**Chapter 8 Calendar** **Name:** ____________________

| Day | Date | Assignment (Due the next class meeting) |
|---|---|---|
| Wednesday<br>Thursday | 2/16/22<br>2/17/22 | **8 Day 1 Notes: Compositions**<br>HW: 8 Day 1 Worksheet |
| Friday<br>Tuesday | 2/18/22<br>2/22/22 | **8 Day 2 Notes: Inverses of Linear and Quadratic Functions**<br>HW: 8 Day 2 Worksheet |
| Wednesday<br>Thursday | 2/23/22<br>2/24/22 | **8 Day 3 Notes: Simplifying Radicals**<br>HW: 8 Day 3 Worksheet |
| Friday<br>Monday | 2/25/22<br>2/28/22 | **8 Day 4 Notes: Solving Radical Equations**<br>HW: 8 Day 4 Worksheet |
| Tuesday<br>Wednesday | 3/1/22<br>3/2/22 | **8 Day 5 Notes: Graphing Square and Cube Root Functions**<br>HW: 8 Day 5 Worksheet |
| Thursday<br>Friday | 3/3/22<br>3/4/22 | **Review Chapter 8**<br>HW: Practice Test |
| Monday<br>Tuesday | 3/7/22<br>3/8/22 | **Chapter 8 Extra Review**<br>HW: Extra Review |
| Wednesday<br>Thursday | 3/9/22<br>3/10/22 | **Ch 8 Test** |

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Try www.khanacademy.org or www.mathguy.us if you need help outside of school hours.
- Students who complete 100% of the assignments for the semester will receive a 2% bonus.

# 8 Day 1 Notes: Composition of Functions

Work with a partner to perform the indicated operations given $f(x) = 2x^2 - x + 11$, $g(x) = 7x - 9, and\ h(x) = -x^2 + 6$.

a) $f(x) + h(x)$

$(2x^2 - x + 11) + (-x^2 + 6)$

$\boxed{x^2 - x + 17}$

c) $g(x) - f(x)$

$(7x - 9) + (-2x^2 + x - 11)$

$\boxed{-2x^2 + 8x - 20}$

b) $g(x) \cdot h(x)$

$(7x - 9)(-x^2 + 6)$

$-7x^3 + 42x + 9x^2 - 54$

$-7x^3 + 9x^2 + 42x - 54$

Functional Notation:

$f(x) = \text{rule}$

f(2) means "what y is when x is 2"

f(6) = 3 means y is 3 when x is 6

What does the word "composition" mean?

Put things together

Use the following worked-out examples as a model to help you with the following examples:

**Example:** Find $f(3)$ if $f(x) = 8x - 1$.

$= 8(3) - 1$

$= 24 - 1$

so $f(3) = 23$

Try the following problems with a partner:

1) Find $g(-2)$ if $g(x) = -7x + 9$.

$g(-2) = -7(-2) + 9$

$= +14 + 9$

$g(-2) = 23$

2) Find $h(5)$ if $h(x) = -2x^2 + 23$.

$h(5) = -2(5)^2 + 23$

$= -2(25) + 23$

$= -50 + 23$

$= -27$

3) Find $d(-8)$ if $d(x) = \frac{3}{4}x + 6$.

$d(-8) = \frac{3}{4}(-8) + 6$

$= -6 + 6$

$= 0$

**Example:** Solve for $x$ if $f(x) = 5x - 3$ and $f(x) = 17$.

$17 = 5x - 3$

$20 = 5x$

so $4 = x$

Try the following problems with a partner:

1) Solve for $x$ if $g(x) = 2x + 9$ and $g(x) = -33$

$-33 = 2x + 9$

$-9 \quad -9$

$\frac{-42}{2} = \frac{2x}{2}$ $x = -21$

2) Solve for $x$ if $h(x) = -2x - 3$ and $h(x) = 6$

$6 = -2x - 3$

$9 = -2x$

$x = -9/2$

3) Solve for $x$ if $d(x) = x^2 + 7$ and $d(x) = 32$

$32 = x^2 + 7$

$-7 \quad -7$

$\sqrt{25} = \sqrt{x^2}$

$x = \pm 5$

**More examples:** If $f(x) = 6x + 18$, $g(x) = 9x - 1$, and $h(x) = -3x + 7$, then find the following compositions.

**Example 4:** Find $f(g(x))$.

$f(g(x)) = 6(9x-1)+18$
$= 54x - 6 + 18$
$= 54x + 12$

**Example 5:** Find $g(f(x))$.

$g(f(x)) = 9(6x+18)-1$
$= 54x + 162 - 1$
$= 54x + 161$

**Example 6:** Find $h(g(x))$.

$h(g(x)) = -3(9x-1)+7$
$= -27x + 3 + 7$
$= -27x + 10$

**Example 7:** Find $h(h(x))$.

$h(h(x)) = -3(-3x+7)+7$
$= 9x - 21 + 7$
$= 9x - 14$

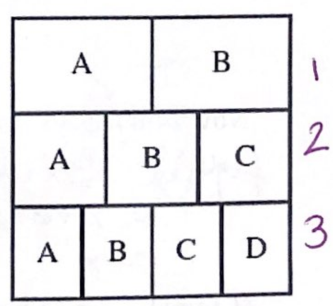
**Example 8:** Find $f(g(-6))$.

$g(-6) = 9(-6)-1$
$= -54-1$
$= -55$

$f(g(-6)) = 6(-55)+18$
$= -330+18$
$= -312$

*Your notes on composition of functions:*

**Example 9:** The square below is divided into 3 rows of equal area. In the top row, the region labeled A has the same area as the region labeled B. In the middle row, the 3 regions have equal areas. In the bottom row, the 4 regions have equal areas. What fraction of the square's area is in a region labeled A?




row 1 $= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

row 2 $= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

row 3 $= \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

$\frac{1}{6} + \frac{1}{9} + \frac{1}{12}$ LCD = 36

$\frac{6}{36} + \frac{4}{36} + \frac{3}{36}$

$\boxed{\frac{13}{36}}$

# 8 Day 2 Notes: Inverses of Linear and Quadratic Functions

What do you think the word "inverse" means in math? Hint: What is the inverse of adding? What is the inverse of dividing?

- Opposite operation
- to "undo" something

**Exploration**: Complete the following input/output tables for the given linear functions. What do you notice?

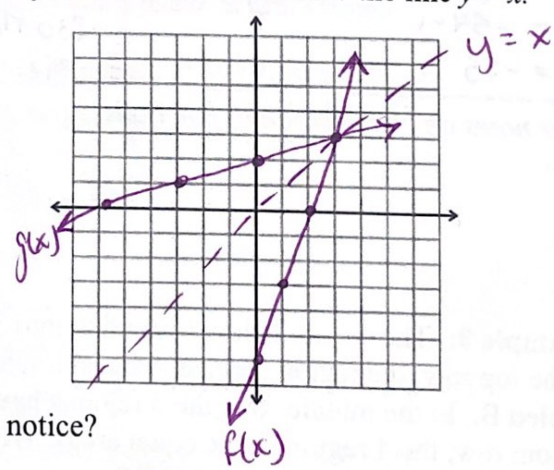
Function A: $f(x) = 3x - 6$

| $x$ | $f(x)$ |
|---|---|
| 3 | $3(3)-6$ <br> 3 |
| 2 | 0 |
| 1 | -3 |
| 0 | -6 |

Function B: $g(x) = \frac{1}{3}x + 2$

| $x$ | $g(x)$ |
|---|---|
| 3 | $\frac{1}{3}(3)+2$ <br> 3 |
| 0 | 2 |
| -3 | 1 |
| -6 | 0 |

Now graph each function on the coordinate system below. Then draw the line $y = x$. What do you notice?




Now find $f(g(x))$ and $g(f(x))$. What do you notice?

$f(g(x)) = 3(\frac{1}{3}x + 2) - 6$
$= x + 6 - 6$
$= x$

$g(f(x)) = \frac{1}{3}(3x - 6) + 2$
$= x - 2 + 2$
$= x$

Both equal x
∴ symmetrical over $y = x$

**Properties of Inverse Functions:**

1.) Symmetrical about line y = x

2.) $f(g(x)) = g(f(x)) = x$

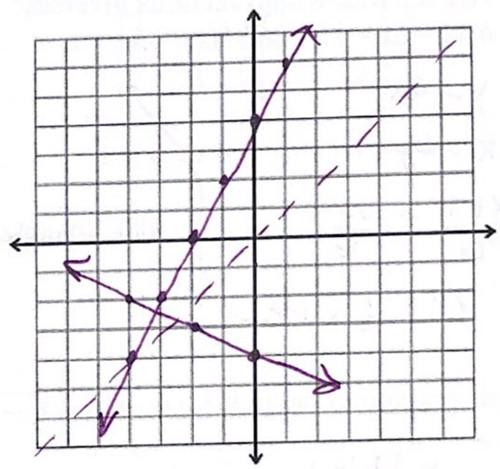
3.) x, y coordinates switches

4.) switch x & y and solve for y

Example 1: Graph the following lines on the same coordinate plane. Are they inverse functions? How do you know?

$y = 2x + 4$ and $y = -\frac{1}{2}x - 4$

No, not Symmetrical about y = x.



**How to find the inverse ($y^{-1}$) of a function:**

- Switch x & y
- solve for y

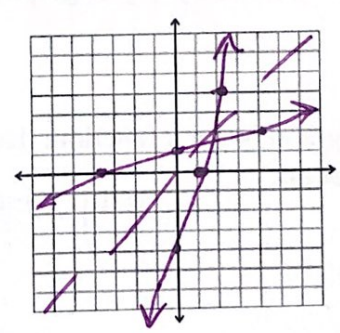
**Examples: Find the inverse of each linear function. Then graph both functions on the same coordinate plane.**

2) $y = 4x - 4$

$x = 4y - 4$

$\frac{x+4}{4} = \frac{4y}{4}$

$y = \frac{1}{4}x + 1$



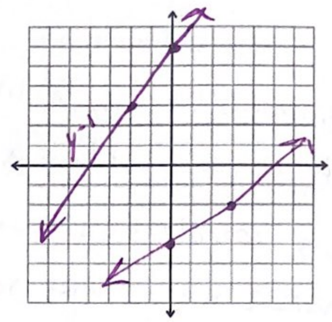
3) $g(x) = \frac{2}{3}x - 4$

$y^{-1} = \frac{3}{2}x + 6$

$y = \frac{2}{3}x - 4$

$x = \frac{2}{3}y - 4$

$\frac{3}{2}(x+4) = \frac{2}{3}y \cdot \frac{3}{2}$




4) Are the following functions inverses? Explain your reasoning.
$h(x) = 6x - 1$ and $k(x) = 6x + 1$

$y = 6x - 1$

$x = 6y - 1$

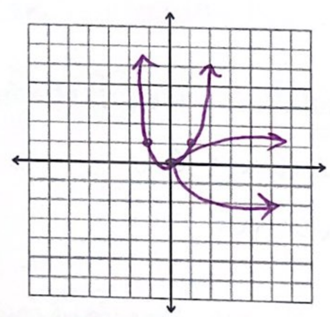
$\frac{x+1}{6} = \frac{6y}{6}$

$y^{-1} = \frac{1}{6}x + \frac{1}{6}$

not inverse bc not =

Now, graph the function $h(x) = x^2$

| x | h(x) |
|---|---|
| -1 | 1 |
| 0 | 0 |
| +1 | 1 |



Switch the input and output values from the graph above and graph the new ordered pairs on the same grid. What did you just graph?

(-1, 1) (1, -1)
(0, 0) (0, 0)
(1, 1) (1, 1)

This new graph is not a function. How can you limit the domain of $h(x)$ so that the inverse is also a function?

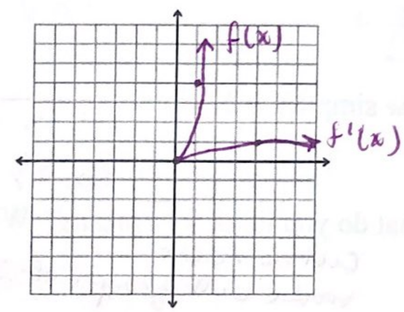
only take the top half, so $x \geq 0$

The inverse of a quadratic function is a Square root function.

Example 6) Find the inverse of $f(x) = 4x^2$ if $x \geq 0$. Then graph both functions on the same grid.

$$\frac{x}{4} = \frac{4y^2}{4}$$

$$y^2 = \sqrt{\frac{x}{4}} = \frac{\sqrt{x}}{2}$$




Example 7) Find the inverse of $y = x^2 - 8$ if $x \geq 0$.

$$x = y^2 - 8$$
$$\sqrt{x+8} = \sqrt{y^2}$$
$$y = \sqrt{x+8}$$

Example 8) Determine if $f(x) = x^2 + 3$ and $g(x) = \sqrt{x-3}$ are inverses if $x \geq 0$. Explain.

$$x = y^2 + 3$$
$$\sqrt{x-3} = \sqrt{y^2}$$
$$y = \sqrt{x-3}$$

Same so yes

could also do:

$$f(g(x)) = (\sqrt{x-3})^2 + 3$$
$$= x - 3 + 3$$
$$= x$$
$$g(f(x)) = \sqrt{(x^2+3)-3}$$
$$g(f(x)) = \sqrt{x^2}$$
$$= x$$

Example 9) Write a function.

$$y = x^2 + 7$$

Now, find its inverse.

$$x = y^2 + 7$$
$$\sqrt{x-7} = \sqrt{y^2}$$
$$y = \sqrt{x-7}$$

Finally, is the inverse a function? Explain your reasoning.

No, but if $x \geq 0$ then yes

## 8 Day 3 Notes: Simplifying Radicals

Work with a partner to simplify $\sqrt{200}$.

$\sqrt{2\cdot 100}$

$\sqrt{2\cdot 10\cdot 10}$

$10\sqrt{2}$

$\sqrt{200}$ → 20 · 10 → 5 4, 5 2 → 2 2

$10\sqrt{2}$

Now simplify $\sqrt{27x^7y^2}$

$\sqrt{3\cdot 3\cdot 3\,xxxxxxx\,yy}$

$3x^3y\sqrt{3x}$

What do you think $\sqrt[3]{\ }$ means? What about $\sqrt[4]{\ }$ ?

cube root
take out groups of 3

4th root
take out groups of 4.

$n^{th}$ root:

$\sqrt[n]{\ }$

index of a radical:

$\sqrt[n]{\ }$

Another way to write a radical is with a rational exponent.

$\sqrt[n]{x^1} = x^{\frac{1}{n}}$ so we can write $\sqrt[2]{64}$ as $64^{1/2}$ or $8^{\frac{1}{3}}$ as $\sqrt[3]{8}$

**Examples: Simplify.**

1) $50^{\frac{1}{2}}$

$\sqrt{50}$

$\sqrt{2\cdot 25}$ $\quad 5\sqrt{2}$

$\sqrt{2\cdot 5\cdot 5}$

2) $(-20x^2y)^{\frac{1}{2}}$

$\sqrt{-1\cdot 2\cdot 2\cdot 5\cdot xxy}$

$2xi\sqrt{5y}$

3) $\sqrt[3]{27a^3b^7}$

$\sqrt[3]{3\cdot 3\cdot 3a^3b^7}$ so $a^3 = 3/3 = 1$

$b^7 = 7/3 = 2r1$

$3ab^2\sqrt[3]{b}$

4) $(-32xy^8z^{10})^{\frac{1}{3}}$

32 → 2 16 → 2 8 → 2 2 2

$\sqrt[3]{-32xy^8z^{10}}$

$\sqrt[3]{-1\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\,xy^8z^{10}}$

$-2y^2z^3\sqrt[3]{4xy^2z}$

5) $\sqrt[4]{625x^{48}y^{36}z^{72}}$

$\sqrt[4]{5\cdot 5\cdot 5\cdot 5x^{48}y^{36}z^{72}}$

$5x^{12}y^{9}z^{18}$

625 → 5, 125 → 5, 25 → 5, 5

$48/4=12$

$36/4=9$

$72/4=18$

6) $2\sqrt[3]{-64a^5}$

$2\sqrt[3]{-2^6a^5}$

$-8a\sqrt[3]{a^2}$

64 → 2, 32 → 2, 16 → 2, 8 → 2, 2, 2

7) $-3\sqrt[4]{162e^{12x}}$

$-3\sqrt[4]{2\cdot 3\cdot 3\cdot 3\cdot 3e^{12x}}$

$-9e^{3x}\sqrt{2}$

162 → 2, 81 → 9, 9 → 3, 3, 3, 3

**You try these!**

a) $\sqrt[3]{256x^4y^6}$

$\sqrt[3]{2^8x^4y^6}$

$2x^2y^2\sqrt{2^2x}$

$4xy^2\sqrt[3]{4x}$

256 → 2, 128 → 2, 64 → 8, 8

b) $\sqrt[5]{-32a^{20}b^{17}c^9}$

$\sqrt[5]{-2a^{20}b^{18}c^9}$

$-2a^4b^3c\sqrt[5]{b^5c^4}$

32 → 2, 16 → 2, 8 → 2, 2, 2

c) $\sqrt[3]{-16a^{13}}$

$\boxed{-2a^4\sqrt[3]{2a}}$

16 → 2, 8 → 2, 2, 2

d) $2(32x^7y^9)^{\frac{1}{2}}$

$2\sqrt{32x^7y^9}$

$2\sqrt{2^5x^7y^9}$

$2.2^2x^3y^4\sqrt{2xy}$

$8x^3y^4\sqrt{2xy}$

e) Create a radical that could be simplified to $2e^{3x}\sqrt[3]{2e^{2x}}$

$\boxed{\sqrt{16e^{11x}}}$

**Adding and Subtracting with Radicals**

With a partner to simplify $\sqrt{6} + 3\sqrt{6}$. Check your answer with a calculator.

outsides w/outsides

insides w/insides

$4\sqrt{6}$

**We can add or subtract radicals as long as they have the same radicand.**

**Examples: Simplify.**

8) $3\sqrt{8} - 4\sqrt{2}$

$3\sqrt{2\cdot 2\cdot 2} - 4\sqrt{2}$

$6\sqrt{2} - 4\sqrt{2}$

$2\sqrt{2}$

9) $10\sqrt[3]{y} - 6\sqrt[3]{y}$

$4\sqrt[3]{y}$

10) $\sqrt[4]{48} - \sqrt[4]{3}$

48
4 12
4 3

$4\sqrt[4]{3} - \sqrt[4]{3}$

$3\sqrt[4]{3}$

11) $7\sqrt[3]{2a^5} - a\sqrt[3]{16a^2}$

16
2 8
2 2 2

$7a\sqrt[3]{2a^2} \cdot 2a\sqrt[3]{2a^2}$

$5a\sqrt[3]{2a^2}$

**You try these!**

a) $3\sqrt[3]{8} + \sqrt[3]{108} - 2\sqrt[3]{32}$

$6 + 3\sqrt[3]{4} - 4\sqrt[3]{4}$

$6 - \sqrt[3]{4}$

108
2 54
2 27
3 9
3 3

b) $7x^5y^{\frac{1}{2}} + 13y^{\frac{1}{2}}x^5$

32
2 16
2 8
2 2 2

$7x^5\sqrt{y} + 13x^5\sqrt{y}$

$20x^5\sqrt{y}$

12) As part of a probability experiment, Elliot is to answer 4 multiple-choice questions. For each question, there are 3 possible answers, only 1 of which is correct. If Elliot randomly and independently answers each question, what is the probability that he will answer 4 questions correctly?

$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{256}$

# 8 Day 4 Notes: Solving Radical Equations

**Examples: Solve the following equations. Check for extraneous solutions.**

Step 1: Isolate the $\sqrt{\phantom{x}}$

Step 2: (square)2 the $\sqrt{\phantom{x}}$

Step 3: Isolate/solve x.

**Work with a partner to solve the following equations for $x$:**

1) $(\sqrt{3x+2})^2 = 3^2$

$3x+2=9$

$\frac{3x}{3} = \frac{7}{3}$

$x = 7/3$

2) $\sqrt{x+6}^2 = \sqrt{2x-3}^2$

$x+6 = 2x-3$

$-x \quad -x$

$6 = x-3$

$+3 \quad +3$

$x = 9$

**Examples: Solve the following equations. Check for extraneous solutions.**

3) $(\sqrt[3]{12x})^3 = 6^3$

$\frac{12x}{12} = \frac{216}{12}$

$\boxed{x = 18}$

check

$\sqrt[3]{12 \cdot 18} = 6$

$\sqrt[3]{216} = 6$

$6 = 6$

4) $2\sqrt{6x-7} + 14 = 4$

$\frac{2\sqrt{6x-7}}{2} = \frac{-10}{2}$

$\sqrt{6x-7} = -5$ NO solution

5) $(x-2)^2 = \sqrt{x+10}^2$

$(x-2)(x-2) = x+10$

$x^2-2x-2x+4 = x+10$

$x^2-4x+4 = x+10$

$-x \quad -10 \quad -x-10$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x = 6, \cancel{-1}$

check $x=6$

$6-2 = \sqrt{6+10}$

$4 = \sqrt{16}$

$4 = 4$

$x=-1$

$-1-2 = \sqrt{-1+10}$

$-3 = \sqrt{9}$

$-3 \neq 3$

NO Solution

Check example 5 by graphing it in your graphing calculator. Graph $f(x) = x - 2$ and $g(x) = \sqrt{x+10}$ on the same screen. What do you notice?

**You try! Solve the following equations. Check for extraneous solutions.**

a) $\sqrt[3]{x-4} + 3 = -1$

$-3 \quad -3$

$\left(\sqrt[3]{x-4}\right)^3 = (-4)^3$

$x - 4 = -64$

$+4 \quad +4$

$x = -60$

check: $\sqrt[3]{-60-4} + 3 = -1$

$\sqrt[3]{-64} + 3 = -1$

$-4 + 3 = -1$

$-1 = -1$

b) $\sqrt{x-3} + 5 = x$

$-5 \quad -5$

$\left(\sqrt{x-3}\right)^2 = (x-5)^2$

$x - 3 = x^2 - 5x - 5x + 25$

$x - 3 = x^2 - 10x + 25$

$-x + 3 \quad -x + 3$

$0 = x^2 - 11x + 28$

$0 = (x-4)(x-7)$

check (x=4): $\sqrt{4-3} + 5 = 4$

$\sqrt{1} + 5 = 4$

$1 + 5 = 4$

$6 \neq 4$

check (x=7): $\sqrt{7-3} + 5 = 7$

$\sqrt{4} + 5 = 7$

$2 + 5 = 7$

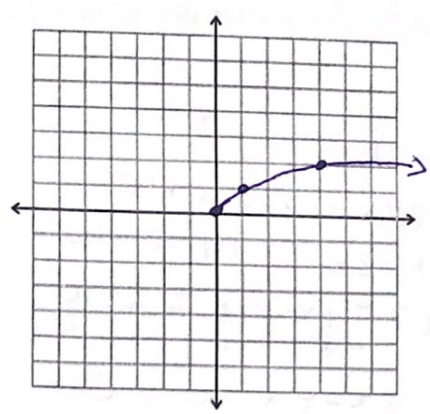
$7 = 7$

## 8 Day 5 Notes: Graphs of Square and Cube Root Functions

Use a table of values to graph the parent radical function: $y = \sqrt{x}$

| x | y |
|---|---|
| 0 | $\sqrt{0} = 0$ |
| 1 | $\sqrt{1} = 1$ |
| 4 | $\sqrt{4} = 2$ |

Identify the following key features:



Endpoint: (0,0)

x-value

Domain: [0, ∞)

y-value

Range: [0, ∞)

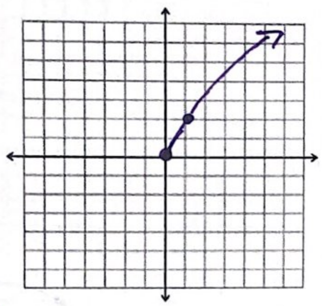
End Behavior: as $x \to \infty$ $y \to \infty$

**Examples: For each radical function $y = a\sqrt{x-h}+k$, describe the transformation from the parent function $y=\sqrt{x}$, identify the domain and range, sketch the graph, and identify the end behavior.**

1) $y = 2\sqrt{x}$

(0,0)

stretched by 2

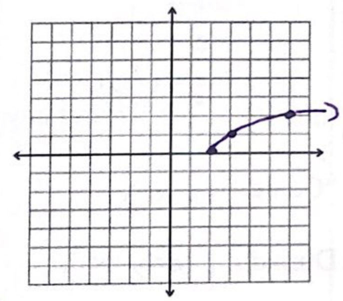


Domain: [0, ∞)
Range: [0, ∞)
End Behavior: x → ∞ y → ∞

2) $y = \sqrt{x-2}$

(2,0)

→2



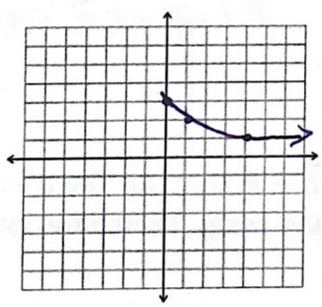
Domain: [2, ∞)
Range: [0, ∞)
End Behavior: x → ∞ y → ∞

3) $y = -\sqrt{x}+3$

reflected

↑3

(0, 3)



Domain: [0, ∞)
Range: (-∞, 3]
End Behavior: x → ∞ y → -∞

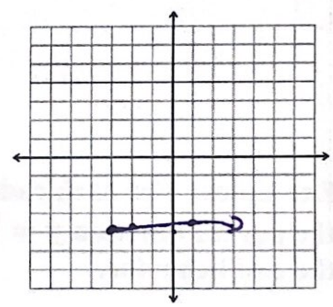
4) $y = \frac{1}{5}\sqrt{x+3} - 4$

comp.

(-3, -4)

→1 ↑ 1·$\frac{1}{5}$

→4 ↑ 2·$\frac{1}{5}$



Domain: [-3, ∞)
Range: [-4, ∞)
End Behavior: x → ∞ , y → ∞

Use a table of values to graph the parent cube root function: $y=\sqrt[3]{x}$

| x | y |
|---|---|
| -2 | -1.26 |
| -1 | -1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 1.26 |

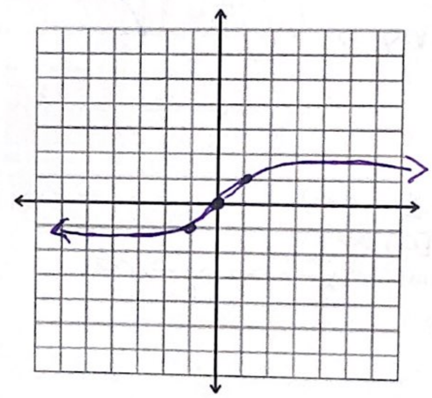
Identify the following key features:

"Center" (0,0)

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

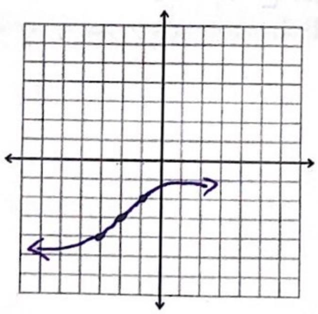
End Behavior: $x \to \infty,\ y \to \infty$
$x \to -\infty,\ y \to -\infty$



**Examples: For each radical function $y = a\sqrt[3]{x-h}+k$, describe the transformation from the parent function $y=\sqrt[3]{x}$, identify the domain and range, sketch the graph, and identify the end behavior.**

5) $y = \sqrt[3]{x+2} - 3$

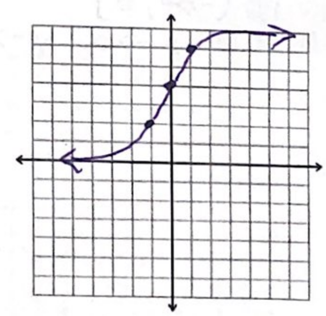
(-2, -3)



Domain: $\mathbb{R}$
Range: $\mathbb{R}$
End Behavior: $x \to \infty,\ y \to \infty$
$x \to -\infty,\ y \to -\infty$

6) $y = 2\sqrt[3]{x} + 4$

(0, 4)

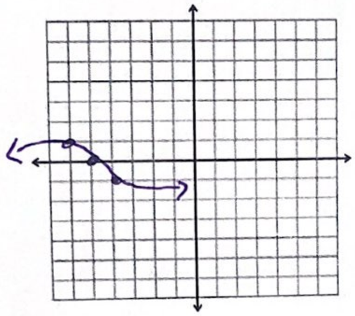


Domain: $\mathbb{R}$
Range: $\mathbb{R}$
End Behavior: $x \to \infty,\ y \to \infty$
$x \to -\infty,\ y \to -\infty$

7) $y = -\sqrt[3]{x+5}$

(-5, 0)

reflected



Domain: $\mathbb{R}$

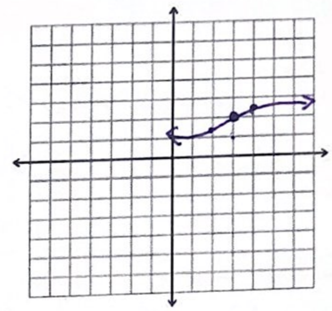
Range: $\mathbb{R}$

End Behavior: $x \to \infty,\ y \to -\infty$

$x \to -\infty,\ y \to \infty$

8) $y = 2 + \frac{1}{2}\sqrt[3]{x-3}$ $+2 \to \frac{1}{2}\sqrt[3]{x-3} + 2$

(3, 2)



Domain: $\mathbb{R}$

Range: $\mathbb{R}$

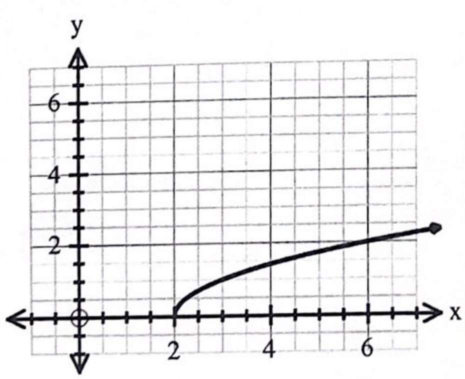
End Behavior: $x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

**Example 9)** Translate the graph of $y = \sqrt{x}$ so that it has a range of $y \geq 3$.

$y = 4\sqrt{x-2} + 3$

**Example 10)** Which of the following statements about the graph of f(x) is correct?




A.) $f(x)$ is increasing over the interval $(-\infty, \infty)$.

B.) $f(x)$ is increasing over the interval $[2, \infty)$.

C.) $f(x)$ is decreasing over the interval $(-\infty, \infty)$.

D.) $f(x)$ is decreasing over the interval $[2, \infty)$.