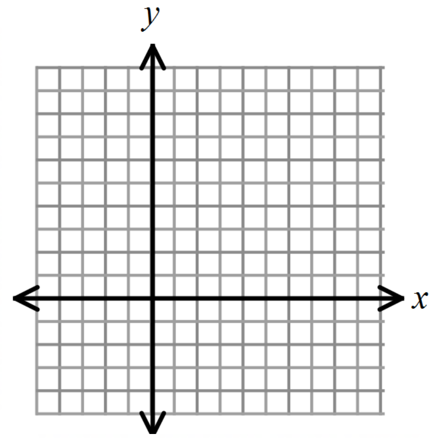
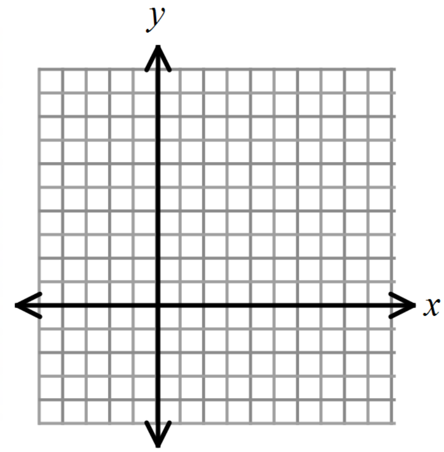
Ch 5 Extra Topics Notes

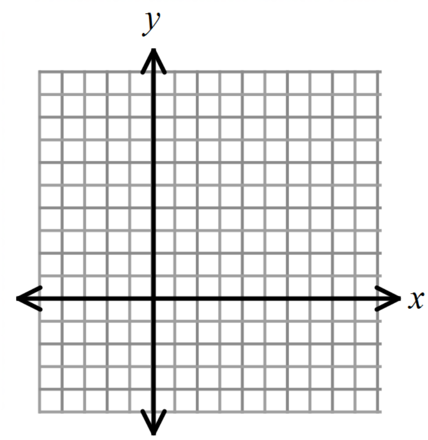
Topic #1: Partitioning a line segment.
Example 1: Given a directed line segment from A (-2, 8) to B (-2, 0).
Point P partitions AB in the ratio of 1:3. Find the coordinates of P.



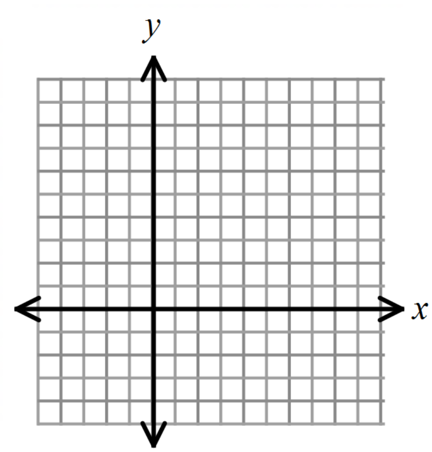
Example 2: Given a directed line segment from A (-1, 7) to B (6, 7).
Point P partitions AB in the ratio of 2:5. Find the coordinates of P.



Example 3: Given a directed line segment from A (9, -3) to B (-1, 2).
Point P partitions AB in the ratio of 3:2. Find the coordinates of P.



Example 4: Given a directed line segment from B to A if A (4, -5) and B(-3, -1). Point P partitions AB in the ratio of 5:3.
Find the coordinates of P.



Example 6: An 80 mile trip is represented on a gridded map by a directed line segment from point $M(3, 2)$ to point $N(9, 14)$. What point represents 70 miles into the trip? Round your answers to the nearest tenth.

Topic #2: Distance between a line and a point.

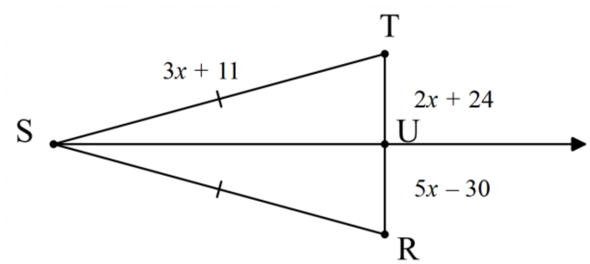
Example 7: Write the equation of the line (in slope-intercept form) that models the distance between the origin and the line $y = -2x + 8$.

Example 8: Write the equation of the line (in slope-intercept form) that models the distance between (-9, -2) and the line $y = 3x$.

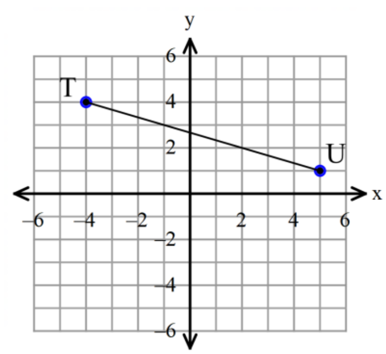
5.1 – 5.2 Notes: Special Segments and Points in Triangles

- **Definition:** A perpendicular bisector is ___________________ to a segment and ___________________ the segment.

Example 1: Find SR if $\overrightarrow{SU}$ is the perpendicular bisector of $\overline{TR}$.

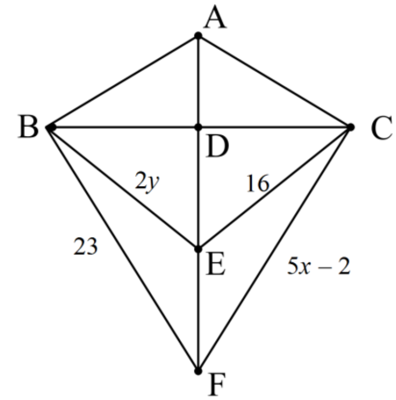


Example 2: A segment has endpoints $T(-4, 4)$ and $U(5, 1)$. Find the equation of the perpendicular bisector of $\overline{TU}$. Write your answer in slope-intercept form.



- **Perpendicular Bisector Theorem:** If a point is on the ⊥ bisector of a segment, then it is equidistant from the endpoints of the segment.

Example 3: Given the diagram where AF is the perpendicular bisector of BC. Find $3x + 2y$.

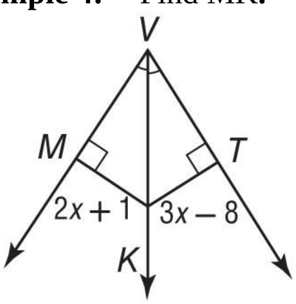


- ❑ **Definition:** An angle bisector is a ray that divides and angle into _______ congruent angles.

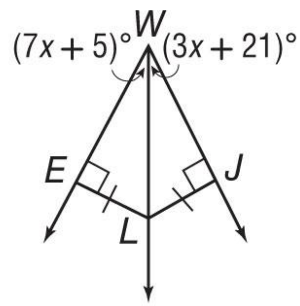
- ❑ **Angle Bisector Theorem:** If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

- ❑ **Converse of the Angle Bisector Theorem:** If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

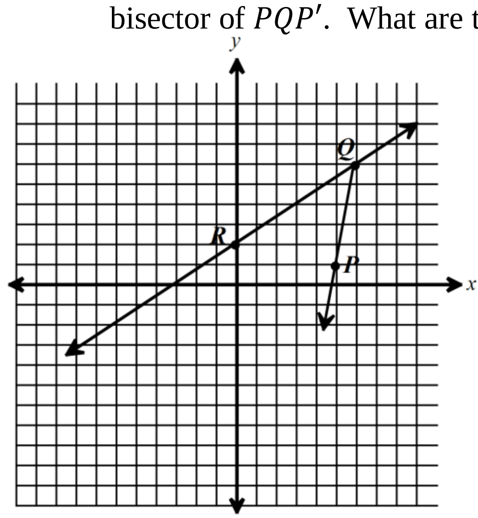
Example 4: Find MK.



Example 5: Find $m\angle EWL$

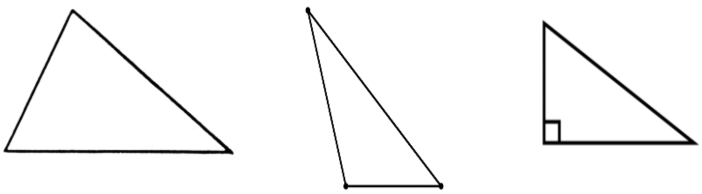


Example 6: On the graph below, $\angle PQR$ is reflected over $\overleftrightarrow{QR}$ so that $\overleftrightarrow{QR}$ is an angle bisector of PQP'. What are the coordinates of P'? Explain your reasoning.

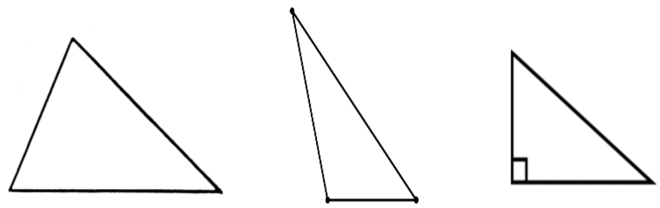


More Special Segments:

- ❑ **Definition of altitude:** If a segment is an altitude, then is extends from a vertex of a triangle and is perpendicular to the line containing the opposite side.



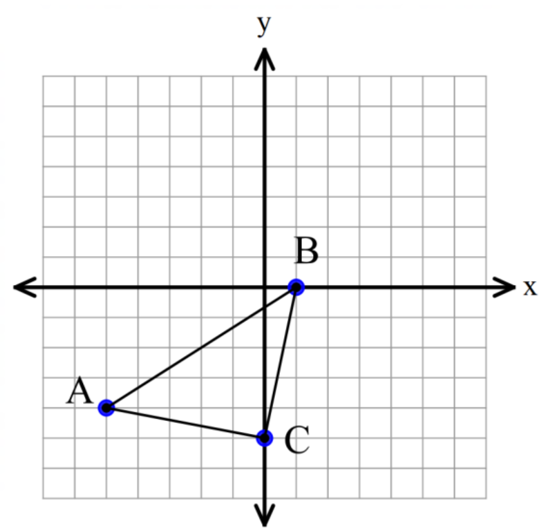
- ❑ **Definition of median:** If a segment is a median, then it extends from a vertex of a triangle to the midpoint of the opposite side.



The shortest distance between a point and a line is a ________________________ segment. Thus, an

____________________ is always shorter than a ____________________ (unless they are the same segment.)

Example 7: Given ΔABC with A (-5, -4), B (1, 0), and C (0, -5).



a) Find the coordinates of D if BD is an altitude.

b) Find the length of BD.

c) Find the coordinates of E if BE is a median.

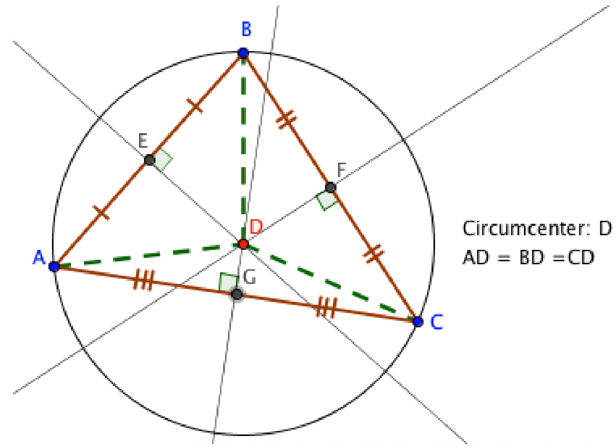
d) Find the length of BE. Which is shorter, the altitude from B, or the median from B?

f) Find the coordinates of F if CF is an altitude.

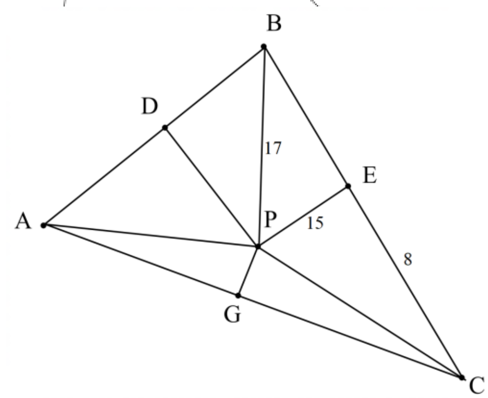
Point of Concurrency: A point where three or more lines ______________________. We will study two types of points of concurrency: circumcenters and centroids.

Circumcenter: The point in a triangle where the perpendicular bisectors of all three sides intersect.

- This point is the ____________ of the circle that is circumscribed about the triangle.
- **Circumcenter Theorem:** If a point is the circumcenter of a triangle, then it is equidistant to the vertices of the triangle.

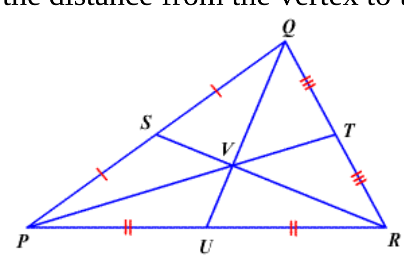


Example 8: Given that P is a circumcenter, find the perimeter of ΔAPC if AG = 9.

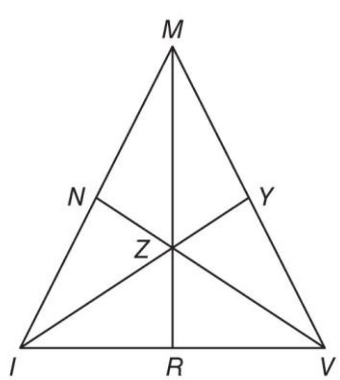


Centroid: The point in a triangle where all the medians of a triangle intersect.

- **Centroid Theorem:** If a point is a centroid, then it is $\frac{2}{3}$ of the distance from the vertex to the midpoint of the opposite side.
  - $QV = \frac{2}{3}QU$; $PV = \frac{2}{3}PT$; $RV = \frac{2}{3}RS$



Example 9: In $\triangle MIV$, Z is the centroid, $MZ = 6$, $YI = 18$, and $NZ = 10$. Find each measure.



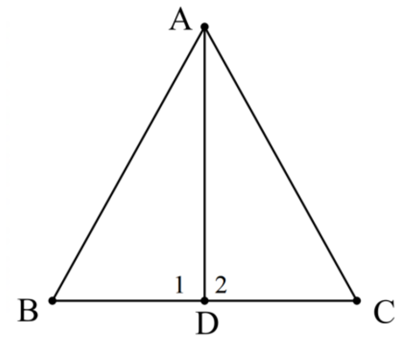
a) ZR b) YZ c) MR

d) ZV e) NV f) IZ

If-time extension:

Theorem: If a triangle is isosceles, then the altitude of the triangle is also the median of a triangle. (note: the converse of this theorem is also true.)

Complete the proof of this theorem below.

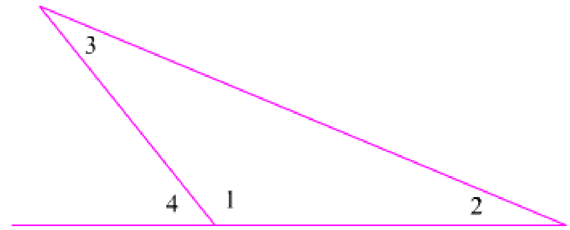


Ex 3: *Given:* ΔABC is isosceles with base $\overline{BC}$
$\overline{AD}$ is an altitude

Prove: $\overline{AD}$ is a median

Statements	Reasons
1) ΔABC is isosceles with base $\overline{BC}$; $\overline{AD}$ is an altitude	1) Given
2) $\overline{AB} \cong \overline{AC}$	2)
3) $\overline{AD} \perp \overline{BC}$	3)
4) $\angle 1$ and $\angle 2$ are right angles.	4)
5) $\overline{AD} \cong \overline{AD}$	5)
6) $\Delta ABD \cong \Delta ACD$	6)
7) $\overline{BD} \cong \overline{DC}$	7)
8) D is the midpoint of $\overline{BC}$.	8)
9) $\overline{AD}$ is a median.	9)

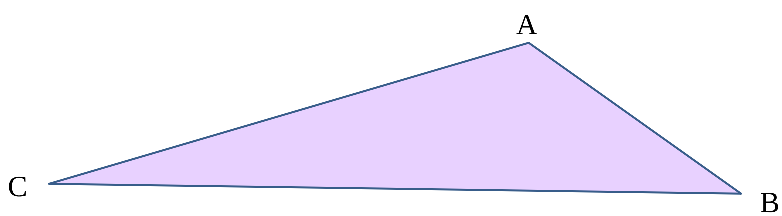
5.3 Guided Notes: Inequalities in One Triangle



Example 1: Find the measure of ∠4 if ∠3 = 23º and ∠2 = 19º. Is it possible for ∠4 to ever be smaller than ∠2 or ∠3? Explain your reasoning.

- **Theorem:** The measure of an exterior angle of a triangle is ____________________ than the measure of either remote interior angle.

Example 2: Use a protractor and ruler to find the measure of the angles and sides (in cm) of the triangle below.



AB = ____________ cm

BC = ____________ cm

AC = ____________ cm

∠A = ________º

∠B = ________º

∠C = ________º

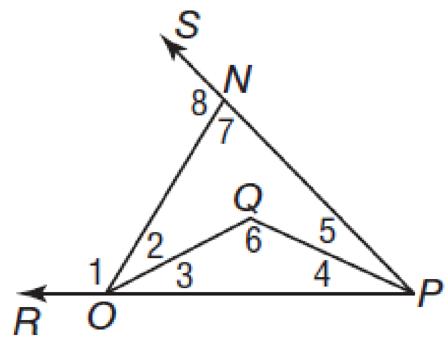
What side is the longest? Which angle is the largest?

What side is the smallest? Which angle is the smallest?

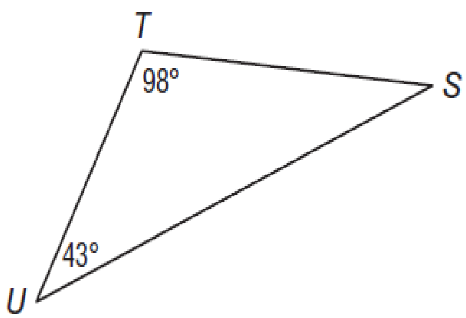
Draw a conclusion about the relationship between side lengths and angle measures in a triangle:

- **Theorems:**
  - ❖ If one side of a triangle is longer than another side, then the angle _________________ the longer side is larger than the angle opposite the shorter side.
  - ❖ If one angle of a triangle is larger than another angle, then the side opposite the larger angle is _________________ than the side opposite the smaller angle.

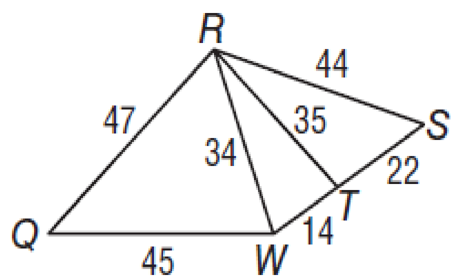
Example 3: Identify any angles that are smaller than <1.



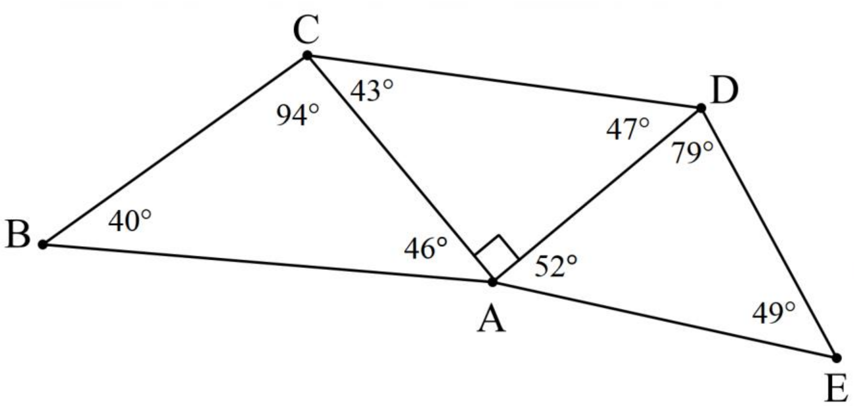
Example 4: List the sides from shortest to longest.



Example 5: Write an inequality comparing the measures of <RWQ and <QRW.



Example 6: Which is the shortest side in the diagram shown below?



5.5 Guided Notes: The Triangle Inequality

Example 7: Use the side lengths given to your group to create a triangle. Can any three side lengths be used to create a triangle? Explain your conclusion.

- **Theorem:** The sum of the lengths of any two sides of a triangle must be ___________________ than the length of the third side

Example 8: Is it possible to form a triangle with the given side lengths? If not, explain why not.

a.) $4 \text{ mm}, 7\text{mm}, 12 \text{ mm}$ b.) $13 \text{ in}, 15 \text{ in}, 28 \text{ in}$

Example 9: Find the range for the measure of the third side of a triangle given the measures of two sides.

a) 12 yds , 15 yds. b) 6 cm, 6 cm

Example 10: Find the range of possible measures of x if each set of expressions represents measures of the sides of a triangle.

$$3x + 2,\ x + 4,\ x + 6$$

5.4 Notes: Indirect Proofs

What is an indirect proof?

Steps for completing an indirect proof:

1. List the possibilities for the conclusion
2. Assume that the negation of the desired conclusion is correct.
3. Write a chain of reasons until you reach an impossibility.
   This will be a contradiction of either
   - Given information or
   - a theorem, definition, or other known fact.
4. State the remaining possibility as the desired conclusion.

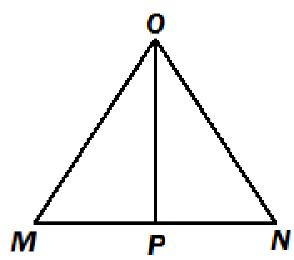
Example 1: State the assumption you would make to start an indirect proof of each statement.

a. **Given:** $\overline{MO} \cong \overline{ON}, \overline{MP} \ncong \overline{NP}$

Prove: $\angle MOP \ncong \angle NOP$

b. Given: $\angle ADB$ is acute.
Prove: $AB \cong BC$

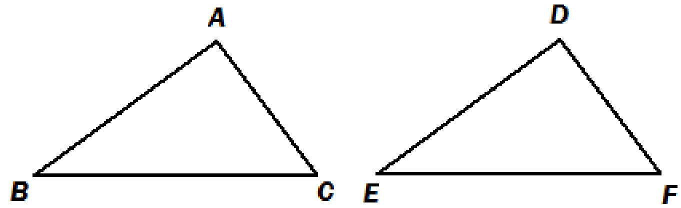
Example 2: Write an indirect proof.



Given: $\overline{MO} \cong \overline{ON}, \overline{MP} \ncong \overline{NP}$

Prove: $\angle MOP \ncong \angle NOP$

Example 3: **Write an indirect proof.**

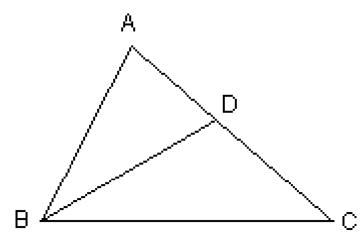


Given: $\angle A \cong \angle D, \overline{AB} \cong \overline{DE}, \overline{AC} \ncong \overline{DF}$

Prove: $\angle B \ncong \angle E$

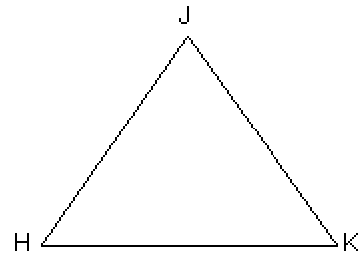
Example 4: **Given**: $\overrightarrow{BD}$ bisects $\angle ABC$, $\angle ADB$ is acute.

Prove: $\overline{AB} \ncong \overline{BC}$



Example 5: **Given**: $\angle H \ncong \angle K$.

Prove: $\overline{JH} \ncong \overline{JK}$



5.6 Notes: The Hinge Theorem

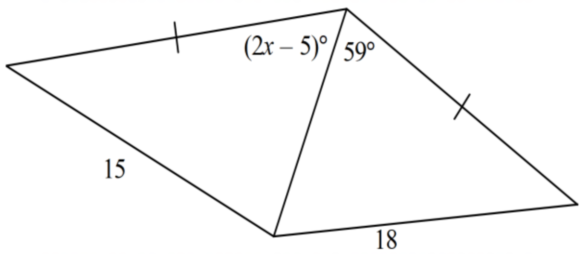
The Hinge Theorem:

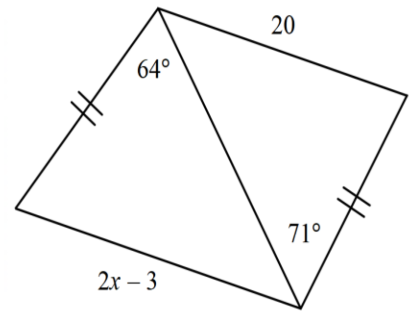
- If 2 sides of a Δ are ≅ to 2 sides of another Δ, and the included ∠ of the first Δ is larger than the included ∠ of the second Δ, then the third side of the first Δ is longer than the third side of the second Δ.

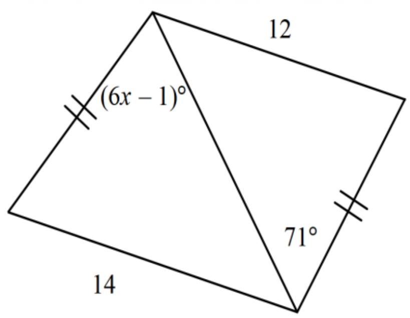
The Converse of the Hinge Theorem:

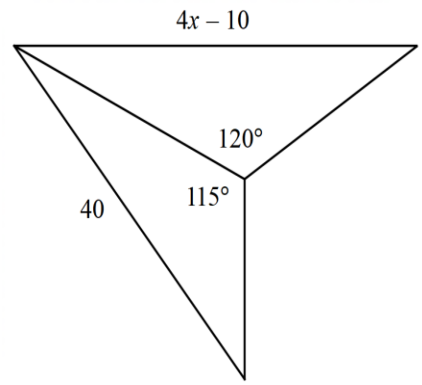
- If 2 sides of a Δ are ≅ to 2 sides of another Δ, and the third side of the first Δ is longer than the third side of the second Δ, then the included ∠ of the first Δ is larger than the included ∠ of the second Δ.

Examples: For each example, find the range of values on the variable.

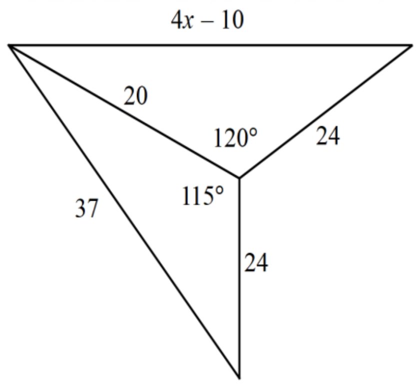
1) 

2) 

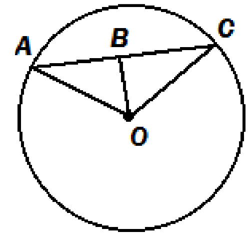
3) 

4) 

5) Note: This one is *slightly* different than #4. Be careful!

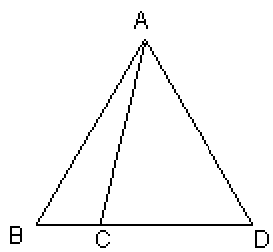


Extra practice for indirect proofs… if time!



Example 4: **Write an indirect proof.**

Given: $\odot O, \overline{AB} \ncong \overline{BC}$
Prove: $\angle AOB \ncong \angle COB$



Example 5: Given: $\overline{AB} \cong \overline{AD}, \angle BAC \ncong \angle DAC$
Prove: $\overline{BC} \ncong \overline{DC}$

Example 6: Given: $\overline{AC} \perp \overline{BD}, \overline{BC} \cong \overline{EC}, \overline{AB} \ncong \overline{ED}$
Prove: $\angle B \ncong \angle CED$