9$

$x^2 - 3x + 3x - 9$

$(x-3)(x+3)$

Greatest Common Factor:	Difference of Squares	Trinomials w/ leading coefficient of 1
$27x^2-81x$ $27x(x-3)$	x^2-81 $(x-9)(x+9)$	$x^2-8x+15$ $(x-3)(x-5)$
Trinomials w/ leading coefficient other than 1.	**Factor by Grouping:**	**Sum/Difference of Cubes**
$2x^2-2x-12$ $(2x+4)(x-3)$	$(x^3+3x^2)(-3x-9)$ $x^2(x+3)-3(x+3)$ $(x^2-3)(x+3)$	x^3-8 x^3-2^3 $a=x$ $b=2$ $(x-2)(x^2+2x+4)$

7.2 Notes: Simplifying Rational Expressions

Factor the following completely:

1) $4x^2 - 25$

$(2x+5)(2x-5)$

2) $x^2 + 8x + 12$

$(x+2)(x+6)$

3) $3x^2 - x - 2$

$(3x+2)(x-1)$

4) $8x^4 + 2x^2$

$2x^2(4x^2+1)$

5) $8x^3 + 16x^2 - 10x$

$2x(4x^2+8x-5)$

$2x(2x-1)(2x+5)$

Things to remember about factoring…

- Always GCF first!
- look for perfect squares

Rational Function:

parent function $y=\frac{1}{x}$

Domain of a Rational Function:

- There is a value x CAN'T equal.
  ↳ Never divided by 0!
- So, in $y=\frac{1}{x}$, $x \neq 0$. Therefore the domain is $\{x \mid x \neq 0\}$

Examples: Find the domain of the following rational functions (write it in words and in set notation).

1) $y = \frac{3}{x}$

$\{x / x \neq 0\}$

2) $f(x) = \frac{x+3}{x-2}$

$x - 2 = 0$

$x = 2$

$\{x / x \neq 2\}$

3) $y = \frac{x+4}{(x+3)(x-1)}$

$x + 3 = 0 \quad x - 1 = 0$

$x = -3 \quad x = 1$

$\{x / x \neq -3, 1\}$

You try! Find the domain of the following rational functions in set notation.

a) $y = \frac{x+2}{5x}$

$\{x / x \neq 0\}$

b) $y = \frac{6}{(x+7)(x-8)}$

$\{x / x \neq -7, 8\}$

Simplified Form of a Rational Expression:

- make sure no factors on top & bottom can reduce
- To see this, you might need to factor.

Examples: Simplify the following and state the domain in set notation (or where the expression is undefined).

1) $\frac{x^2+7x+10}{x^2-4}$

$\frac{(x+5)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

Always find Domain from original equation

$D: \{x / x \neq \pm 2\}$

$\boxed{\frac{x+5}{x-2}}$

2) $\frac{x^2+5x+4}{x^2+x-12}$

$\frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-3)}$

$D: \{x / x \neq -4, 3\}$

hole asymptote

$\boxed{\frac{(x+1)}{x-3}}$

3) $\frac{4x^2-1}{6x^2+5x-4}$

$\frac{(2x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}(3x+4)}$

D: $\{x \mid x \neq \frac{1}{2}, -\frac{4}{3}\}$

hole asymptote

$\boxed{\frac{2x+1}{3x+4}}$

4) $\frac{3x^2-24x}{3x^3-15x^2-72x}$

$\frac{\cancel{3x}(x-8)}{\cancel{3x}(x^2-5x-24)}$

D: $\{x \mid x \neq 0, 8, -3\}$

$\frac{\cancel{x-8}}{\cancel{(x-8)}(x+3)}$

$\boxed{\frac{1}{x+3}}$

You try! Simplify the following and state the domain in set notation.

Choose one of the following to complete from a – c.

a) $\frac{4x^2+8x}{x^2-4}$

$\frac{4x\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

D: $\{x \mid x \neq \pm 2\}$

$\frac{4x}{x-2}$

b) $\frac{x^2-x-12}{2x^2-7x-4}$

$\frac{\cancel{(x-4)}(x+3)}{(2x+1)\cancel{(x-4)}}$

D: $\{x \mid x \neq -\frac{1}{2}, 4\}$

$\boxed{\frac{x+3}{2x+1}}$

c) $\frac{2x+3}{2x^2-11x-21}$

$\frac{\cancel{2x+3}}{\cancel{(2x+3)}(x-7)}$

D: $\{x \mid x \neq -\frac{3}{2}, 7\}$

$\boxed{\frac{1}{x-7}}$

d) Create a rational function where the numerator and denominator are quadratic functions that reduce to $\frac{x+6}{x-2}$.

$\frac{(x+6)(x+3)}{(x-2)(x+3)} = \frac{x^2+3x+6x+18}{x^2+3x-2x-6} = \boxed{\frac{x^2+9x+18}{x^2+x-6}}$

e) Write a rational function that has the following restrictions on the domain: $x \neq -4, 1$.

$\frac{1}{(x+4)(x-1)}$ or $\frac{1}{x^2+3x-4}$

7.3 Notes: Multiplying and Dividing Rational Functions

Explain how you would multiply $\frac{3}{8} \cdot \frac{12}{9}$. How would you divide $\frac{3}{8} \div \frac{12}{9}$?

$\frac{\cancel{3}}{\cancel{8}} \cdot \frac{\cancel{12}}{\cancel{9}} = \boxed{\frac{1}{2}}$ cross reduce

Skip, flip and multiply

$\frac{\cancel{3}}{8} \cdot \frac{9}{\cancel{12}} = \boxed{\frac{9}{32}}$

Multiplying Rational Expressions:

1) **Factor if possible**

1.5) Find Domain

2) **Reduce any common factors from the numerator and denominator**

3) **Multiply the numerators and denominators**

Examples: Simplify and state any restrictions on the domain.

1) $\frac{x^2+4x}{x^2-4x-12} \cdot \frac{x^2-9x+18}{2x}$

$\frac{\cancel{x}(x+4)}{(x-6)(x+2)} \cdot \frac{(x-3)(x-6)}{2\cancel{x}}$

$\boxed{\frac{(x+4)(x-3)}{2(x+2)}}$

Get Domain first

$D: \{x \mid x \neq 6, -2, 0\}$

2) $\frac{6x^2+18x}{x^2+x-6} \bullet \frac{x^2-x-2}{3x^2+x-2}$

$\frac{6x\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}\cancel{(x+1)}}{(3x-2)\cancel{(x+1)}}$

$D: \{x \mid x \neq -3, 2, \frac{2}{3}, -1\}$

$\boxed{\frac{6x}{3x-2}}$

You try! Simplify and state any restrictions on the domain.

a) $\frac{2x-7}{x^2-3x-4} \cdot \frac{x+1}{x^2-16}$

$\frac{2x-7}{(x-4)\cancel{(x+1)}} \cdot \frac{\cancel{x+1}}{(x+4)(x-4)}$

D: $\{x / x \neq \pm 4, -1\}$

$\boxed{\frac{2x-7}{(x+4)(x-4)^2}}$

b) $\frac{5x^2-28x-12}{x^2+11x+30} \bullet \frac{x^2-2x-35}{x+4}$

$\frac{(5x+2)(x-6)}{\cancel{(x+5)}(x+6)} \cdot \frac{(x-7)\cancel{(x+5)}}{x+4}$

D: $\{x / x \neq -5, -6, -4\}$

$\frac{(5x+2)(x-6)(x-7)}{(x+6)(x+4)}$

Dividing Rational Expressions:

1) **Take the** reciprocal **of the fraction after the division sign**

2) **Multiply** (same idea as the previous examples)

Examples: Simplify and state any restrictions on the domain.

4) $\frac{2x^2+x-3}{8x^2+12x} \div \frac{x^2-1}{2x+3}$ goes in Domain restrictions

$\frac{(2x-3)\cancel{(x+1)}}{4x\cancel{(2x+3)}} \cdot \frac{\cancel{2x+3}}{\cancel{(x+1)}(x-1)}$

D: $\{x / x \neq 0, \pm 1, -\frac{3}{2}\}$

$\boxed{\frac{2x-3}{4x(x-1)}}$

5) $\frac{x^2-25}{x^2+2x-3} \div \frac{x+5}{x^2-3x-18}$

$\frac{\cancel{(x+5)}(x-5)}{\cancel{(x+3)}(x-1)} \cdot \frac{(x-6)\cancel{(x+3)}}{\cancel{x+5}}$

D: $\{x / x \neq -3, 1; -5, 6\}$

$\boxed{\frac{(x-5)(x-6)}{(x-1)}}$

You try! Simplify and state any restrictions on the domain.

a) $\frac{x^2-9}{x^2-4x-12} \div \frac{x^2+2x-3}{2x^2-15x+18}$

$\frac{\cancel{(x+3)}(x-3)}{\cancel{(x-6)}(x+2)} \cdot \frac{(2x-3)\cancel{(x-6)}}{\cancel{(x+3)}(x-1)}$

D: {x / x ≠ 6, −2, −3, 1, $\frac{3}{2}$}

$\boxed{\frac{(x-3)(2x-3)}{(x+2)(x-1)}}$

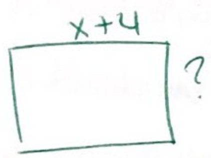
b) $\frac{3x^2-11x-4}{3x-27} \div \frac{x-4}{6}$

$\frac{(3x+1)\cancel{(x-4)}}{\cancel{3}(x-9)} \cdot \frac{\cancel{6}^{\,2}}{\cancel{x-4}}$

$\boxed{D: \{x / x \neq 9, 4\}}$

$\boxed{\frac{2(3x+1)}{x-9}}$

Example 6) The area of a rectangle is $x^2 + 13x + 36$ units squared and the height of the rectangle is x + 4 units. Write an expression to represent the base of the rectangle.



$(?)(x+4) = x^2+13x+36$

$\underline{(?)(x+4)} = (x+9)(x+4)$

so ? = (x+9)

7.4 Notes: Adding and Subtracting Rational Functions

Explain how you would simplify: $\frac{3}{3} \cdot \frac{2}{3} + \frac{1}{9}$

Common Denominator!

$\frac{6}{9} + \frac{1}{9} = \boxed{\frac{7}{9}}$

Explain how you would simplify: $\frac{2}{2} \cdot \frac{3}{2} - \frac{5}{4}$

$\frac{6}{4} - \frac{5}{4} = \boxed{\frac{1}{4}}$

Find a rational expression that when added to $\frac{x+1}{5x}$ would give a common denominator of $5x(x+3)$.

$\frac{x+2}{x+3}$

Adding/Subtracting Rational Expressions:

1) **Factor the** denominator **if possible**
2) **Get a** common **denominator**
3) **Add or subtract the numerators**
4) **Reduce**

Examples: Simplify and state where the domain is undefined.

1) $\frac{9}{x+7} + \frac{2}{x+7}$

$\frac{11}{x+7}$

$\{x \mid x \neq -7\}$

2) $\frac{2x}{x+6} - \frac{5}{x+6}$

Examples: Find the Least Common Multiple

3) $2x$ and $3x^2$

$3 \cdot 2x \cdot x \quad 3x\,x \cdot 2$

$\boxed{6x^2}$

4) $2x-2$ and x^2-1

$2(x-1)(x+1) \quad 2\cdot(x+1)(x-1)$

$\boxed{2(x+1)(x-1)}$

Examples: Simplify and state where the domain is undefined.

5) $\frac{3x}{3x}\cdot\frac{7}{2x} - \frac{4x+12}{3x^2}\cdot\frac{2}{2}$

$\frac{21x-8x-24}{6x^2} = \boxed{\frac{13x-24}{6x^2}}$

$\{x \mid x \neq 0\}$

6) $(x+1)\cdot\frac{x}{2x-2} - \frac{x}{x^2-1}\cdot 2$

$2(x-1) \quad (x+1)(x-1)$

$\frac{x^2+x-2x}{2(x-1)(x+1)}$

$\frac{x^2-x}{2(x-1)(x+1)}$

$\frac{x\cancel{(x-1)}}{2\cancel{(x-1)}(x+1)} = \boxed{\frac{x}{2(x+1)}}$

D: $\{x \mid x \neq \pm 1\}$

7) $(x-3)\cdot\dfrac{x+1}{x^2+6x+9} + \dfrac{6}{x^2-9}\cdot(x+3)$

$(x+3)(x+3) \quad (x+3)(x-3)$

$$\frac{(x-3)(x+1)+6(x+3)}{(x+3)(x+3)(x-3)}$$

$$\frac{x^2-2x-3+6x+18}{(x+3)(x+3)(x-3)}$$

$$\frac{x^2+4x+15}{(x+3)^2(x-3}$$ $\{x/x\neq\pm3\}$

5) $\dfrac{3}{x-4}\cdot(x+4) + \dfrac{2x}{x-4}\cdot(x+4) - \dfrac{11}{x^2-16}$

$(x+4)(x-4)$

$$\frac{3x+12+2x^2+8x-11}{(x+4)(x-4)}$$

$$\boxed{\frac{2x^2+11x+1}{(x+4)(x-4)}}$$

D: $\{x|x\neq\pm4\}$

You try! Simplify and state any restrictions on the domain.

a) $\dfrac{x}{x+6}\cdot(x-6) + \dfrac{-72}{x^2-36}$

$(x+6)(x-6)$

$$\frac{x^2-6x-72}{(x+6)(x-6)}$$

$$\frac{(x-12)\cancel{(x+6)}}{\cancel{(x+6)}(x-6)} = \boxed{\frac{x-12}{x-6}}$$

D: $\{x/x\neq\pm6\}$

b) $\dfrac{2}{x+6} + \dfrac{-3x+4}{x+6}$

$$\frac{-3x+6}{x+6}$$

$$\boxed{\frac{-3(x-2)}{x+6}}$$

$\{x/x\neq-6\}$

7.5 Notes: Solving Rational Equations

Strategies for solving rational equations:

1.) Cross-Multiply i.e. $\frac{2x}{5} = \frac{5}{6} \Rightarrow \frac{12x}{12} = \frac{25}{12}$ $\boxed{x = \frac{25}{12}}$

2.) Find a common Denominator i.e. $\frac{6}{6} \cdot \frac{2x}{5} = \frac{5}{6} \cdot \frac{5}{5}$

$\frac{12x}{30} = \frac{25}{30} \Rightarrow \boxed{x = \frac{25}{12}}$

Reminder: Rational expressions *cannot* equal zero on the denominator.
Any value of *x* that would give a zero on a denominator is a domain restriction.

Examples: Solve each equation for the variable. Include the restrictions on the domain.

1) $(x-1) \cdot \frac{x}{x+3} = \frac{6}{x-1}(x+3)$

$x \neq 1, -3$

$x^2 - x = 6x + 18$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$x = 9, -2$

check

2) $x+9 \cdot \frac{x-1}{x+4} = \frac{x+3}{x+9} \cdot x+4$

$x \neq -4, -9$

$x^2 + 9x - x - 9 = x^2 + 3x + 4x + 12$

$8x - 9 = 7x + 12$

$-7x + 9 \quad -7x + 9$

$x = 21$

check

4) $4 \cdot \frac{5}{x} + \frac{7}{4} \cdot x = \frac{-9}{x} \cdot 4$

$x \neq 0$

$\frac{20}{4x} + \frac{7x}{4x} = \frac{-36}{4x}$

$20 + 7x = -36$

$\frac{7x}{7} = \frac{-56}{7}$

$\boxed{x = -8}$

check

5) $(x-3) \cdot \frac{1}{x+3} - \frac{8}{x-3}(x+3) = \frac{2}{x-3}(x+3)$

$x \neq \pm 3$

$x - 3 - 8x - 24 = 2x + 6$

$-7x - 27 = 2x + 6$

$-33 = 9x$

$\boxed{x = \frac{-33}{9}}$

check

6) $(x-2) \cdot \frac{1}{x+1} - \frac{8}{x-2}(x+1) = \frac{2}{x^2-x-2}$

$(x-2)(x+1)$

$x \neq 2, -1$

$x - 2 - 8x - 8 = 2$

$-7x - 10 = 2$

$-7x = 12$

$\boxed{x = \frac{-12}{7}}$

check

7) $\frac{7x^2}{x^2-16} = \frac{3x}{x+4} \cdot (x-4) + \frac{4x}{x-4}$

$x = \pm 4$

$7x^2 = 3x^2 - 12x + 4x^2 + 16x$

$7x^2 = 7x^2 - 12x + 16x$

$0 = 4x$

$\boxed{0 = x}$

check

8) $(x-4)\frac{2x}{x+3} + \frac{3x}{x-4}(x+3) = \frac{5x^2-7x+2}{x^2-x-12}$

$(x+3)(x-4)$

$2x^2 - 8x + 3x^2 + 9x = 5x^2 - 7x + 2$

$x \neq -3, 4$

$5x^2 + x = 5x^2 - 7x + 2$

$\frac{8x}{8} = \frac{2}{8}$

$\boxed{x = \frac{1}{4}}$

9) $(x-2) \cdot \frac{x}{x+2} - \frac{8}{x^2-4} = \frac{2}{x-2} \cdot (x+2)$

$(x+2)(x-2)$

$x \neq \pm 2$

$x^2 - 2x - 8 = 2x + 4$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6, \cancel{-2}$

check

so

$\boxed{x = 6}$

6.3 Notes: Dividing Polynomials

Long Division

Divide using long division:

$473 \div 12$

$$\begin{array}{r} 39 \\ 12\overline{)473} \\ -36\downarrow \\ \hline 113 \\ 108 \\ \hline 5 \end{array}$$

$39\frac{5}{12}$

What steps did you do?

① Decide goes into
② multiply
③ subtract
④ Bring down Repeat
⑤ Find r → write as fraction

Divide $f(x) = x^3 + 3x^2 - 7$ by $x^2 - x - 2$

$$\begin{array}{r} x + 4 + \frac{6x+1}{x^2-x-2} \\ x^2-x-2\overline{)x^3+3x^2+0x-7} \\ -\cancel{x^3}+x^2+2x\downarrow \\ \hline 4\cancel{x^2}+2x-7 \\ -4\cancel{x^2}+4x+8 \\ \hline 6x+1 \end{array}$$

$\boxed{x+4+\frac{6x+1}{x^2-x-2}}$

1) Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$

$$\begin{array}{r} 3x^2+4x-3 + \frac{-25x+9}{x^2-3x+5} \\ x^2-3x+5\overline{)3\cancel{x^4}-5x^3+0x^2+4x-6} \\ -3\cancel{x^4}+9x^3+15x^2\downarrow \\ \hline 4\cancel{x^3}+15x^2+4x \\ -4\cancel{x^3}+12x^2-20x\downarrow \\ \hline -3x^2-16x-6 \\ +3x^2-9x+15 \\ \hline -25x+9 \end{array}$$

2) $(2x^3 + 10x^2 + 6x - 18) \div (2x + 6)$

$$\begin{array}{r} x^2+2x-3 \\ 2x+6\overline{)2x^3+10x^2+6x-18} \\ -\cancel{2x^3}-6x^2\downarrow \\ \hline 4x^2+6x \\ -4x^2-12x \\ \hline -6x-18 \\ +6x+18 \\ \hline 0 \end{array}$$

Write about it: Imagine that you had to get in front of the class and explain how to do long division with polynomials. Write a paragraph describing what you would say to the class to help them understand the process. List the steps

Synthetic Division

A shorthand method for dividing a polynomial by $x - a$ is called synthetic division. It is similar to long division, but you use only the coefficients.

3) Divide $(2x^3 + x^2 - 8x + 5)$ by $(x + 3)$

$x+3=0$

$x=-3$ root

$$
\begin{array}{r|rrrr}
-3 & 2 & 1 & -8 & 5 \\
 & & -6 & 15 & -21 \\
\hline
 & 2 & -5 & 7 & \boxed{-16}
\end{array}
$$

coefficients

Divide by power of 1 so each exponent goes down by 1

$2x^2 - 5x + 7 + \frac{-16}{x+3}$

4) Divide $(4x^3 - 3x + 7)$ by $(x - 1)$

$$
\begin{array}{r|rrrr}
1 & 4 & 0 & -3 & 7 \\
 & & 4 & 4 & 1 \\
\hline
 & 4 & 4 & 1 & \boxed{8}
\end{array}
$$

$4x^2 + 4x + 1 + \frac{8}{x-1}$

Factor Theorem: a polynomial $f(x)$ has a factor $x - a$ if and only if $f(a)=0$ (REMAINDER = 0)

* check for missing terms

5) Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

$$
\begin{array}{r|rrrr}
-2 & 3 & -4 & -28 & -16 \\
 & & -6 & 20 & 16 \\
\hline
 & 3 & -10 & -8 & \boxed{0}
\end{array}
$$

$3x^2 - 10x - 8$

$(3x+2)(x-4)(x+2)$

**Because you know $x+2$ is a factor, you know that $f(-2) = 0$ or that $x = -2$ is a root. Use synthetic division to find the other factors.*

no remainder means its a factor/root

6) Factor the polynomial completely given that $f(x) = x^3 - 6x^2 + 5x + 12$ and that $x - 4$ is a factor.

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & \boxed{0} \end{array}$$

$(x-3)(x+1)(x-4)$

7) Find the other zeros of f given that $f(2) = 0$ and $f(x) = x^3 - x^2 - 22x + 40$

zero not factor

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -22 & 40 \\ & & 2 & 2 & -40 \\ \hline & 1 & 1 & -20 & \boxed{0} \end{array}$$

-5 & 4

$(x+5)(x-4)$

8) Find the other solutions of f given that $x = -7$ is a root and $f(x) = x^3 + 8x^2 + 5x - 14$

$(x+7)$ is the factor

$$\begin{array}{r|rrrr} -7 & 1 & 8 & 5 & -14 \\ & & -7 & -7 & 14 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$(x+2)(x-1)(x+7)$

$-2 \quad 1$

6.4 Notes: Rational Root Theorem

The Fundamental Theorem of Algebra: Any polynomial of degree **n** has at most **n** roots, both real and complex.

1) How many x-intercepts does the following function have? $f(x) = 7x^5 - 4x^2 + 1$

Degree 5: 5 or less ex: $(x+3)^3(x-1)(x+2)$

or imaginary roots also

2) With a partner try to solve (find the roots of) the polynomial, $x^3 + 7x^2 + 15x + 9 = 0$.

4 terms: Try grouping → doesn't work

up to 3 roots so could be 3 factors ( )( )( )

Can it be factored? Any other ways to solve it?

If we know <u>possible roots</u> we can use synthetic division to tell if they are <u>actual roots</u>! So how do we find a possible root?

The Rational Zeros Theorem: If $f(x)$ is a polynomial with integer coefficients and if $\frac{p}{q}$ is a zero of $f(x)$, then p is a factor of the constant term of $f(x)$ and q is a factor of the leading coefficient of $f(x)$.

3) Make a list of all possible rational zeros of $f(x)$ given below.

$1x^3 + 7x^2 + 15x + 9 = 0$

leading coeff q

constant term p

$\frac{\pm p}{q} = \frac{\pm 1, 3, 9}{\pm 1}$

Steps for finding possible roots:
1. Write down all the factors of the constant term (p)

Plug in or use synthetic

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 15 & 9 \\ & & -3 & -12 & -9 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$(x+3)^2(x+1) = 0$ -3 & -1

$f(x) = 2x^4 + x^3 - 19x^2 - 9x + 9$

q   p

$\frac{\pm 1, 3, 9}{\pm 1, 2}$

$\pm 1, \frac{1}{2}, 3, \frac{3}{2}, 9, 9/2$

2. Write down all the factors of the leading coefficient (q)	
3. Write down all the possible value of $\frac{p}{q}$. Remember to include both positive and negative factors.	
4. Remove any duplicate values.	

4) Make a list of all possible roots for the function, $f(x) = x^3 + x^2 - 8x - 12$.

$\frac{\pm 1, 2, 3, 4, 6, 12}{\pm 1} \Rightarrow \pm 1, 2, 3, 4, 6, 12$

Now use synthetic division to see which possible roots are actual solutions to $f(x)$. If it is a solution the remainder needs to be equal to 0.

Try: 1 $1+1-8-12 \neq 0$

Try: -1 $-1+1+8-12 \neq 0$

Try: 2 $8+4-16-12 \neq 0$

Try: -2 $-8+4+16-12 = 0$ ✓

So -2 is a root

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -12 \\ & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$(x-3)(x+2)$

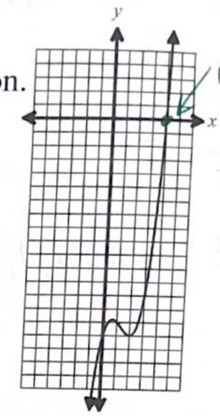
$\boxed{3 \quad -2}$

5) Which of the following are not possible solutions to the function below. Choose all that apply! $f(x) = 2x^4 + 5x^3 - 5x^2 - 5x + 3$

$\frac{\pm 1, 3}{\pm 1, 2}$

A. 2   B. $-\frac{3}{2}$ C. 1   D. $\frac{1}{2}$ E. $-\frac{2}{3}$ F. 3   G. $\frac{1}{3}$

6) The equation $x^3 - 4x^2 + 4x - 16 = 0$ is graphed to the right. Use the graph to help solve the equation and find all the roots of the function.



$$\begin{array}{r|rrrr} 4 & 1 & -4 & 4 & -16 \\ & & 4 & 0 & 16 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array}$$

x^2+4 — imaginary roots

$(x+2i)(x-2i)$

$\pm 2i$

7) a. Find all possible roots of the function, $g(x) = 2x^4 - 3x^3 + 7x^2 + 12x$.

$x(2x^3 - 3x^2 + 7x + 12)$

q p

$\downarrow$

0 $\quad \dfrac{\pm 1, 2, 3, 4, 6, 12}{\pm 1, 2}$ $\qquad \pm 1, \frac{1}{2}, 2, 3, \frac{3}{2}, 4, 6, 12$

b. Use the possible roots and synthetic division to find the solutions.

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 7 & 12 \\ & & -2 & 5 & -12 \\ \hline & 2 & -5 & 12 & \boxed{0} \end{array}$$

* on Final how can calculator help you?

* 4 solutions
- 2 real
- 2 imaginary

$g(x) = x(x+1)(2x^2-5x+12)$ use quadratic

$\downarrow$ 0 $\quad \downarrow$ -1

$$\frac{5 \pm \sqrt{25-4(24)}}{4} = \frac{5 \pm \sqrt{-71}}{4} = \frac{5 \pm i\sqrt{71}}{4}$$

7.6 Notes: Inverse Variation and the Reciprocal Function

Inverse Variation:

Inverse variation is a relation between two variables such that as one variable increases, the other decreases proportionally. ($xy = k$)

Ex 1) In an inverse variation, $x = 6$ and $y = \frac{1}{2}$. What is the value of y, when $x = 15$.

$6(\frac{1}{2}) = 3$ $k = 3$ $\frac{15y}{15} = \frac{3}{15}$ $\boxed{y = \frac{1}{5}}$

Ex 2) If x and y vary inversely and $x = 4$ when $y = 32$. What is the value of x when $y = 16$.

$4 \cdot 32 = k$
$128 = k$

$xy = k$
$16x = 128$

$\boxed{x = 8}$

Ex 3) Determine if the following table of values represent inverse variation?

a)

x	1	2	3	4	6	12
y	12	6	4	3	2	1

12 12 12

yes

$k = 12$

b)

x	1	2	3	4	5	6
y	20	17	14	11	8	5

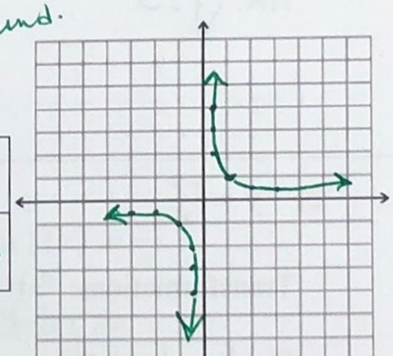
20 34

NO

Parent Function for Rational Functions: $f(x) = \frac{1}{x}$ $= \frac{1}{0} = $ und.

Graph by using a table of values.

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	-3	-4	∅	4	3	2	1	1/2	1/3



Key Features

Domain: $\{x \mid x \neq 0\}$

Range: $\{y \neq 0\}$

Vertical Asymptote (VA): $x = 0$

Horizontal Asymptote (HA): $y = 0$

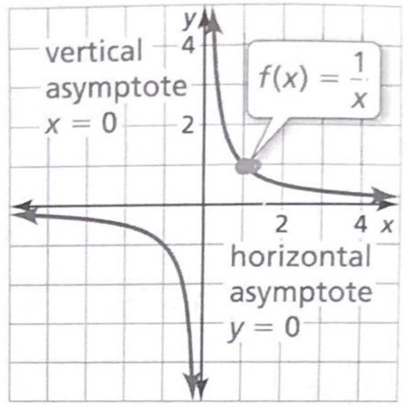
End Behavior: as $x \to \infty, f(x) \to$ 0
as $x \to -\infty, f(x) \to$ 0

Graphing the Simple Rational function:

$$y = \frac{a}{x}$$

The graph of the parent function $f(x) = \frac{1}{x}$ is a hyperbola, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.

Any function of the form $f(x) = \frac{1}{x}$ (a ≠ 0) has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.



Examples: Graph the following and state the transformation from the parent function $f(x) = \frac{1}{x}$, the domain, range (in set notation), and the vertical and horizontal asymptotes.

1) $y = \frac{4}{x}$

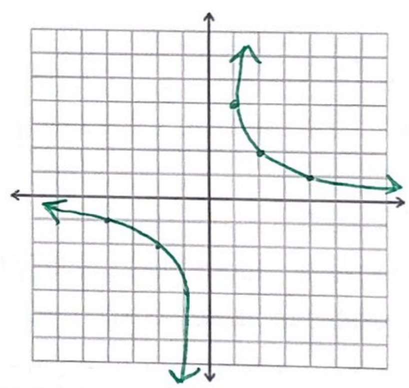
Transformations: Stretch 4

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y \neq 0\}$

VA: $x=0$

HA: $y=0$



2) $y = \frac{-3}{x}$

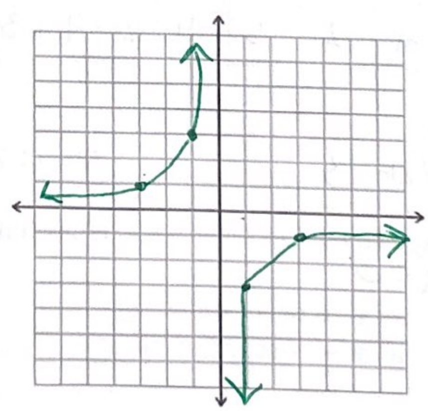
Transformations: Stretch 3 reflect

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y \neq 0\}$

VA: $x=0$

HA: $y=0$



Translations of Simple Rational Functions

$$y = \frac{a}{x - h} + k$$ ↑↓ keep

← → opp

Example 3: Translate the graph of $f(x) = \frac{1}{x}$ to the left 5 units and up one unit. Write the equation of the function after the translation.

$f(x) = \frac{1}{x+5} + 1$

Example 4: Identify the transformations from the parent function $f(x) = \frac{1}{x}$

a. $f(x) = \frac{1}{x-6} + 2$

→ 6 ↑ 2

b. $f(x) = \frac{-1}{x+2} - 4$

reflect

← 2 ↓ 4

c. $f(x) = \frac{5}{x} + 1$

stretch 5

↑ 1

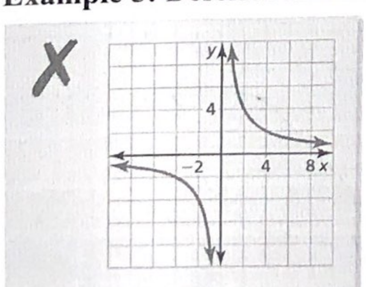
d. $f(x) = \frac{-2}{x-3}$

reflect

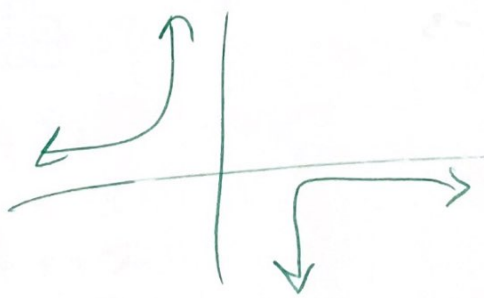
stretch 2

→ 3

Example 5: Describe and correct the error in graphing the function $f(x) = \frac{-8}{x}$



Didn't reflect



7.7: Graphing Rational Functions in the Form $y = \frac{a}{x-h} + k$

stretch/compress

changes VA: ← → opp
changes D:

keep ↑↓
moves HA
changes Rang

Examples: Graph the following and state the transformation from the parent function $f(x) = \frac{1}{x}$, the domain, range (in set notation), and the vertical and horizontal asymptotes.

1) $y = \frac{4}{x-3} + 2$ (3, 2)

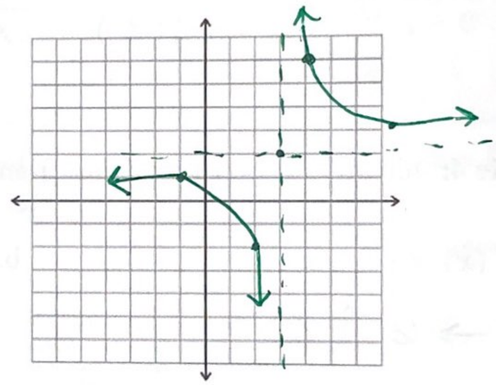
Transformations: → 3, ↑ 2, stretched 4

D: {x | x ≠ 3}

R: {y | y ≠ 2}

VA: x = 3

HA: y = 2



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2) $y = \frac{-6}{x+1} - 3$ (-1, -3)

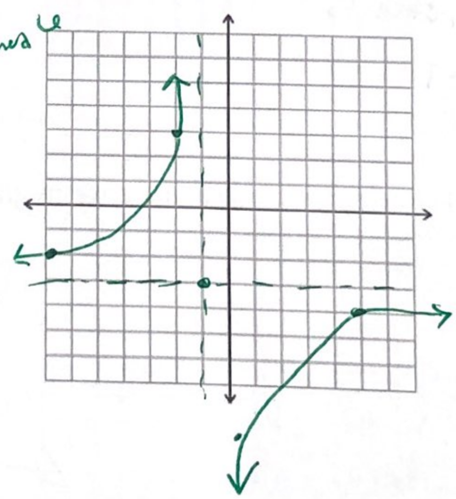
Transformations: ← 1, ↓ 3, Reflected, stretched 6

D: {x | x ≠ -1}

R: {y | y ≠ -3}

VA: x = -1

HA: y = -3



Changing Rational Expressions of the form $y = \frac{ax+b}{cx+d}$ to Graphing Form $y = \frac{a}{x-h} + k$

Stretch/comp.

Center @ (h,k)

Example 4: How could you change $f(x) = \frac{5x-7}{x-4}$ to graphing form?

Use Synthetic or long division

Write the following rational expression in graphing form: $f(x) = \frac{5x-7}{x-4}$. Then identify the HA and VA.

$5 + \frac{13}{x-4}$

$x-4 \enclose{longdiv}{5x-7}$

$\underline{-5x+20}$

13

$\frac{13}{x-4} + 5$

(4,5)

VA: x=4

HA: y=5

Example 5: Write the following rational expression in graphing form: $f(x) = \frac{-2x+1}{x+5}$. Then identify the HA and VA.

$$\begin{array}{r|rr} -5 & -2 & 1 \\ & & 10 \\ \hline & -2 & \boxed{11} \end{array}$$

$-2 + \frac{11}{x+5}$

or

$\frac{11}{x+5} - 2$

(-5,-2)

VA: x=-5

HA: y=-2

Example 6: Which is the graph of $f(x) = \frac{5x-8}{x-3}$?

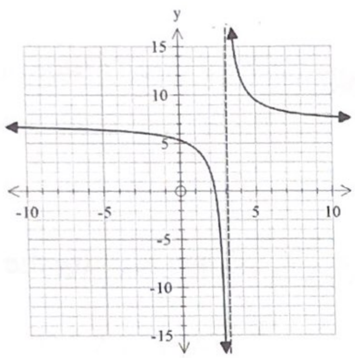
$\rightarrow 3 \uparrow 5$

$3\,|\; 5 \quad -8$
$\quad\quad\;\; 15$
$\quad\; 5 \quad \boxed{7}$

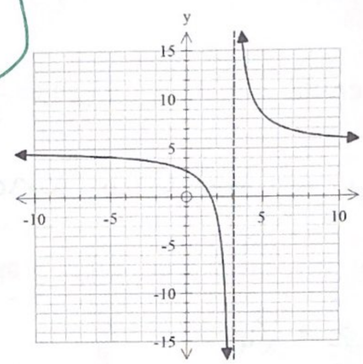
$\frac{7}{x-3} + 5$

$(3,5)$

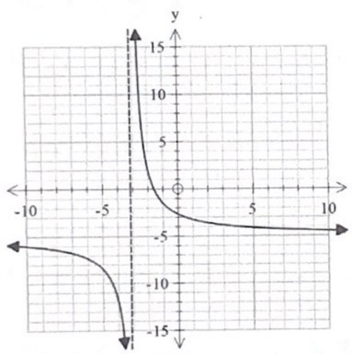
A)



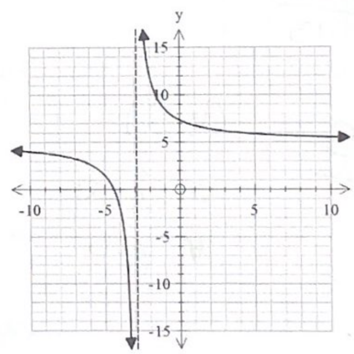
B)



C)



D)



Example 7: Find the end behavior of $y = \frac{3x+5}{x+4}$.

$-4\,|\; 3 \quad 5$
$\quad\quad\;\; -12$
$\quad\; 3 \quad -7$

$\frac{-7}{x+4} + 3 \quad (-4, 3) \quad HA: y = 3$

as $x \to \pm\infty$ $y \to 3$