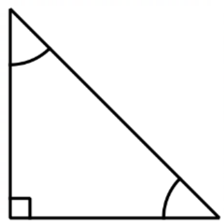
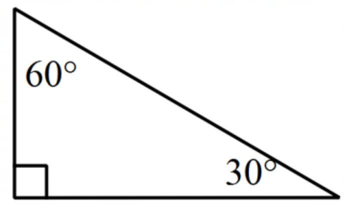
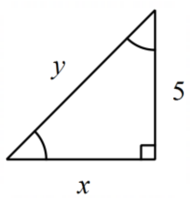
4.0 Notes: Review of Special Right Triangles

	45 – 45 – 90 Triangles	30 – 60 – 90 Triangles
Special Right Triangles		

For #1 – 12, find the length of each missing side for each special right triangle.

1)


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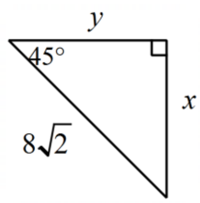
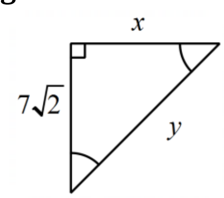


3)


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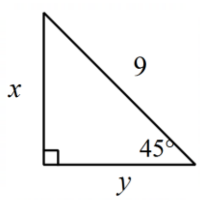

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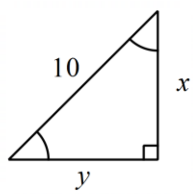

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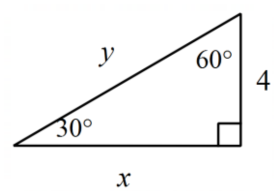

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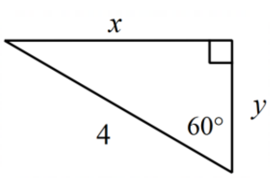

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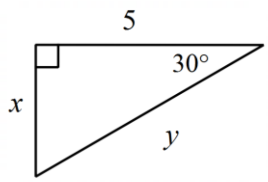
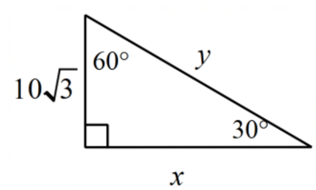
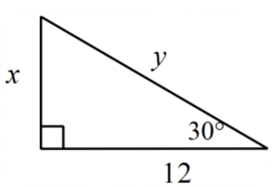
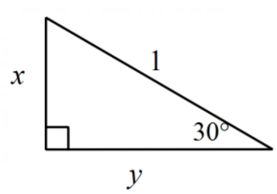
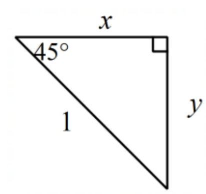
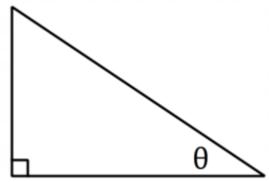


9)


10)


11)


12)


Right Triangle Trigonometry	$S\frac{o}{h}\quad C\frac{a}{h}\quad T\frac{o}{a}$ 

For #13 – 21, find the requested trig ratio by using special right triangles. Do NOT use a calculator.

13) sin 30

14) cos 30

15) tan 30

16) sin 60

17) cos 60

18) tan 60

19) sin 45

20) cos 45

21) tan 45

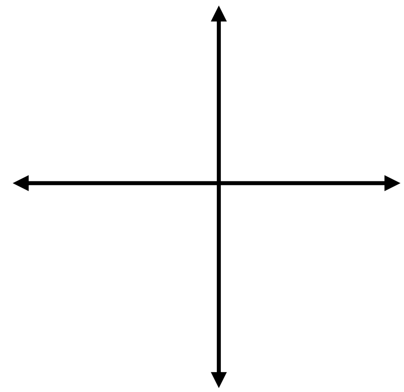
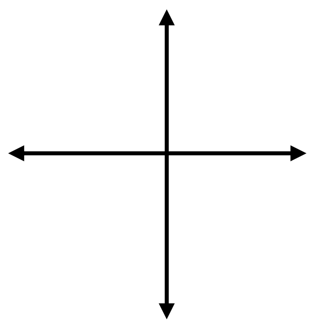
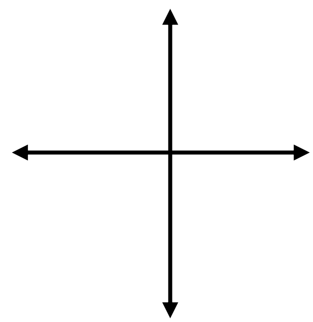
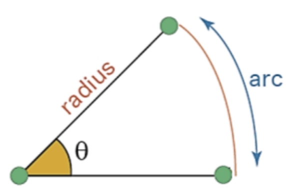
Examine your results for #13 and #17, as well as for #14 and #16. What do you notice? Why would this be so?

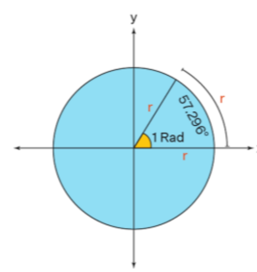
Exam your results for #15 and #18. What do you notice? Why is this happening?

4.1 Notes: Angles and Radian Measure

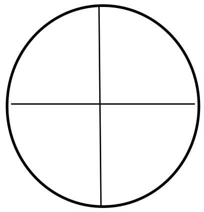
<table>
<tr><th colspan="3">Key Terminology</th></tr>
<tr><td rowspan="2">Standard Position of an Angle</td><td>An angle is in standard position if its vertex is located at the ______________and one ray is on the positive side of the _____-axis.

Initial Side (ray):

Terminal Side (ray):</td><td></td></tr>
<tr><td>Positive Angles
</td><td>Negative Angles
</td></tr>
<tr><td>Converting Negative Angles to Positive Angles</td><td colspan="2"></td></tr>
<tr><td>Radian Measure</td><td>The radian measure of any central angle (θ) is the length of the intercepted arc divided by the circle's radius.
$$\theta = \frac{\text{arc length}}{\text{radius}}$$
Note: when angles are measured in radians, no units are used.</td><td>What is one radian?
This is the measure of the central angle where the radius is ________________ to the intercepted arclength.

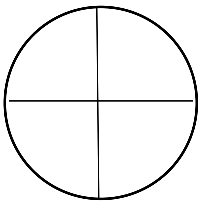
For any circle, one radian is equivalent to 57.296°.
</td></tr>
</table>

Example 1: A central angle θ in a circle of radius 12ft intercepts an arc of length 42ft. What is the radian measure of θ?

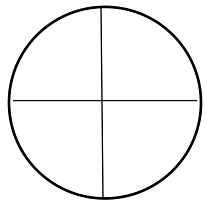
Radians and Degrees		
Comparing Degrees and Radians	**Angle Measurement for One Circle** $2\pi = 360°$ and $\pi = 180°$ 	**Conversions Between Degrees and Radians** $\frac{\pi}{180} = \frac{\text{radians}}{\text{degrees}}$

For Examples 2 – 4: Convert each angle measurement from degrees to radians. Also, sketch the terminal ray for each angle.

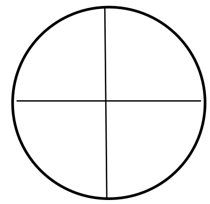
2) 60^{o}



3) 270^{o}

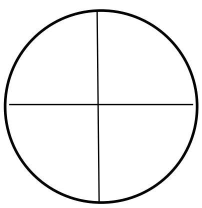


4) -300^{o}

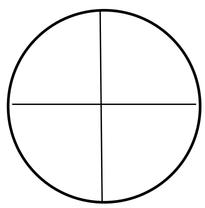


For Examples 5 – 7: Convert each angle measurement from radians to degrees. Also, sketch the terminal ray for each angle.

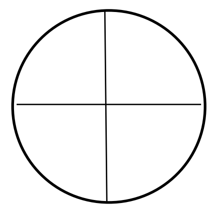
5) $\frac{\pi}{4}$

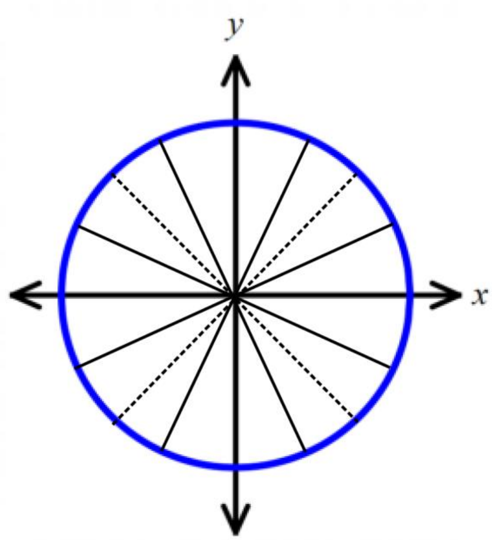


6) $-\frac{4\pi}{3}$



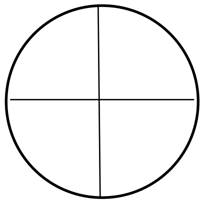
7) 6



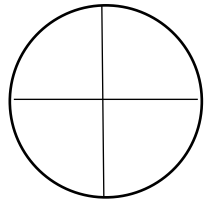
Angles in Standard Position (measured in radians and degrees)	 You will need to memorize these angle positions.

For Examples 8 – 10: Draw each angle in standard position.

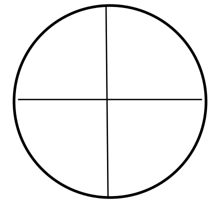
8) $\theta = \frac{3\pi}{4}$



9) $\theta = -\frac{\pi}{4}$

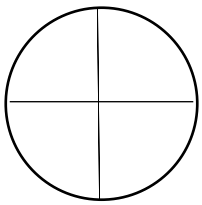


10) $\theta = \frac{5\pi}{6}$

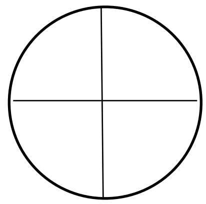


For Examples 11 – 13: Draw each angle in standard position.

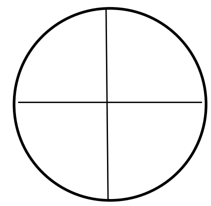
11) $\theta = \frac{7\pi}{4}$

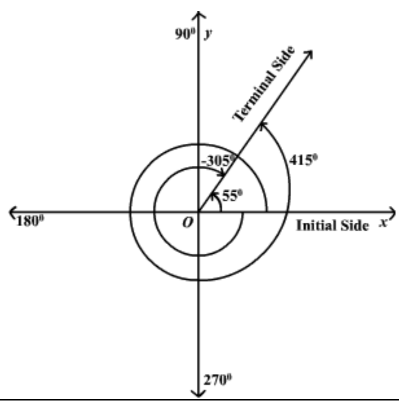


12) $\theta = -\frac{7\pi}{4}$



13) $\theta = -\frac{2\pi}{3}$



Coterminal Angles	
What are Coterminal Angles? Angles in standard position that have the _____________ terminal sides. 	**How to Find Coterminal Angles** **In degrees:** add or subtract _________ degrees **In radians:** add or subtract _________

For #14 – 19: Assume the following angles are in standard position. Find a positive angle less than 360° or 2π that is coterminal with each of the following:

14) $\frac{13\pi}{6}$

15) $-\frac{2\pi}{3}$

16) 400°

17) $-135°$

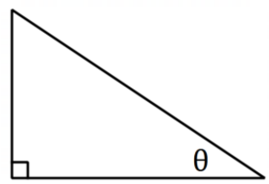
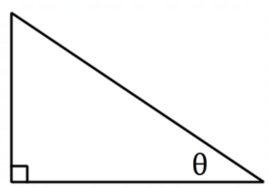
18) $\frac{17\pi}{3}$

19) $-885°$

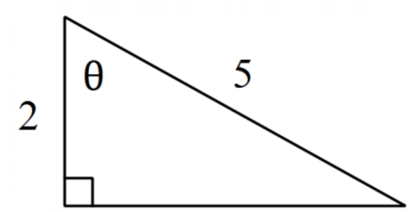
<table>
<tr><td rowspan="2">Arc Length</td><td colspan="2">To find the length of an arc with a central angle of θ, use one of the following formulas.</td></tr>
<tr><td>In Degrees:
$l = \frac{\theta}{360} \cdot 2\pi r$</td><td>In Radians:
$l = \frac{\theta}{2\pi} \cdot 2\pi r = \theta r$</td></tr>
</table>

Example 20: A circle has a radius of 6 inches. Find the length of the arc intercepted by a central angle of 45°. Also express this arc length in terms of π. Then round your answer to two decimal places.

4.3 Notes: Right Triangle Trigonometry

Right Triangle Trigonometry	
Basic Trig Functions	$S\frac{o}{h}$ $C\frac{a}{h}$ $T\frac{o}{a}$  $\sin\theta =$ $\cos\theta =$ $\tan\theta =$
Reciprocal Trig Functions	 $\csc\theta =$ $\sec\theta =$ $\cot\theta =$

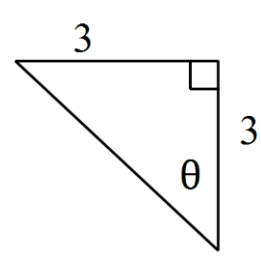
Example 1: Find the requested ratios for the triangle shown. Simplify your answer.



a) $\cos\theta$ b) $\sec\theta$ c) $\tan\theta$

d) $\sin\theta$ e) $\csc\theta$ f) $\cot\theta$

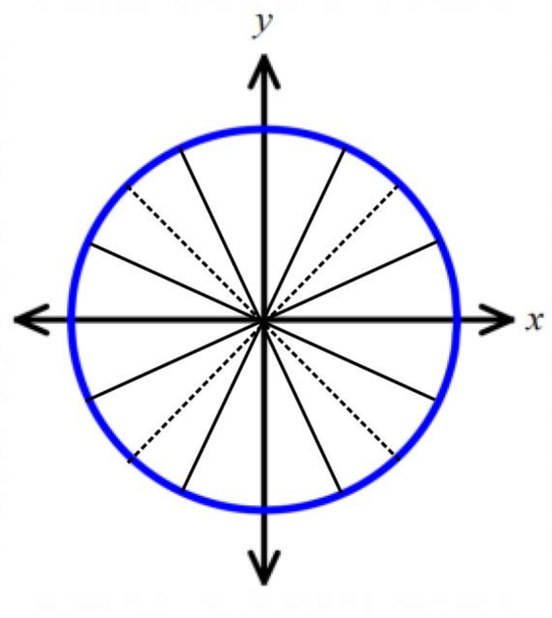
Example 2: Find the requested ratios for the triangle shown. Simplify your answer.



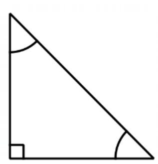
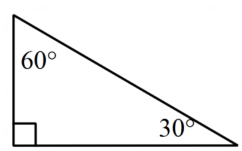
a) $\cos\theta$ b) $\sec\theta$ c) $\tan\theta$

d) $\sin\theta$ e) $\csc\theta$ f) $\cot\theta$

Review: Consider the circle. Label the angle measurements for all multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{2}$. Also, include the label measurements for each angle in degrees.



Example 3: Use the special triangles below to find the requested trig values in the table below.

<table>
<tr><td rowspan="2">Special Right Triangles</td><td>45 – 45 – 90 Triangles</td><td>30 – 60 – 90 Triangles</td></tr>
<tr><td></td><td></td></tr>
</table>

θ	$30^o = \frac{\pi}{6}$	$45^o = \frac{\pi}{4}$	$60^o = \frac{\pi}{3}$
sin θ			
cos θ			
tan θ			

Draw conclusions about trig functions of complementary angles. (Complementary angles have a sum of 90° or $\frac{\pi}{2}$.)

	Definition	Specifics
Cofunctions	The cofunction of an angle is the trigonometric function of the angle's ____________ that produces the same value to another trig function of the original angle.	Using Degree Measure $\sin(90° - \theta) = \cos\theta$, $\csc(90° - \theta) = \sec\theta$ $\cos(90° - \theta) = \sin\theta$, $\sec(90° - \theta) = \csc\theta$ $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$ Using Radian Measure $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$, $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$, $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$, $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Examples 4 – 6: Find a cofunction with the same value as the given expression.

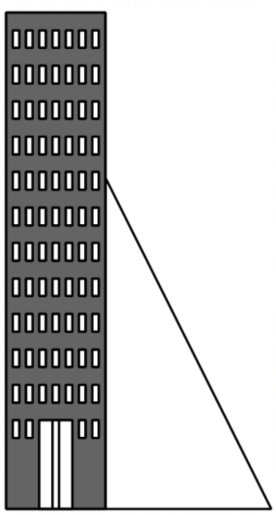
4) $\sin 46^{o}$ 5) $\cot\frac{\pi}{4}$ 6) $\sec\frac{\pi}{3}$

	Finding a Side	Finding an Angle
Solving Trigonometric Equations with Right Triangles	1) Use two sides and one angle to identify a trig function to use. 2) Set up an equation in the form below: **trig function** (**angle**) = **ratio** 3) Use algebra to isolate the variable.	1) Use two sides and one angle to identify a trig function to use. 2) Set up an equation in the form below: **trig function** (**angle**) = **ratio** 3) Use an inverse trig function to isolate the variable.

Example 7) A surveyor is standing 115 ft from the base of the Washington Monument. The surveyor measures the angle of elevation to the top of the monument as 78.3^{o}. How tall is the Washington Monument? If needed, round to 3 decimal places.

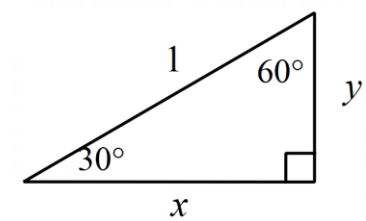
Example 8) A flagpole that is 14 meters tall casts a shadow 10 meters long. Find the angle of elevation of the sun to the nearest degree.

Example 9) A bird is on the street outside of the Empire State Building. The angle of elevation from the bird to the top of the 86^{th} floor is 82^{o}. If the 86^{th} floor is at a height of 320 feet, then how far away is the bird to the 86^{th} floor of the Empire State Building? Round to 2 decimal places.

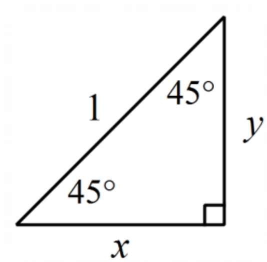


4.2 Part 1 Notes: The Unit Circle

Exploration A: Consider a 30 – 60 – 90 triangle with a hypotenuse of 1.

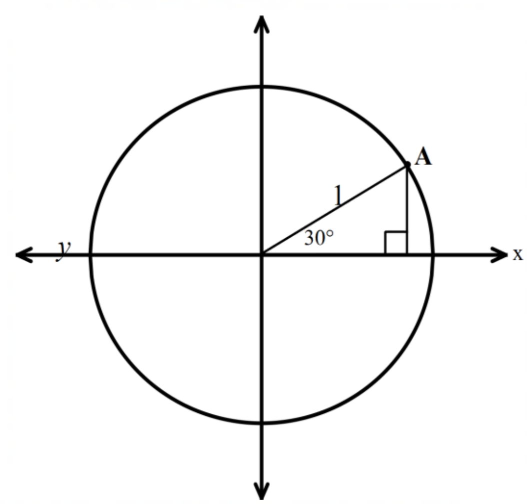


- Find the length of each leg.
  - $x =$
  - $y =$
- Find the following ratios:
  - $\sin 30$
  - $\cos 30$
  - $\sin 60$
  - $\cos 60$



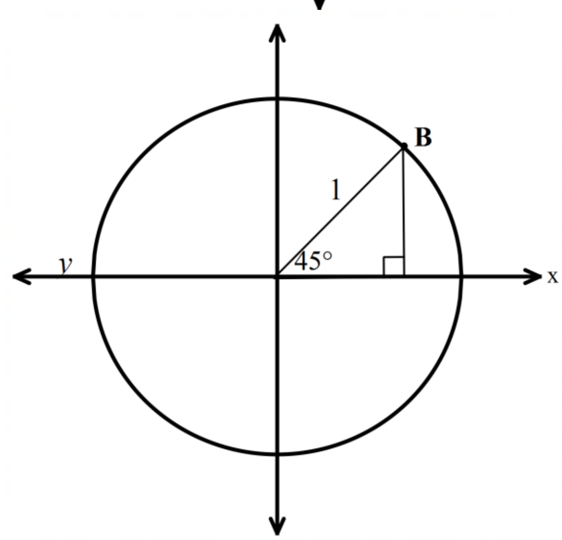
Exploration B: Consider a 45 – 45 – 90 triangle with a hypotenuse of 1.

- Find the length of each leg.
  - $x =$
  - $y =$
- Find the following ratios:
  - $\sin 45$
  - $\cos 45$



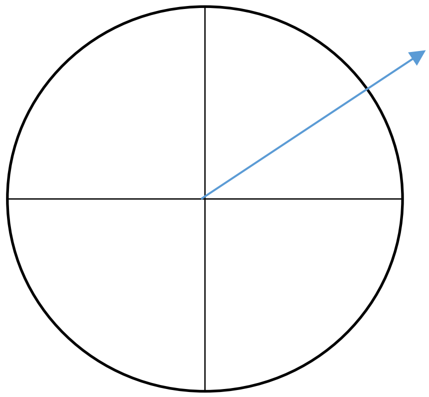
Exploration C: Consider a circle with a radius of 1, as shown.

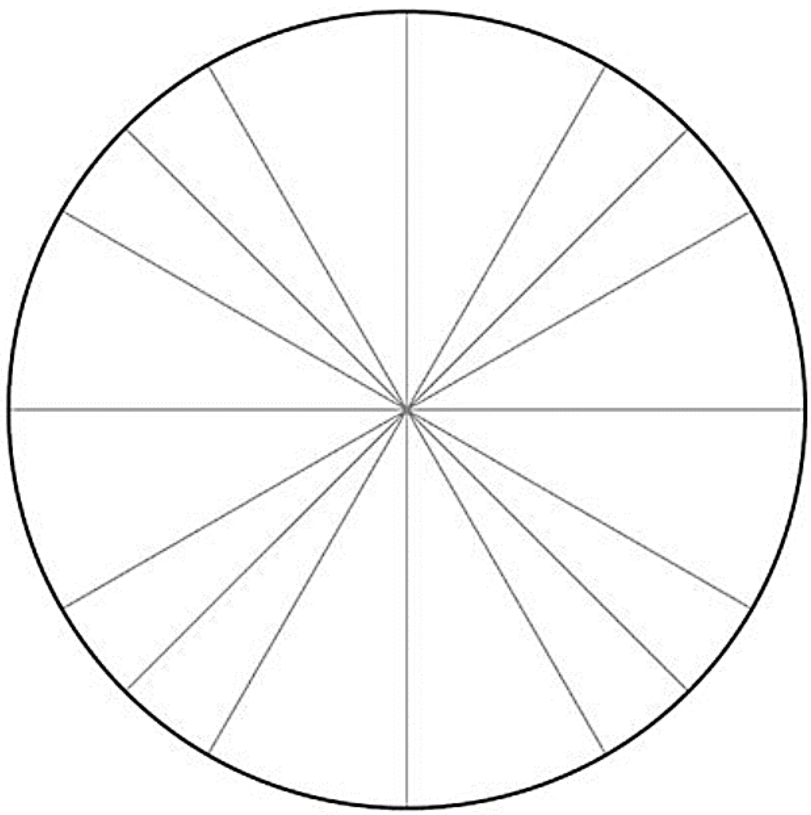
- Find the coordinates of point A.
- Compare these values with the sine and cosine of 30°. What do you notice?

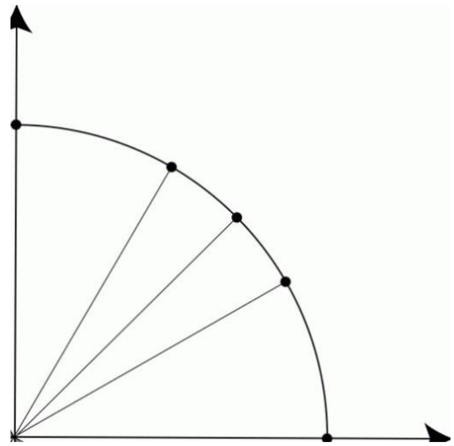


Exploration D: Consider a circle with a radius of 1, as shown.

- Find the coordinates of point B.
- Compare these values with the sine and cosine of 45°. What do you notice?

<table>
<tr><th colspan="2">The Unit Circle</th></tr>
<tr><td>What is the Unit Circle?</td><td>The unit circle is a circle with a radius of _________.

$\sin\theta =$
$\cos\theta =$
$\tan\theta =$
$\csc\theta =$
$\sec\theta =$
$\cot\theta =$</td></tr>
<tr><td>Values for the Unit Circle
(in degrees and radians)

Note: memorize these values!</td><td></td></tr>
</table>

Using the Unit Circle	
The First Quadrant	

For #1 – 15: Use the unit circle to find the exact value of each trig function at the indicated real number, or state the expression is undefined. No calculator allowed.

1) $\cos\frac{\pi}{6}$

2) $\sin\frac{\pi}{4}$

3) $\sin\frac{5\pi}{6}$

4) $\tan\frac{\pi}{2}$

5) $\csc\frac{8\pi}{3}$

6) $\sec\frac{7\pi}{6}$

7) $\cot\frac{5\pi}{4}$

8) $\sin\pi$

9) $\sec\frac{3\pi}{2}$

10) $\cos 2\pi$

11) $\cot 0$

12) $\sec\frac{11\pi}{3}$

13) $\csc\frac{\pi}{6}$

14) $\sec\frac{3\pi}{4}$

15) $\cot\frac{\pi}{3}$

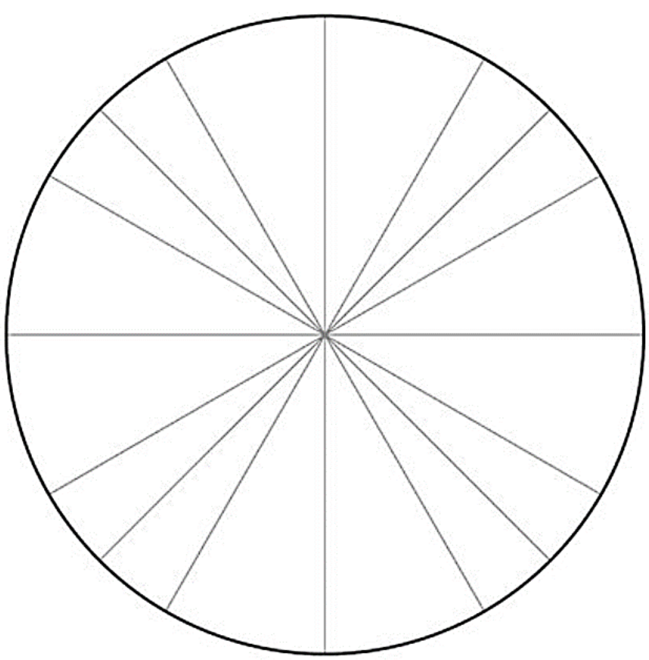
For examples 16 – 18, find the value of all six trig functions with the given information about a point or angle on the unit circle.

16) Given point $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

17) Given $\theta = \pi$.

18) Given point $P\left(-\frac{8}{17}, \frac{15}{17}\right)$.

Practice filling out the unit circle for degrees and radians. You will have a quiz on this next class!



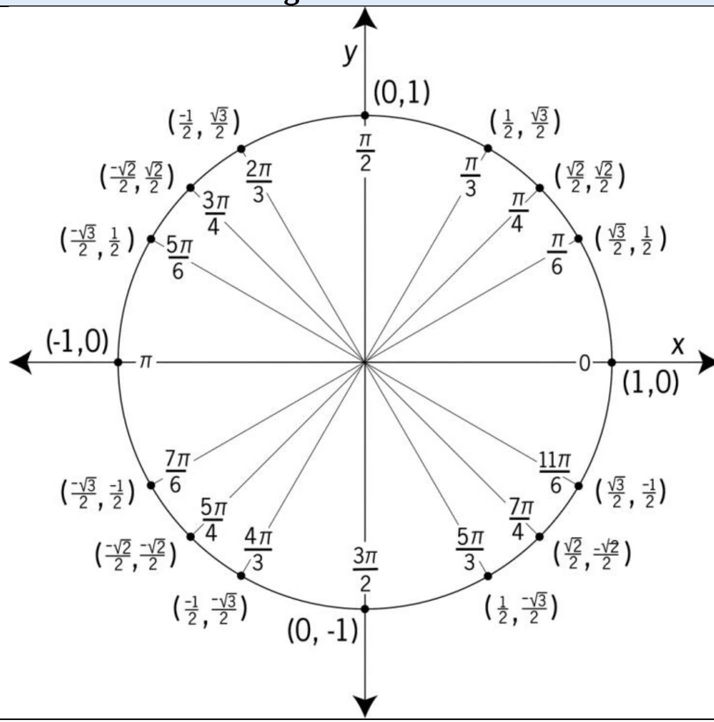
4.2 Part 2 Notes – Trig Identities, Domain, and Range

Even Functions	Odd Functions
A function is even when $f(-x) =$__________.	A function is odd when $f(-x) =$__________.

Even and Odd Trig Functions	Using the Unit Circle

Trigonometric Functions

Even Functions $f(-x)=f(x)$	Odd Functions $f(-x)=-f(x)$
$\cos(-x)=\cos x$ $\sec(-x)=\sec x$	$\sin(-x)=-\sin x$ $\csc(-x)=-\csc x$ $\tan(-x)=-\tan x$ $\cot(-x)=-\cot x$



For Examples 1 – 3: Use odd and even functions to find the value of each trig function.

1) $\sin\left(-\frac{\pi}{4}\right)$ 2) $\sec\left(-\frac{\pi}{4}\right)$ 3) $\tan\left(-\frac{\pi}{3}\right)$

Trig Identities

Quotient Identities

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$
$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\cot\theta=\frac{1}{\tan\theta}$$
$$\csc\theta=\frac{1}{\sin\theta}$$
$$\sec\theta=\frac{1}{\cos\theta}$$

Pythagorean Identities

$$\sin^2\theta+\cos^2\theta=1$$
$$\tan^2\theta+1=\sec^2\theta$$
$$1+\cot^2\theta=\csc^2\theta$$

Examples:

4) Given that sin t = $\frac{2}{3}$ and cos t = $\frac{\sqrt{5}}{3}$, find the values of each of the four remaining trig functions.

5) Given that sin t = $\frac{1}{2}$ and $0 \le t \le \frac{\pi}{2}$, find the value of $\cos t$ by using a trig identity.

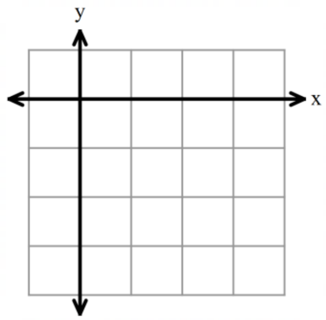
Periodic Properties
The period of a function is the smallest number that can be added to the function while still keeping it the same. For sine and cosine (and their reciprocals), the period is 2π. For tangent and cotangent, the period is π. Periodic Properties show that values stay the same when the period is added to the argument of a trig function. $\sin(\theta + 2\pi) = \sin\theta$ $\quad$ $\csc(\theta + 2\pi) = \csc\theta$ $\cos(\theta + 2\pi) = \cos\theta$ $\quad$ $\sec(\theta + 2\pi) = \sec\theta$ $\tan(\theta + \pi) = \tan\theta$ $\quad$ $\cot(\theta + \pi) = \cot\theta$

For Examples 6 – 8: Find the value of each trig function using periodic properties.

6) cot $\frac{5\pi}{4}$ 7) cos $(-\frac{9\pi}{4})$ 8) sin $\frac{14\pi}{3}$

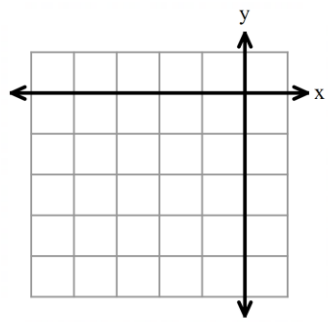
4.4 Notes: Trigonometric Functions of Any Angle

Exploration A: Let $P(2,-3)$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

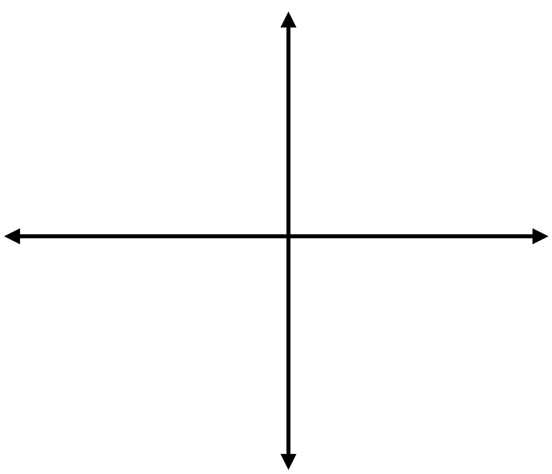
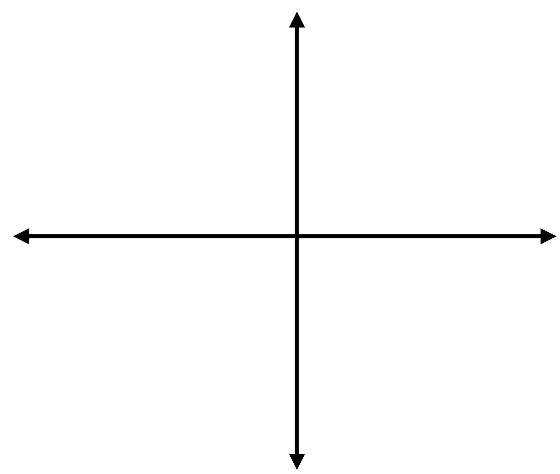


$\sin\theta =$	$\csc\theta =$
$\cos\theta =$	$\sec\theta =$
$\tan\theta =$	$\cot\theta =$

Exploration B: Let $P(-3,-5)$ be a point on the terminal side of θ. Find each of the six trig functions.



$\sin\theta =$	$\csc\theta =$
$\cos\theta =$	$\sec\theta =$
$\tan\theta =$	$\cot\theta =$

Signs of Trig Functions	Quadrantal Angles
The sign of a trig function depends on the quadrant that the terminal side lands in. 	A quadrantal angle is an angle whose terminal side is on either the *x*- or *y*-axis. 

Example 1: Find the sin θ, cos θ, and tan θ at the four quadrantal angles listed below.

a. $\theta = 0^{o}$ b. $\theta = \frac{\pi}{2}$ c. $\theta = \pi$ d. $\theta = \frac{3\pi}{2}$

For Examples 2 – 3, use the given information to identify the quadrant for θ.

2) $\sin\theta < 0$ and $\cos\theta < 0$

3) $\tan\theta < 0$ and $\cos\theta > 0$

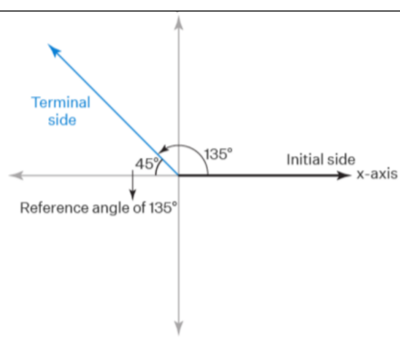
Examples 3 – 5: Use the given information to find the requested trig functions.

3) Given $\tan\theta = -\frac{1}{3}$ and $\cos\theta < 0$, find $\sin\theta$ and $\sec\theta$.

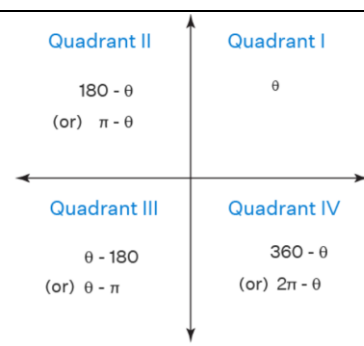
4) Given $\cos\theta = \frac{1}{2}$ and $\csc\theta < 0$, find $\tan\theta$ and $\csc\theta$.

5) Given $\tan\theta = -\frac{4}{3}$ and $\sin\theta > 0$, find $\csc\theta$ and $\cos\theta$.

Reference Angles



A **reference angle** $\hat{\theta}$ is the smallest ________________ , ______________ angle between the terminal side of an angle and the ______-axis.



(Note: A reference angle is always acute unless it is exactly 90 degrees.)

For Examples 6 – 11: Find the reference angle $\hat{\theta}$ for each given angle.

6) $\theta = 210^{\circ}$

7) $\theta = \frac{7\pi}{4}$

8) 580°

9) $\theta = \frac{-11\pi}{3}$

10) $\theta = \frac{15\pi}{4}$

11) $\theta = \frac{5\pi}{6}$

For #12 – 15: Use reference angles to find each requested value without using a calculator.

12) $\sin 300^{\circ}$

13) $\sec\left(-\frac{\pi}{6}\right)$

14) $\cos\left(\frac{17\pi}{6}\right)$

15) $\sin\left(-\frac{22\pi}{3}\right)$