#1, 2, 6, 7, 11, 13, 15, 21, 23, 25, 27, 29, 31, 35, 39, 41, 43

5.1 Day 1 Solutions - Ault

① $\sin x \sec x = \tan x$

$\sin x \cdot \frac{1}{\cos x}$

$\frac{\sin x}{\cos x}$

$\boxed{\tan x = \tan x}$

② $\cos x \csc x = \cot x$

$\cos x \cdot \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}$

$\boxed{\cot x = \cot x}$

⑥ $\cot x \sec x \sin x = 1$

$\frac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\sin x}}{1}$

$\boxed{1 = 1}$

⑦ $\sec x - \sec x \sin^2 x = \cos x$

$\sec x(\underbrace{1 - \sin^2 x})$

$\frac{1}{\cancel{\cos x}} \cdot \frac{\cos^2 x}{1}$

$\boxed{\cos x = \cos x}$

⑪ $\csc\theta - \sin\theta = \cot\theta\cos\theta$

$\frac{1}{\sin\theta} - \frac{\sin\theta \cdot \sin\theta}{1 \cdot \sin\theta}$ (get a common denom)

$\frac{1-\sin^2\theta}{\sin\theta}$

$\frac{\cos^2\theta}{\sin\theta}$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{\cos\theta}{1}$

$\boxed{\cot\theta\cos\theta = \cot\theta\cos\theta}$

⑬ $\frac{\tan\theta\cot\theta}{\csc\theta} = \sin\theta$

$\frac{\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}}{\frac{1}{\sin\theta}}$

$1 \cdot \frac{\sin\theta}{1}$

$\boxed{\sin\theta = \sin\theta}$

⑮ $\sin^2\theta(\underbrace{1+\cot^2\theta}) = 1$

$\sin^2\theta(\csc^2\theta)$

$\cancel{\sin^2\theta} \cdot \frac{1}{\cancel{\sin^2\theta}}$

$\boxed{1 = 1}$

(21) $\frac{\tan^2 t}{\sec t} = \sec t - \cos t$

$\frac{\sec^2 t - 1}{\sec t}$

$\frac{\sec^{\cancel{2}} t}{\cancel{\sec t}} - \frac{1}{\sec t}$

$\boxed{\sec t - \cos t = \sec t - \cos t}$

(23) $\frac{1-\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$

$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$\boxed{\csc\theta - \cot\theta = \csc\theta - \cot\theta}$

(25) $\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = 1$

$\frac{\sin t}{\frac{1}{\sin t}} + \frac{\cos t}{\frac{1}{\cos t}}$

$\sin t \cdot \sin t + \cos t \cdot \cos t$

$\sin^2 t + \cos^2 t$

$\boxed{1 = 1}$

(27) $\tan t + \frac{\cos t}{1+\sin t} = \sec t$

$\frac{\sin t}{\cos t} + \frac{\cos t}{1+\sin t}$

$\frac{\sin t(1+\sin t) + \cos t \cdot \cos t}{\cos t(1+\sin t)}$

$\frac{\sin t + \sin^2 t + \cos^2 t}{\cos t(1+\sin t)}$

$\frac{(1 + \cancel{\sin t})}{\cos t(1\cancel{+\sin t})}$

$\frac{1}{\cos t}$

$\boxed{\sec t = \sec t}$

(29) $1 - \frac{\sin^2 x}{1+\cos x} = \cos x$

$\frac{1+\cos x}{1+\cos x} - \frac{(1-\cos^2 x)}{1+\cos x}$

$\frac{\cancel{1}+\cos x \cancel{-1} + \cos^2 x}{1+\cos x}$

$\frac{\cos x + \cos^2 x}{1+\cos x}$

$\frac{\cos x(\cancel{1+\cos x})}{\cancel{1+\cos x}} = \boxed{\cos x = \cos x}$

(31) $\frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x} = 2\sec x$

$\frac{\cos^2 x + (1-\sin x)^2}{(1-\sin x)\cos x}$ get a common denom

$\frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{(1-\sin x)\cos x}$

$= \frac{1+1-2\sin x}{(1-\sin x)\cos x} \rightarrow \frac{2-2\sin x}{(1-\sin x)\cos x}$

$= \frac{2\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}\cos x} = \frac{2}{\cos x} = \boxed{2\sec x}$

(35) $\dfrac{\sec x - \csc x}{\sec x + \csc x} = \dfrac{\tan x - 1}{\tan x + 1}$

$= \dfrac{\frac{1}{\cos x} - \frac{1}{\sin x}}{\frac{1}{\cos x} + \frac{1}{\sin x}} = \dfrac{\frac{\sin x}{\cos x} - 1}{\frac{\sin x}{\cos x} + 1}$

$= \dfrac{\frac{\sin x - \cos x}{\cancel{\cos x \sin x}}}{\frac{\sin x + \cos x}{\cancel{\cos x \sin x}}} = \dfrac{\frac{\sin x - \cos x}{\cancel{\cos x}}}{\frac{\sin x + \cos x}{\cancel{\cos x}}}$

$= \boxed{\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{\sin x - \cos x}{\sin x + \cos x}}$

(39) $\tan^2 2x + \underbrace{\sin^2 2x + \cos^2 2x}_{1} = \sec^2 2x$

$\tan^2 2x + 1$

$\boxed{\sec^2 2x = \sec^2 2x}$

✓

(41) $\dfrac{\tan 2\theta + \cot 2\theta}{\csc 2\theta} = \sec 2\theta$

$= \dfrac{\frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\sin 2\theta}}$

$= \dfrac{\frac{\sin^2 2\theta + \cos^2 2\theta}{\cos 2\theta \cancel{\sin 2\theta}}}{\frac{1}{\cancel{\sin 2\theta}}}$

$= \dfrac{1}{\cos 2\theta}$

$= \boxed{\sec 2\theta = \sec 2\theta}$

(43) $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

$= \dfrac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}}$

$= \dfrac{\frac{\sin x \cos y + \cos x \sin y}{\cancel{\cos x \cdot \cos y}}}{\frac{\cos x \cos y - \sin x \sin y}{\cancel{\cos x \cos y}}}$

$= \boxed{\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}}$ ✓

#3, 4, 8, 10, 14, 16, 20, 24, 26, 30, 32, 36, 40, 44, 47, 53, 59*, 60*, 61, 62, 63, 64*

*Bonus

5.1 Day 2 Assignment

(3) $\tan(-x)\cos x = -\sin x$

$-\tan x \cos x$

$\frac{-\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1}$

$\boxed{-\sin x = -\sin x}$

(4) $\cot(-x)\sin x = -\cos x$

$-\cot x \sin x$

$\frac{-\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}$

$\boxed{-\cos x = -\cos x}$

(8) $\csc x - \csc x \cos^2 x = \sin x$

$\csc x(1-\cos^2 x)$

$\csc x \cdot \sin^2 x$

$\frac{1}{\cancel{\sin x}} \cdot \sin^{\cancel{2}} x$

$\boxed{\sin x = \sin x}$

(10) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$\cos^2 x - (1-\cos^2 x)$

$\cos^2 x - 1 + \cos^2 x$

$\boxed{2\cos^2 x - 1 = 2\cos^2 x - 1}$

(14) $\frac{\cos\theta \sec\theta}{\cot\theta} = \tan\theta$

$\frac{\cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}}}{\cot\theta}$

$\frac{1}{\cot\theta}$

$\boxed{\tan\theta = \tan\theta}$

(16) $\cos^2\theta(\underbrace{1+\tan^2\theta}) = 1$

$\cos^2\theta \cdot \sec^2\theta$

$\cancel{\cos^2\theta} \cdot \frac{1}{\cancel{\cos^2\theta}}$

$\boxed{1 = 1}$

(20) $\frac{\sec^2 t}{\tan t} = \sec t \csc t$

$\frac{\frac{1}{\cos^2 t}}{\frac{\sin t}{\cos t}}$

$\frac{1}{\cos^{\cancel{2}} t} \cdot \frac{\cancel{\cos t}}{\sin t}$

$\frac{1}{\cos t} \cdot \frac{1}{\sin t}$

$\boxed{\sec t \csc t = \sec t \csc t}$

(24) $\frac{1-\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

$\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$\boxed{\sec\theta - \tan\theta = \sec\theta - \tan\theta}$

(26) $\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \sin t + \cos t$

$\frac{\sin t}{\frac{\sin t}{\cos t}} + \frac{\cos t}{\frac{\cos t}{\sin t}}$

$\frac{\cancel{\sin t}\cdot\cos t}{\cancel{\sin t}} + \frac{\cancel{\cos t}\cdot\sin t}{\cancel{\cos t}}$

$\boxed{\cos t + \sin t = \sin t + \cos t}$

(30) $1 - \frac{\cos^2 x}{1+\sin x} = \sin x$

$\frac{1+\sin x - \cos^2 x}{1+\sin x}$

$\frac{1+\sin x - (1-\sin^2 x)}{1+\sin x}$

$\frac{\cancel{1} + \sin x - \cancel{1} + \sin^2 x}{1+\sin x}$

$\frac{\sin x + \sin^2 x}{1+\sin x}$

$\frac{\sin x\,\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}}$

$\boxed{\sin x = \sin x}$

(32) $\frac{\sin x}{\cos x+1} + \frac{\cos x - 1}{\sin x} = 0$

$\frac{\sin^2 x + (\cos x - 1)(\cos x + 1)}{(\cos x + 1)\sin x}$

$\frac{\sin^2 x + \cos^2 x - 1}{(\cos x + 1)\sin x}$

$\frac{1-1}{(\cos x+1)\sin x}$

$\boxed{0 = 0}$

(36) $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} = \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1}$

$\frac{\frac{\cos x - \sin x}{\cancel{\sin x \cos x}}}{\frac{\cos x + \sin x}{\cancel{\sin x \cos x}}} = \frac{\frac{\cos x - \sin x}{\cancel{\sin x}}}{\frac{\cos x + \sin x}{\cancel{\sin x}}}$

$\boxed{\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\cos x - \sin x}{\cos x + \sin x}}$

(40) $\cot^2(2x) + \cos^2(2x)$

$+ \sin^2(2x) = \csc^2(2x)$

$\cot^2 2x + 1$

$\boxed{\csc^2 2x = \csc^2 2x}$

(44) $\dfrac{\cot x + \cot y}{1 - \cot x \cot y} = \dfrac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y}$

$$\dfrac{\dfrac{\cos x}{\sin x} + \dfrac{\cos y}{\sin y}}{1 - \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos y}{\sin y}}$$

$$\dfrac{\dfrac{\cos x \sin y + \cos y \sin x}{\cancel{\sin x \sin y}}}{\dfrac{\sin x \sin y - \cos x \cos y}{\cancel{\sin x \sin y}}}$$

$$\boxed{\dfrac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y} = \dfrac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y}}$$

(47) $\dfrac{\sec t + 1}{\tan t} = \dfrac{\tan t}{\sec t - 1}$

cross-mult

$(\sec t + 1)(\sec t - 1) = \tan^2 t$

$\sec^2 t - 1 = \tan^2 t$

$\boxed{\tan^2 t = \tan^2 t}$

(53) $\dfrac{\sin\theta - \cos\theta}{\sin\theta} + \dfrac{\cos\theta - \sin\theta}{\cos\theta} = 2 - \sec\theta\csc\theta$

$$\dfrac{\cos\theta(\sin\theta - \cos\theta) + \sin\theta(\cos\theta - \sin\theta)}{\sin\theta\cos\theta}$$

$$\dfrac{\sin\theta\cos\theta - \cos^2\theta + \sin\theta\cos\theta - \sin^2\theta}{\sin\theta\cos\theta}$$

$$\dfrac{2\sin\theta\cos\theta - (\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta}$$

$$\dfrac{2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$\dfrac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} - \dfrac{1}{\sin\theta\cos\theta}$$

$$\boxed{2 - \sec\theta\csc\theta = 2 - \sec\theta\cos\theta}$$

per graph

(61) $\dfrac{(\sec x + \tan x)(\sec x - \tan x)}{\sec x} = \cos x$

$\dfrac{\sec^2 x - \tan^2 x}{\sec x}$

$\dfrac{\sec^2 x - (\sec^2 x - 1)}{\sec x}$

$\dfrac{\cancel{\sec^2 x} - \cancel{\sec^2 x} + 1}{\sec x}$

$\dfrac{1}{\sec x}$

$\boxed{\cos x = \cos x}$

---

(62) $\dfrac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x} = \sin x$

$\dfrac{\frac{1}{\cos^2 x} \cdot \frac{1}{\sin x}}{\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}}$

$\dfrac{\frac{1}{\cos^2 x \cdot \sin x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}}$

$\dfrac{\frac{1}{\cos^2 x \sin x}}{\frac{1}{\cos^2 x \sin^2 x}}$

$\dfrac{1}{\cancel{\cos^2 x}\, \cancel{\sin x}} \cdot \dfrac{\cancel{\cos^2 x} \sin^{\cancel{2}} x}{}$

$\boxed{\sin x = \sin x}$

(63) $\dfrac{\cos x + \cot x \sin x}{\cot x} = \boxed{2\sin x}$ from graph

$\dfrac{\cos x + \frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}}{\cot x}$

$\dfrac{\cos x + \cos x}{\cot x}$

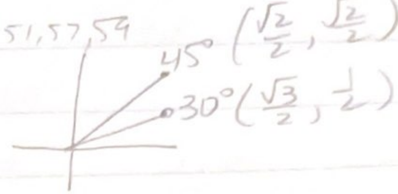
$\dfrac{2\cos x}{\frac{\cos x}{\sin x}}$

$2\cancel{\cos x} \cdot \dfrac{\sin x}{\cancel{\cos x}}$

$2\sin x = 2\sin x$

5.2 Day 1 Alternate Assgn
Solutions - Ault

#1, 5, 9, 11, 13, 15, 25, 27, 29, 33, 37, 41, 49, 51, 57, 59



① $\cos(45-30) =$

$\cos 45 \cos 30 + \sin 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

⑤ $\cos 50 \cos 20 + \sin 50 \sin 20$

a) $\alpha = 50,\ \beta = 20$

b) $\cos(50-20) = \boxed{\cos 30}$

c) $\cos 30 = \boxed{\frac{\sqrt{3}}{2}}$

⑨ $\frac{\cos(\alpha-\beta)}{\cos\alpha \sin\beta} = \tan\alpha + \cot\beta$

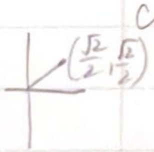
$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha \sin\beta}$

$\frac{\cancel{\cos\alpha}\cos\beta}{\cancel{\cos\alpha}\sin\beta} + \frac{\sin\alpha\cancel{\sin\beta}}{\cos\alpha\cancel{\sin\beta}}$

$\boxed{\cot\beta + \tan\alpha = \tan\alpha + \cot\beta}$

⑪ $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$

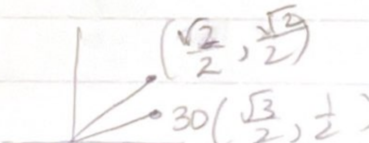
$\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}$



$\cos x \cdot \frac{\sqrt{2}}{2} + \sin x \cdot \frac{\sqrt{2}}{2}$

$\boxed{\frac{\sqrt{2}}{2}(\cos x + \sin x) = \frac{\sqrt{2}}{2}(\cos x + \sin x)}$

⑬ Find $\sin(45-30)$



$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

⑮ $\sin 105 = \sin(60+45)$

$\sin 60 \cos 45 + \cos 60 \sin 45$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

㉕ $\sin 25 \cos 5 + \cos 25 \sin 5$

$\sin(25+5) = \sin 30 = \boxed{\frac{1}{2}}$

㉗ $\frac{\tan 10 + \tan 35}{1 - \tan 10 \tan 35}$

$= \tan(10+35) = \tan 45 = \boxed{1}$

㉙ $\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{4}\right)$

$\sin\left(\frac{5\pi}{12} - \frac{\pi}{4}\right)$

$\sin\left(\frac{5\pi}{12} - \frac{3\pi}{12}\right)$

$\sin\left(\frac{2\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$

$= \boxed{\frac{1}{2}}$

5.2 Day 1

(33) $\sin(x + \frac{\pi}{2}) = \cos x$

$\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}$

$\sin x \cdot 0 + \cos x \cdot 1$

$\boxed{\cos x = \cos x}$

(37) $\tan(2\pi - x) = -\tan x$

$\dfrac{\tan 2\pi - \tan x}{1 + \tan 2\pi \tan x}$

$\dfrac{0 - \tan x}{1 + 0 \cdot \tan x}$

$\boxed{-\tan x = -\tan x}$

(41) $\dfrac{\sin(\alpha - \beta)}{\cos\alpha \cos\beta} = \tan\alpha - \tan\beta$

$\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$\dfrac{\sin\alpha \cancel{\cos\beta}}{\cos\alpha \cancel{\cos\beta}} - \dfrac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}$

$\boxed{\tan\alpha - \tan\beta = \tan\alpha - \tan\beta}$

(51) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

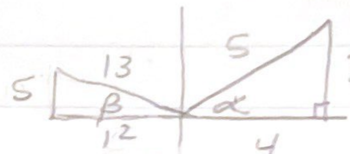
$\sin(\alpha + \alpha)$

$\sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\boxed{2\sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha}$

(57) $\sin\alpha = \frac{3}{5}$ Quad I

$\sin\beta = \frac{5}{13}$ Quad II



$\cos\beta = \frac{-12}{13}$ $\cos\alpha = \frac{4}{5}$

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\frac{4}{5} \cdot \frac{-12}{13} - \frac{3}{5} \cdot \frac{5}{13}$

$\frac{-48}{65} \; \frac{-15}{65} = \boxed{\frac{-63}{65}}$

b) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{3}{5} \cdot \frac{-12}{13} + \frac{4}{5} \cdot \frac{5}{13} = \frac{-36}{65} + \frac{20}{65}$

$\boxed{\frac{-16}{65}}$

c) $\tan(\alpha+\beta) = \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$= \dfrac{\frac{-16}{\cancel{65}}}{\frac{-63}{\cancel{65}}} = \boxed{\frac{16}{63}}$

(49) $\dfrac{\cos(x+h) - \cos x}{h} = \cos x \dfrac{\cos h - 1}{h} - \sin x \dfrac{\sin h}{h}$

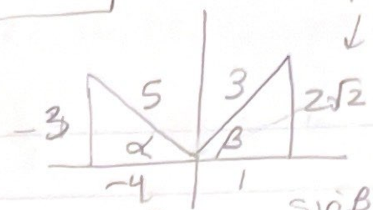
$\dfrac{\cos x\cos h - \sin x \sin h - \cos x}{h}$

$\dfrac{\cos x \cos h - \cos x}{h} - \dfrac{\sin x \sin h}{h}$

$\boxed{\cos x \dfrac{(\cos h - 1)}{h} - \sin x \cdot \dfrac{\sin h}{h} = \cos x \dfrac{\cos h - 1}{h} - \sin x \dfrac{\sin h}{h}}$

5.2 Day 1, cont'd

$1^2 + b^2 = 3^2$
$b^2 = 8$
$b = 2\sqrt{2}$



$\sin\alpha = \frac{3}{5}$ $\quad \sin\beta = \frac{2\sqrt{2}}{3}$

$\cos\alpha = -\frac{4}{5}$ $\quad \tan\beta = \frac{2\sqrt{2}}{1}$

5a) $\tan\alpha = -\frac{3}{4}$ Quad II

$\cos\beta = \frac{1}{3}$ Quad I

a) $\cos(\alpha+\beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\frac{-4}{5}\cdot\frac{1}{3} - \frac{3}{5}\cdot\frac{2\sqrt{2}}{3}$

$\frac{-4}{15} - \frac{6\sqrt{2}}{15} = \boxed{\frac{-4-6\sqrt{2}}{15}}$

b) $\sin(\alpha+\beta)$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\frac{3}{5}\cdot\frac{1}{3} + -\frac{4}{5}\cdot\frac{2\sqrt{2}}{3}$

$\boxed{\frac{3-8\sqrt{2}}{15}}$

c) $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\frac{3-8\sqrt{2}}{\cancel{15}}}{\frac{-4-6\sqrt{2}}{\cancel{15}}} = \frac{(3-8\sqrt{2})}{(-4-6\sqrt{2})}\cdot\frac{(-4+6\sqrt{2})}{(-4+6\sqrt{2})}$

$= \frac{-12+18\sqrt{2}+32\sqrt{2}-48\cdot 2}{+16-72} = \frac{(-108+50\sqrt{2})}{(-56)}$ reduce by $\frac{-2}{-2}$ $= \boxed{\frac{54-25\sqrt{2}}{28}}$

5.2 Day 2 Alternate Assign
Solutions (Ault)

2, 3, 6–8, 10, 12, 14, 16, 19, 22, 26, 28, 32, 48, 50, 52, 56, 58, 60, 63

② $\cos(120° - 45°)$

$\cos 120 \cos 45 + \sin 120 \sin 45$

$-\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{2}+\sqrt{6}}{4}}$

③ $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

$\cos \frac{3\pi}{4} \cdot \cos \frac{\pi}{6} + \sin \frac{3\pi}{4} \sin \frac{\pi}{6}$

$-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

⑥ $\cos 50 \cos 5 + \sin 50 \sin 5$

a) $\alpha = 50$; $\beta = 5$

b) $\cos(50 - 5) = \boxed{\cos 45}$

c) $\boxed{\frac{\sqrt{2}}{2}}$

⑦ $\cos\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$

a) $\alpha = \frac{5\pi}{12}$; $\beta = \frac{\pi}{12}$

b) $\cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) = \cos\left(\frac{4\pi}{12}\right) = \boxed{\cos\left(\frac{\pi}{3}\right)}$

c) $\boxed{\frac{1}{2}}$

8) $\cos \frac{5\pi}{18} \cos \frac{\pi}{9} + \sin \frac{5\pi}{18} \sin \frac{\pi}{9}$

a) $\alpha = \frac{5\pi}{18}$; $\beta = \frac{\pi}{9}$

b) $\cos\left(\frac{5\pi}{18} - \frac{\pi}{9}\right) = \cos\left(\frac{5\pi}{18} - \frac{2\pi}{18}\right)$

$= \cos\left(\frac{3\pi}{18}\right) = \boxed{\cos \frac{\pi}{6}}$

c) $\boxed{\frac{\sqrt{3}}{2}}$

⑩ $\frac{\cos(\alpha - \beta)}{\sin\alpha \sin\beta} = \cot\alpha \cot\beta + 1$

$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta}$

$\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$

$\boxed{\cot\alpha \cot\beta + 1 = \cot\alpha\cot\beta + 1}$

⑫ $\cos\left(x - \frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$

$\cos x \cos \frac{5\pi}{4} + \sin x \sin \frac{5\pi}{4}$

$\cos x \cdot -\frac{\sqrt{2}}{2} + \sin x \cdot -\frac{\sqrt{2}}{2}$

$\boxed{-\frac{\sqrt{2}}{2}(\cos x + \sin x) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)}$

⑭ $\sin(60 - 45)$

$\sin 60 \cos 45 - \cos 60 \sin 45$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

(16) $\sin 75 = \sin(30+45)$

$\sin 30 \cos 45 + \cos 30 \sin 45$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

(20) $\cos 105 = \cos(60+45)$

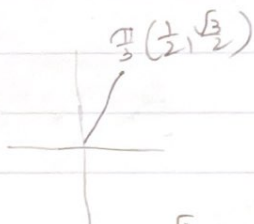
$\cos 60 \cos 45 - \sin 60 \sin 45$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

(22) $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$



$\tan\frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$\tan\frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$

$= \frac{(\sqrt{3}+1)}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$

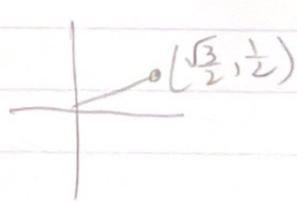
$= \frac{2\sqrt{3}+\sqrt{9}+1}{1-3} = \frac{2\sqrt{3}+4}{-2} = \boxed{-\sqrt{3}-2}$

(26) $\sin 40 \cos 20 + \cos 40 \sin 20$

$= \sin(40+20) = \sin 60$

$= \boxed{\frac{\sqrt{3}}{2}}$

28) $\frac{\tan 50 - \tan 20}{1 + \tan 50 \cdot \tan 20}$



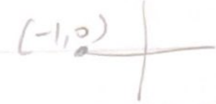
$\tan(50-20) = \tan 30$

$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{\sqrt{3}}{3}}$

(32) $\frac{\tan\frac{\pi}{5} + \tan\frac{4\pi}{5}}{1 - \tan\frac{\pi}{5}\tan\frac{4\pi}{5}}$

$= \tan\left(\frac{\pi}{5} + \tan\frac{4\pi}{5}\right)$

$= \tan(\pi)$



$= \frac{0}{-1} = \boxed{0}$

(48) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{1 - \tan\alpha\tan\beta}{1 + \tan\alpha\tan\beta}$

$= \frac{1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{1 + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$= \frac{\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cancel{\cos\alpha\cos\beta}}}{\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cancel{\cos\alpha\cos\beta}}}$

$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$

$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

(50) $\frac{\sin(x+h) - \sin x}{h} = \cos x \cdot \frac{\sin h}{h} + \sin x \cdot \frac{\cos h - 1}{h}$

$\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

$\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$

$\frac{\sin x(\cos h - 1)}{h} + \cos x \cdot \frac{\sin h}{h}$

$= \cos x \cdot \frac{\sin h}{h} + \sin x \cdot \frac{\cos h - 1}{h}$

5.2 Day 2, cont'd

52) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos(\alpha + \alpha)$

$\cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\boxed{\cos^2\alpha - \sin^2\alpha = \cos^2\alpha - \sin^2\alpha}$

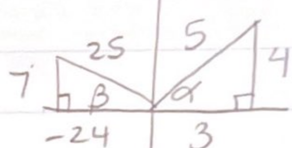
56) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(\alpha + -\beta)$

$\dfrac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)}$

(note: $\tan{-\beta} = -\tan\beta$)

$\boxed{\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$



58) $\sin\alpha = \frac{4}{5}$ Quad I

$\sin\beta = \frac{7}{25}$ Quad II

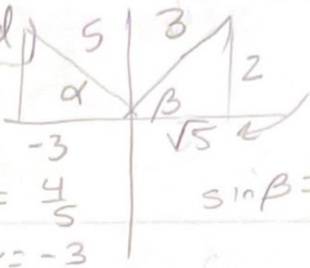
$\cos\beta = -\frac{24}{25}$ $\cos\alpha = \frac{3}{5}$

a) $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$= \frac{3}{5} \cdot -\frac{24}{25} - \frac{4}{5} \cdot \frac{7}{25} = -\frac{72}{125} - \frac{28}{125} = -\frac{100}{125} = \boxed{-\frac{4}{5}}$

b) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= \frac{4}{5} \cdot -\frac{24}{25} + \frac{3}{5} \cdot \frac{7}{25} = -\frac{96}{125} + \frac{21}{125} = -\frac{75}{125} = \boxed{-\frac{3}{5}}$

c) $\tan(\alpha + \beta) = \dfrac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \dfrac{-\frac{3}{8}}{-\frac{4}{8}} = \boxed{\frac{3}{4}}$

5.2 Day 2, cont'd



$2^2+b^2=3^2$

(60) $\tan\alpha = \frac{-4}{3}$ Quad II

$\cos\beta = \frac{2}{3}$ Quad I

$\sin\alpha = \frac{4}{5}$

$\cos\alpha = -\frac{3}{5}$

$\sin\beta = \frac{\sqrt{5}}{3}$

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\frac{3}{5}\cdot\frac{2}{3} - \frac{4}{5}\cdot\frac{\sqrt{5}}{3} = \boxed{\frac{-6-4\sqrt{5}}{15}}$

b) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{4}{5}\cdot\frac{2}{3} + \frac{-3}{5}\cdot\frac{\sqrt{5}}{3} = \boxed{\frac{8-3\sqrt{5}}{15}}$

c) $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$= \frac{\frac{8-3\sqrt{5}}{\cancel{15}}}{\frac{-6-4\sqrt{5}}{\cancel{15}}} = \frac{(8-3\sqrt{5})}{(-6-4\sqrt{5})}\cdot\frac{(-6+4\sqrt{5})}{(-6+4\sqrt{5})}$

$= \frac{-48+32\sqrt{5}+18\sqrt{5}-60}{36-80} = \frac{-108+50\sqrt{5}}{-44}$

$= \boxed{\frac{54-25\sqrt{5}}{22}}$

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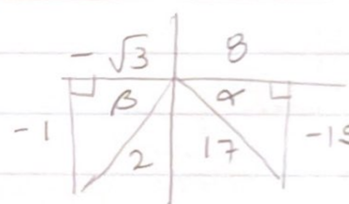
(61) $\cos\alpha = \frac{8}{17}$; Quad IV

$\sin\beta = -\frac{1}{2}$; Quad III



$\cos\beta = -\frac{\sqrt{3}}{2}$ $\sin\alpha = \frac{-15}{17}$

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\frac{8}{17}\cdot\frac{-\sqrt{3}}{2} - \left(\frac{-15}{17}\right)\cdot\frac{-1}{2}$

$\boxed{\frac{-8\sqrt{3}-15}{34}}$

b) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

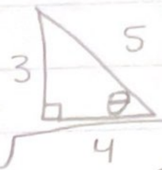
$= \frac{-15}{17}\cdot\frac{-\sqrt{3}}{2} + \frac{8}{17}\cdot\frac{-1}{2} = \boxed{\frac{15\sqrt{3}-8}{34}}$

c) $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\frac{15\sqrt{3}-8}{\cancel{34}}}{\frac{-8\sqrt{3}-15}{\cancel{34}}} = \frac{(15\sqrt{3}-8)}{(-8\sqrt{3}-15)}\cdot\frac{(-8\sqrt{3}+15)}{(-8\sqrt{3}+15)}$

$= \frac{-120\cdot 3+225\sqrt{3}+64\sqrt{3}-120}{64\cdot 3-225} = \boxed{\frac{-480+289\sqrt{3}}{-33} \text{ OR } \frac{480-289\sqrt{3}}{33}}$

Day 1 — 5.3 Alternate Assignment Solutions - Ault

#1, 5, 15, 17, 21, 23, 27, 40, 41, 47–49, 53, 55, 56, 73, 74

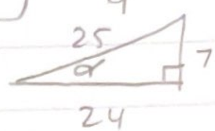


$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

① $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \boxed{\frac{24}{25}}$



$\sin\alpha = \frac{7}{25}$

$\cos\alpha = \frac{24}{25}$

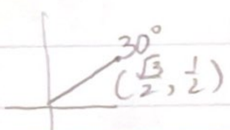
⑤ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2$

$= \frac{576}{625} - \frac{49}{625} = \boxed{\frac{527}{625}}$

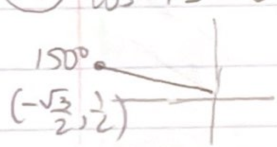
⑮ $2\sin 15^\circ \cos 15^\circ = \boxed{\sin 30}$

$= \boxed{\frac{1}{2}}$



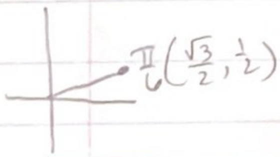
⑰ $\cos^2 75 - \sin^2 75 = \boxed{\cos 150}$

$= \boxed{-\frac{\sqrt{3}}{2}}$



㉑ $\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}} = \tan\left(2\cdot\frac{\pi}{12}\right)$

$= \boxed{\tan\frac{\pi}{6}}$



$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{\sqrt{3}}{3}}$

㉓ $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2\cdot\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

common denom

$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$

$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$

$= \frac{2\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cos^{\cancel{2}}\theta}{1}$

$= 2\sin\theta\cos\theta$

$\boxed{\sin 2\theta = \sin 2\theta}$

㉗ $\cancel{\sin^2 x} + \cos 2x = \cos^2 x$

$-\sin^2 x \qquad\qquad -\sin^2 x$

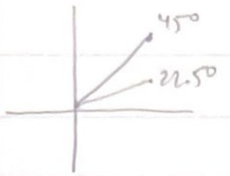
$\cos 2x = \cos^2 x - \sin^2 x$

$\boxed{\cos 2x = \cos 2x}$

✓

5.3 Day 1, cont'd

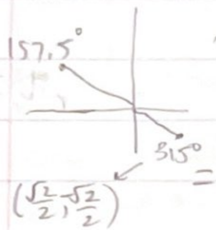
(40) $\cos 22.5 = \cos\left(\frac{45}{2}\right) \to$ quad I



$$= +\sqrt{\frac{1+\cos 45}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$= \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

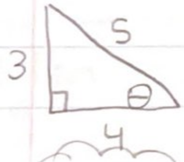
(41) $\cos 157.5 = \cos\left(\frac{315}{2}\right) \to$ quad II



$$= \ominus\sqrt{\frac{1+\cos 315}{2}} = -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}}$$

$$= \boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$$

(47) $\sin\left(\frac{\theta}{2}\right) \to$ all values of θ are pos $\to$ θ in Quad I $\to \frac{\theta}{2}$ in quad I

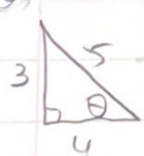


$\cos\theta = \frac{4}{5}$

$$\oplus\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{5}{5}-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}}$$

$$= \sqrt{\frac{1}{10}} = \boxed{\frac{\sqrt{10}}{10}}$$

(48) $\cos\left(\frac{\theta}{2}\right) \to$ Quad I (see reasoning in #47)



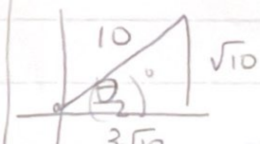
$\cos\theta = \frac{4}{5}$

$$\oplus\sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{\frac{2}{1}}} = \sqrt{\frac{9}{5}\cdot\frac{1}{2}} = \sqrt{\frac{9}{10}}$$

$$= \frac{3}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$$

or use $\Delta \to$

Note:

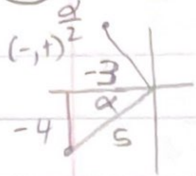


from $\sin\left(\frac{\theta}{2}\right)$ & Pythag Thm

(49) $\tan\left(\frac{\theta}{2}\right) = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{\frac{1\cancel{\sqrt{10}}}{\cancel{10}}}{\frac{3\cancel{\sqrt{10}}}{\cancel{10}}} = \boxed{\frac{1}{3}}$

5.3 Day 1, cont'd

(55) $\tan\alpha = \frac{4}{3}$; $\frac{180°}{2} < \frac{\alpha}{2} < \frac{270°}{2} \rightarrow 90 < \frac{\alpha}{2} < 135° \rightarrow$ Quad II $\frac{\alpha}{2}$



$\cos\alpha = -\frac{3}{5}$

$\sin\alpha = -\frac{4}{5}$

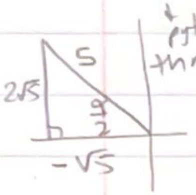
a) $\sin\left(\frac{\alpha}{2}\right) = \oplus\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(\frac{-3}{5}\right)}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{\frac{2}{1}}}$

$= \sqrt{\frac{8}{5}\cdot\frac{1}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$

b) $\cos\left(\frac{\alpha}{2}\right) = \ominus\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1-\frac{3}{5}}{2}} = -\sqrt{\frac{\frac{5}{5}-\frac{3}{5}}{2}} = -\sqrt{\frac{\frac{2}{5}}{\frac{2}{1}}}$

$= -\sqrt{\frac{\cancel{2}}{5}\cdot\frac{1}{\cancel{2}}} = -\frac{1}{\sqrt{5}} = \boxed{-\frac{\sqrt{5}}{5}}$

Note from $\sin\left(\frac{\alpha}{2}\right)$ + pythag thm



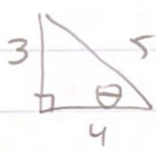
c) $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\frac{2\sqrt{5}}{\cancel{5}}}{\frac{-\sqrt{5}}{\cancel{5}}} = -\frac{2\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}} = \boxed{-2}$

---

(53) oops! out of order! :) $2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \sin\left(\cancel{2}\cdot\frac{\theta}{\cancel{2}}\right)$

$= \sin(\theta)$

$= \boxed{\frac{3}{5}}$



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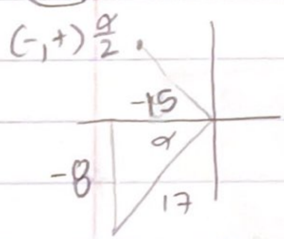
(56) $\tan\alpha = \frac{8}{15}$; $\frac{180°}{2} < \frac{\alpha}{2} < \frac{270°}{2} \rightarrow 90° < \frac{\alpha}{2} < 135°$ Quad II for $\left(\frac{\alpha}{2}\right)$



$\cos\alpha = \frac{-15}{17}$

a) $\sin\left(\frac{\alpha}{2}\right) = \oplus\sqrt{\frac{1-\cos\alpha}{2}} = +\sqrt{\frac{1-\left(\frac{-15}{17}\right)}{2}}$

$= \sqrt{\frac{\frac{17}{17}+\frac{15}{17}}{2}} = \sqrt{\frac{\frac{32}{17}}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}} = \boxed{\frac{4\sqrt{17}}{17}}$

b) $\cos\left(\frac{\alpha}{2}\right) = \ominus\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1-\frac{15}{17}}{2}} = -\sqrt{\frac{\frac{17}{17}-\frac{15}{17}}{\frac{2}{1}}}$

$= -\sqrt{\frac{\cancel{2}}{17}\cdot\frac{1}{\cancel{2}}} = \frac{-1}{\sqrt{17}} = \boxed{\frac{-\sqrt{17}}{17}}$

c) $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\frac{4\cancel{\sqrt{17}}}{\cancel{17}}}{\frac{-1\cancel{\sqrt{17}}}{\cancel{17}}} = \boxed{-4}$

5.3 Day 1, cont'd

(73) $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \ ? = \sec x$ (from graph)

$\frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} - \frac{(\cos^2 x - \sin^2 x)}{\cos x}$

$\frac{2\cos x}{1} - \frac{(\cos^2 x - \sin^2 x)}{\cos x}$ get common denom

$\frac{2\cos^2 x - \cos^2 x + \sin^2 x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\cos x}$

$\frac{1}{\cos x}$

$\boxed{\sec x = \sec x}$

(74) $\sin 2x \sec x = \ ? = 2\sin x$ (from graph

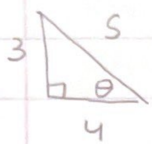
$2\sin x \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}}$

$\boxed{2\sin x = 2\sin x}$

5.3 Day 2 Alternate Solutions → Ault

#2, 3, 7, 11, 13, 25, 29, 39, 43, 50–52, 54, 57, 58

(2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$



$= \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

$\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

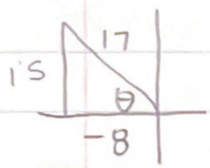
(3) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

or use $\frac{\sin 2\theta}{\cos 2\theta}$ ← often easier

$\frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{16}{16}-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}}$

$= \frac{3}{\cancel{2}}\cdot\frac{\cancel{16}8}{7} = \boxed{\frac{24}{7}}$

(7) $\sin\theta = \frac{15}{17}$ → θ in quad II



$\cos\theta = \frac{-8}{17}$

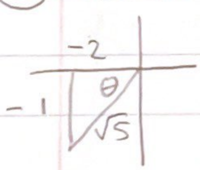
a) $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{15}{17}\cdot\frac{-8}{17} = \boxed{\frac{-240}{289}}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 = \frac{64}{289} - \frac{225}{289}$

$= \boxed{\frac{-161}{289}}$

c) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-\frac{240}{\cancel{289}}}{\frac{-161}{\cancel{289}}} = \boxed{\frac{240}{161}}$

(11) $\cot\theta = 2$; θ in quad III



$\tan\theta = \frac{1}{2}\frac{\text{opp}}{\text{adj}}$

$\sin\theta = -\frac{1}{\sqrt{5}}$

$\cos\theta = -\frac{2}{\sqrt{5}}$

a) $\sin 2\theta = 2\sin\theta\cos\theta$

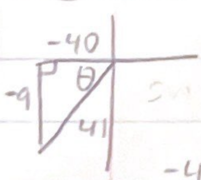
$= 2\cdot\frac{-1}{\sqrt{5}}\cdot\frac{-2}{\sqrt{5}} = \boxed{\frac{4}{5}}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{-1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$

c) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{4}{\cancel{5}}}{\frac{3}{\cancel{5}}} = \boxed{\frac{4}{3}}$

(13) $\sin\theta = \frac{-9}{41}$, Quad III



$\cos\theta = \frac{-40}{41}$

a) $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{-9}{41}\cdot\frac{-40}{41} = \boxed{\frac{720}{1681}}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-40}{41}\right)^2 - \left(\frac{-9}{41}\right)^2 =$

$\frac{1600}{1681} - \frac{81}{1681} = \boxed{\frac{1519}{1681}}$

c) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{720}{\cancel{1681}}}{\frac{1519}{\cancel{1681}}} = \frac{720}{1519}$

5.3 Day 2, cont'd

(25) $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$

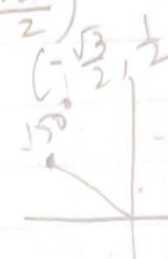
$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$1 + 2\sin\theta\cos\theta$

$\boxed{1 + \sin 2\theta = 1 + \sin 2\theta}$

(43) $\tan(75°) = \tan\left(\frac{150}{2}\right)$

$= \frac{1 - \cos(150)}{\sin(150)}$



$= \frac{1 - \frac{-\sqrt{3}}{2}}{\frac{1}{2}}$

$= \frac{\frac{2+\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \boxed{2+\sqrt{3}}$

(29) $\cot x = \frac{\sin 2x}{1 - \cos 2x}$

$= \frac{2\sin x \cos x}{1 - (2\cos^2 x - 1)}$

$= \frac{2\sin x \cos x}{1 - 2\cos^2 x + 1}$

$= \frac{2\sin x \cos x}{2 - 2\cos^2 x}$

$= \frac{\cancel{2}\sin x \cos x}{\cancel{2}(1 - \cos^2 x)}$

$= \frac{\cancel{\sin x}\cos x}{\sin^{\cancel{2}} x}$

$= \frac{\cos x}{\sin x}$

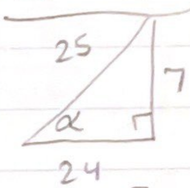
$\boxed{\cot x = \cot x}$

(50) $\sin\left(\frac{\alpha}{2}\right)$

$\sqrt{\frac{1 - \cos\alpha}{2}} = \sqrt{\frac{1 - \frac{24}{25}}{2}}$

$= \sqrt{\frac{\frac{25}{25} - \frac{24}{25}}{2}} = \sqrt{\frac{1}{50}}$

$= \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$



$\sin\alpha = \frac{7}{25}$

$\cos\alpha = \frac{24}{25}$

(51) $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos\alpha}{2}}$

$\sqrt{\frac{1 + \frac{24}{25}}{2}} = \sqrt{\frac{\frac{49}{25}}{2}} = \sqrt{\frac{49}{50}}$

$= \frac{7}{5\sqrt{2}} = \boxed{\frac{7\sqrt{2}}{10}}$

(52) $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\frac{1\cancel{\sqrt{2}}}{\cancel{10}}}{\frac{7\cancel{\sqrt{2}}}{\cancel{10}}} = \boxed{\frac{1}{7}}$

(39) $\sin 15° = \sin\left(\frac{30}{2}\right)$ Quad I

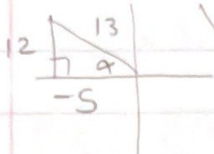
$= (+)\sqrt{\frac{1 - \cos 30}{2}} = (+)\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2-\sqrt{3}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

5.3 Day 2, cont'd

(54) $2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right) = \sin\left(\cancel{2}\cdot\frac{\alpha}{\cancel{2}}\right) = \sin\alpha = \boxed{\frac{7}{25}}$

(57) $\sec\alpha = \frac{-13}{5}$, $\frac{\pi}{2} < \alpha < \pi \rightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2} \rightarrow$ quad #1



$\cos\alpha = -\frac{5}{13}$

a) $\sin\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}} = \sqrt{\frac{\frac{13}{13}+\frac{5}{13}}{2}}$

$= \sqrt{\frac{18}{13}\cdot\frac{1}{2}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$

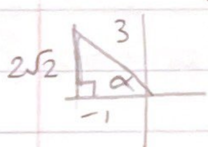
b) $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+-\frac{5}{13}}{2}} = \sqrt{\frac{\frac{13}{13}-\frac{5}{13}}{2}} = \sqrt{\frac{\left(\frac{8}{13}\right)}{\left(\frac{2}{1}\right)}}$

$= \sqrt{\frac{\cancel{8}^{4}}{13}\cdot\frac{1}{\cancel{2}}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}} = \boxed{\frac{2\sqrt{13}}{13}}$

c) $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\frac{3\cancel{\sqrt{13}}}{\cancel{13}}}{\frac{2\cancel{\sqrt{13}}}{\cancel{13}}} = \boxed{\frac{3}{2}}$

(58) $\sec\alpha = -3$; $\frac{\pi}{2} < \alpha < \pi \rightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$ Quad I

$\cos\alpha = -\frac{1}{3}$



a) $\sin\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{1}{3}\right)}{2}} = \sqrt{\frac{\frac{3}{3}+\frac{1}{3}}{2}} = \sqrt{\frac{\frac{4}{3}}{2}}$

$= \sqrt{\frac{\cancel{4}^{2}}{3}\cdot\frac{1}{\cancel{2}}} = \frac{\sqrt{2}}{\sqrt{3}} = \boxed{\frac{\sqrt{6}}{3}}$

b) $\cos\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1-\frac{1}{3}}{2}} = \sqrt{\frac{\frac{3}{3}-\frac{1}{3}}{2}} = \sqrt{\frac{\frac{2}{3}}{2}}$

$= \sqrt{\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

c) $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\frac{\sqrt{6}}{\cancel{3}}}{\frac{\sqrt{3}}{\cancel{3}}} = \boxed{\sqrt{2}}$

PowerReducing Alternate Assigment
(from 5.3)

Solutions → Aul4

#35-38, 59-62, 71, 72, 75, 76, 78

(35) $6\sin^4 x = 6\cdot\left(\frac{1-\cos(2x)}{2}\right)^2 = 6\left(\frac{1-2\cos 2x+\cos^2(2x)}{4}\right)$

$= \frac{3}{2} - 3\cos 2x + \frac{3}{2}\cos^2(2x) = \frac{3}{4} - 3\cos 2x + \frac{3}{2}\left(\frac{1+\cos 4x}{2}\right)$

$= \frac{3}{2} - 3\cos 2x + \frac{3}{4} + \frac{3}{4}\cos 4x = \boxed{\frac{9}{4} - 3\cos 2x + \frac{3}{4}\cos 4x}$

(36) $10\cos^4 x = 10\left(\frac{1+\cos 2x}{2}\right)^2 = 10\,\frac{(1+2\cos 2x+\cos^2 2x)}{4}$

$= \frac{5}{2} + 5\cos 2x + \frac{5}{2}\cos^2(2x) = \frac{5}{2} + 5\cos 2x + \frac{5}{2}\left(\frac{1+\cos 4x}{2}\right)$

$= \frac{5}{2} + 5\cos 2x + \frac{5}{4} + \frac{5}{4}\cos 4x = \boxed{\frac{15}{4} + 5\cos 2x + \frac{5}{4}\cos 4x}$

(37) $\sin^2 x\cos^2 x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) = \frac{1}{4}\left(1-\cos^2(2x)\right)$

$= \frac{1}{4} - \frac{1}{4}\cos^2(2x) = \frac{1}{4} - \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)$

$= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4x = \boxed{\frac{1}{8} - \frac{1}{8}\cos 4x}$

or $\boxed{\frac{1}{8}\left(1-\cos 4x\right)}$

(38) $8\sin^2 x\cos^2 x = 8$ (solution from #37)

$= \cancel{8}\cdot\frac{1}{\cancel{8}}\left(1-\cos 4x\right)$

$= \boxed{1-\cos 4x}$

Power Reducing, cont'd

(59) $\sin^2\left(\frac{\theta}{2}\right) = \frac{\sec\theta - 1}{2\sec\theta}$

$\frac{1-\cos(\cancel{2}\cdot\frac{\theta}{\cancel{2}})}{2} = \frac{\frac{1}{\cos\theta} - 1}{2\cdot\frac{1}{\cos\theta}}$

$\frac{1-\cos\theta}{2} = \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{2}{\cos\theta}}$

$\frac{1-\cos\theta}{2} = \frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{2}{\cancel{\cos\theta}}}$

$\boxed{\frac{1-\cos\theta}{2} = \frac{1-\cos\theta}{2}}$

(60) $\sin^2\left(\frac{\theta}{2}\right) = \frac{\csc\theta - \cot\theta}{2\csc\theta}$

$\frac{1-\cos(\cancel{2}\cdot\frac{\theta}{\cancel{2}})}{2} = \frac{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}{2\cdot\frac{1}{\sin\theta}}$

$\frac{1-\cos\theta}{2} = \frac{\frac{1-\cos\theta}{\cancel{\sin\theta}}}{\frac{2}{\cancel{\sin\theta}}}$

$\boxed{\frac{1-\cos\theta}{2} = \frac{1-\cos\theta}{2}}$

(61) $\cos^2\left(\frac{\theta}{2}\right) = \frac{\sin\theta + \tan\theta}{2\tan\theta}$

$\frac{1+\cos(\cancel{2}\cdot\frac{\theta}{\cancel{2}})}{2} = \frac{\frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}}{2\cdot\frac{\sin\theta}{\cos\theta}}$

$\frac{1+\cos\theta}{2} = \frac{\frac{\cos\theta\sin\theta + \sin\theta}{\cancel{\cos\theta}}}{\frac{2\sin\theta}{\cancel{\cos\theta}}}$

$= \frac{\cancel{\sin\theta}(\cos\theta + 1)}{2\cancel{\sin\theta}}$

$\boxed{\frac{1+\cos\theta}{2} = \frac{1+\cos\theta}{2}}$

(62) $\cos^2\left(\frac{\theta}{2}\right) = \frac{\sec\theta + 1}{2\sec\theta}$

$\frac{1+\cos(\cancel{2}\cdot\frac{\theta}{\cancel{2}})}{2} = \frac{\frac{1}{\cos\theta} + 1}{2\cdot\frac{1}{\cos\theta}}$

$\frac{1+\cos\theta}{2} = \frac{\frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{2}{\cos\theta}}$

$= \frac{\frac{1+\cos\theta}{\cancel{\cos\theta}}}{\frac{2}{\cancel{\cos\theta}}}$

$\boxed{\frac{1+\cos\theta}{2} = \frac{1+\cos\theta}{2}}$

(71) $\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2 = ? = \sin x + 1$

$\sin^2\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right)$

$\frac{1-\cos(\cancel{2}\cdot\frac{x}{\cancel{2}})}{2} + \sin(\cancel{2}\cdot\frac{x}{\cancel{2}}) + \frac{1+\cos(\cancel{2}\cdot\frac{x}{\cancel{2}})}{2}$

$\frac{1}{2} - \cancel{\frac{1}{2}\cos x} + \sin x + \frac{1}{2} + \cancel{\frac{1}{2}\cos x}$

$\boxed{1 + \sin x = \sin x + 1}$

(72) $\sin^2\left(\frac{x}{2}\right) - \cos^2\left(\frac{x}{2}\right) = ? = -\cos x$

$-1\left(\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)\right)$

$-1\cdot\cos(\cancel{2}\cdot\frac{x}{\cancel{2}})$

$\boxed{-\cos x = -\cos x}$

Power-Reducing, cont'd

(76) $\tan x + \cot x \overset{?}{=} 2\csc 2x$ from graph

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$\frac{1}{\sin x \cos x}$

$\frac{1}{\frac{1}{2}\sin 2x}$

$1 \cdot \frac{2}{\sin 2x}$

$\boxed{2\csc 2x = 2\csc 2x}$

(75) oops! out of order! :)

$\frac{\csc^2 x}{\cot x} = 2\csc(2x)$ (from graph)

$\frac{\left(\frac{1}{\sin^2 x}\right)}{\left(\frac{\cos x}{\sin x}\right)}$

$\frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x}$

$\frac{1}{\sin x \cos x}$

$\frac{2}{2\sin x \cos x}$

$\frac{2}{\sin 2x}$

$\boxed{2\csc 2x = 2\csc 2x}$

(78) $1 - 8\sin^2 x \cos^2 x = ? = \cos 4x$

$1 - 8\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

$1 - 8\left(\frac{1-\cos^2(2x)}{4}\right)$

$1 - 2(1 - \cos^2(2x))$

$1 - 2 + 2\cos^2(2x)$

$-1 + 2\left(\frac{1+\cos 4x}{2}\right)$

$-1 + 1 + \cos 4x$

$\boxed{\cos 4x = \cos 4x}$