Ch 7 Review Wk with answers

Friday, March 1, 2024 11:25 AM

Ch 7 Review Wk with...

Math 127 **Ch 7 Review Worksheet** **Name** Ault Solutions

For #1 – 4, solve each system of equations by using the method of your choice.

1) $\begin{cases} x - y + z = -1 \\ x + y + z = -9 \\ x + y - z = -3 \end{cases}$

① $x - y + z = -1$
③ $x + y - z = -3$

$2x = -4$
$x = -2$

① $x - y + z = -1$
② $x + y + z = -9$

$2x + 2z = -10$

$-4 + 2z = -10$
$2z = -6$
$z = -3$

② $x + y + z = -9$
$-2 + y - 3 = -9$
$y - 5 = -9$
$y = -4$

$(2, -4, -3)$

2) $\begin{cases} x + y = 6 \\ y = x^2 - 8x + 16 \end{cases}$ → $y = 6 - x$

$6 - x = x^2 - 8x + 16$
$0 = x^2 - 7x + 10$
$0 = (x - 5)(x - 2)$
$x = 5$ $x = 2$

$x = 5$	$x = 2$
$y = 6 - 5$	$y = 6 - 2$
$y = 1$	$y = 4$

$(5, 1)$ and $(2, 4)$

3) $\begin{cases} x + y = -15 \\ xy = 56 \end{cases}$ → $y = -15 - x$

$x(-15 - x) = 56$
$-15x - x^2 = 56$
$0 = x^2 + 15x + 56$
$0 = (x + 7)(x + 8)$
$x = -7$ $x = -8$

if $x = -7$	if $x = -8$
$y = -15 - (-7)$	$y = -15 - (-8)$
$y = -8$	$y = -7$

$(-7, -8)$ and $(-8, -7)$

4) $\begin{cases} 2x^2 + y^2 = 66 \\ x^2 + y^2 = 41 \end{cases}$

① $2x^2 + y^2 = 66$
+ ② $-x^2 - y^2 = -41$

$x^2 = 25$
$x = \pm 5$

if $x = 5$	if $x = -5$
$5^2 + y^2 = 41$	$(-5)^2 + y^2 = 41$
$25 + y^2 = 41$	same
$y^2 = 16$	$y = \pm 4$
$y = \pm 4$	
$(5, 4)$	$(-5, 4)$
$(5, -4)$	$(-5, -4)$

or $(\pm 5, \pm 4)$

For #5 – 7, write the form of the partial fraction decomposition for each rational expression. Do not solve for the constants.

5) $\frac{2x+3}{(x-6)(x+6)}$

$\frac{A}{x-6} + \frac{B}{x+6}$

6) $\frac{(3x-1)}{(x+5)(x+7)^2}$

$\frac{A}{x+5} + \frac{B}{x+7} + \frac{C}{(x+7)^2}$

7) $\frac{2x-5}{(x+2)(x^2+x-4)}$

$\frac{A}{x+2} + \frac{Bx+C}{x^2+x-4}$

For #8 – 10, write the partial fraction decomposition of each rational expression.

8) $\frac{15x-39}{(x-1)(x-5)}$

$\frac{A}{x-1} + \frac{B}{x-5} \rightarrow$

$A(x-5) + B(x-1) = 15x - 39$

if $x=5$: $4B = 36$, $B = 9$

if $x=1$: $-4A = -24$, $A = 6$

$\frac{6}{x-1} + \frac{9}{x-5}$

9) $\frac{36-7x}{x(x-3)^2}$

$\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$A(x-3)^2 + Bx(x-3) + Cx = 36 - 7x$

if $x=3$: $3C = 15$, $C = 5$

if $x=0$: $9A = 36$, $A = 4$

$A(x^2-6x+9) + B(x^2-3x) + Cx$

$Ax^2 - 6Ax + 9A + Bx^2 - 3Bx + Cx = 36 - 7x$

Quadratic: $Ax^2 + Bx^2 = 0x^2$

$4 + B = 0$

$B = -4$

$\frac{4}{x} + \frac{-4}{x-3} + \frac{5}{(x-3)^2}$

10) $\frac{10x+2}{(x-1)(x^2+x+1)}$

$\frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

$A(x^2+x+1) + (Bx+C)(x-1) = 10x + 2$

if $x=1$: $3A = 12$, $A = 4$

$Ax^2 + Ax + A + Bx^2 - Bx + Cx - C = 10x + 2$

Quadratic: $4x^2 + Bx^2 = 0x^2$

$4 + B = 0$

$B = -4$

Constant: $4 - C = 2$

$-C = -2$

$C = 2$

$\frac{4}{x-1} + \frac{-4x+2}{x^2+x+1}$

11) Write the partial fraction decomposition for $\frac{x^2+3x+1}{(x^2+4)^2}$.

$\frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}$

$\frac{1}{x^2+4} + \frac{3x-3}{(x^2+4)^2}$

$(Ax+B)(x^2+4) + Cx + D = x^2+3x+1$

$Ax^3 + 4Ax + Bx^2 + 4B + Cx + D = x^2 + 3x + 1$

cubic: $Ax^3 = 0x^3$, $A=0$

Quadratic: $Bx^2 = 1x^2$, $B=1$

Linear: $4Ax + Cx = 3x$, $Cx = 3x$, $C=3$

Constant: $4B + D = 1$, $4 + D = 1$, $D = -3$

12) Solve by any method of your choice: $\begin{cases} x^2 + y^2 = 29 \\ 4x + y^2 = 17 \end{cases}$

$-1 \cdot$ ① $-x^2 - y^2 = -29$

② $4x + y^2 = 17$

$-x^2 + 4x = -12$

$0 = x^2 - 4x - 12$

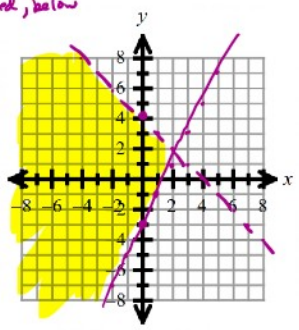
$0 = (x-6)(x+2)$

$x = 6, -2$

if $x=6$	if $x=-2$
$24 + y^2 = 17$	$-8 + y^2 = 17$
$\sqrt{y^2} = \sqrt{-7}$	$\sqrt{y^2} = \sqrt{25}$
$y = \pm i\sqrt{7}$	$y = \pm 5$
$(6, i\sqrt{7})$	$(-2, 5)$
$(6, -i\sqrt{7})$	$(-2, -5)$

For #13 – 16, graph each solution set of the system of inequalities, or indicate that there is no solution.

13) $\begin{cases} y < -x + 4 \\ y \geq 2x - 3 \end{cases}$ dotted, below; solid, above

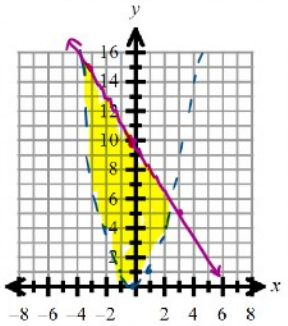


14) $\begin{cases} y > x^2 \\ 10x + 6y \leq 60 \end{cases}$ dotted, above

$6y \leq -10x + 60$

$y \leq -\frac{5}{3}x + 10$

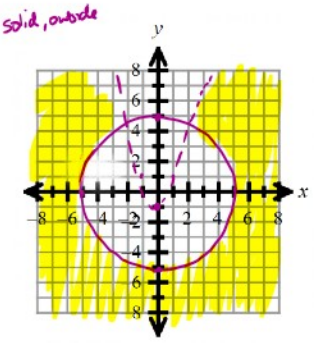
solid below



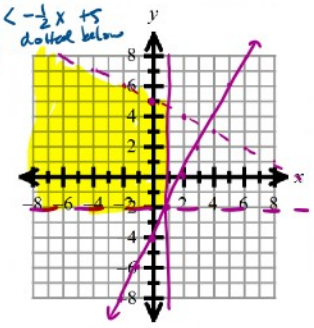
15) $\begin{cases} x^2 + y^2 \geq 25 \\ y - 2x^2 < -1 \end{cases}$ solid, outside

$y < 2x^2 - 1$

dotted below



16) $\begin{cases} y \geq 2x - 4 \\ x + 2y < 10 \\ y > -2 \\ x \leq 1 \end{cases}$ solid, above; $y < -\frac{1}{2}x + 5$ dotted below; dotted, above; solid, left



17) Graph the system of inequalities on the provided coordinate system.

$$\begin{cases} y > 1 \\ 2 \le x < 7 \\ x - 2y \ge -2 \\ x + y < 6 \end{cases}$$

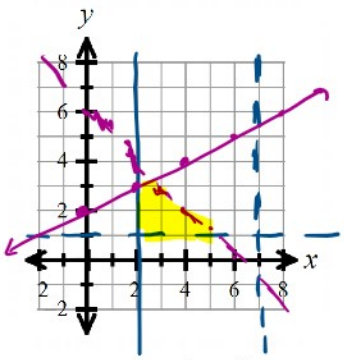
$y > 1$ dotted above

$2 \le x < 7$ between (solid, dotted)

$x - 2y \ge -2 \rightarrow -2y \ge -x - 2$

$y \le \frac{1}{2}x + 1$; solid, below

$y < -x + 6$ dotted below



For #18 – 20, solve problem by using a system of equations.

18) A system for tracking ships indicated that a ship lies on a hyperbolic path described by $5x^2 - y^2 = 20$. The process is repeated, and the ships is also found to lie on the hyperbolic path described by $y^2 - 2x^2 = 7$. It is known that the ship is located in the first quadrant of the coordinate system. Find the coordinates of its exact location.

$5x^2 - y^2 = 20$

$+ \quad -2x^2 + y^2 = 7$

$3x^2 = 27$

$x^2 = 9$

$x = \pm 3$

use $x = +3$

if $x = 3$

$y^2 - 2(3)^2 = 7$

$y^2 - 18 = 7$

$y^2 = 25$

$y = \pm 5$

use $y = 5$

$(3, 5)$

19) The sum of two numbers is -7, and their product is -144. Find the two numbers.

$x + y = -7 \rightarrow y = -x - 7$

$x \cdot y = -144$

$x(-x - 7) = -144$

$-x^2 - 7x = -144$

$0 = x^2 + 7x - 144$

$0 = (x + 16)(x - 9)$

if $x = -16$	if $x = 9$
$y = -(-16) - 7$	$y = -9 - 7$
$y = 9$	$y = -16$

$(-16, 9)$ or $(9, -16)$

20) The sum of the squares of two numbers is 37. The sum of the two numbers is -5. Find the two numbers.

$x^2 + y^2 = 37$

$x + y = -5 \rightarrow x = -5 - y$

$(-5 - y)^2 + y^2 = 37$

$(-5 - y)(-5 - y)$

$25 + 10y + y^2 + y^2 = 37$

$2y^2 + 10y - 12 = 0$

divide by 2

$y^2 + 5y - 6 = 0$

$(y + 6)(y - 1)$

if $y = -6$	if $y = 1$
$x = -5 - (-6)$	$x = -5 - 1$
$x = 1$	$x = -6$

$(1, -6)$ or $(-6, 1)$

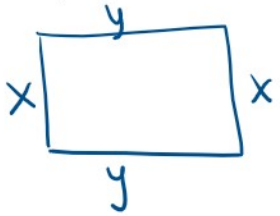
For #21 – 23, solve each problem by using a system of equations.

21) Find the dimensions of a rectangle with perimeter of 42 ft and area of 90 ft^2.

$2x + 2y = 42 \rightarrow x + y = 21 \rightarrow y = 21 - x$

For #21 – 25, solve each problem by using a system of equations.

21) Find the dimensions of a rectangle with perimeter of 42 ft and area of 90 ft^2.



$2x+2y=42 \rightarrow x+y=21 \rightarrow y=21-x$

$xy=90$

$x(21-x)=90$

$21x-x^2=90$

$0=x^2-21x+90$

$0=(x-15)(x-6)$

$x=15 \quad x=6$

if $x=15$	if $x=6$
$y=21-15$	$y=21-6$
$y=6$	$y=15$

(15,6) or (6,15)

22) In a chemistry class, 9 liters of a 4% silver iodide solution must be mixed with a 10% solution to get a 6% solution. How many liters of the 10% solution are needed?

x = liters of 10%

$9+x$ = total # of liters

$9(.04)+.10x=.06(9+x)$

$0.36+.10x=0.54+.06x$

$.04x=.18$

$x=4.5$ liters

23) A chemist needs 170 milliliters of a 27% solution but only has 17% and 51% solutions available. How many milliliters of each should be mixed to obtain the desired solution?

x y

① $x+y=170$

② $.17x+.51y=.27(170)$

$-.17\cdot$ ① $-.17x-.17y=-28.9$

② $.17x+.51y=45.9$

$.34y=17$

$y=50$

so $x=120$

120 ml of 17% sol.
50 ml of 51% sol.

Ch 7 Review Wk continued on next page…

24) Bottled water and medical supplies are to be shipped to survivors of an earthquake by plane. Each container of water bottles will serve 10 people, and each medical kit will aid 6 people. Each plane can carry no more than 80,000 pounds. The bottled water weighs 20 pounds per container, and each medical kit weighs 10 pounds. Each plane can carry a total volume of supplies that does not exceed 6000 ft^3. Each water bottle is 1 ft^3, as is each medical kit. Determine how many bottles of water and how many medical kits should be sent on each plane to maximize the number of earthquake survivors who can be helped.

ft^3, as is each medical kit. Determine how many bottles of water and how many medical kits should be sent on each plane to maximize the number of earthquake survivors who can be helped.

Let x = # bottles of water
Let y = # med kits

Objective Function: $10x + 6y$

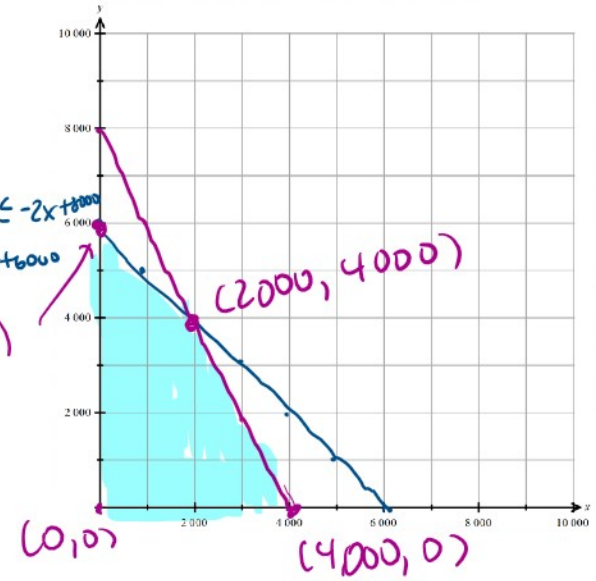
Constraints:

pounds: $20x + 10y \leq 80{,}000 \rightarrow y \leq -2x + 8000$

volume: $x + y \leq 6000 \rightarrow y \leq -x + 6000$

$x \geq 0 \quad y \geq 0$

Solution:



2000 water bottles & 4000 med kits

	$10x + 6y$
(0,0)	0
(0,6000)	36,000
(4000,0)	40,000
(2000,4000)	44,000 ← max

Ch 7 Rev Wk answers are on the next page.

Ch 7 Review Worksheet Answers:

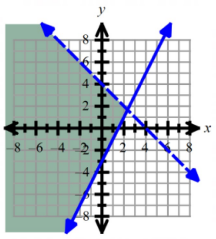
1) $(-2,-4,-3)$ 2) $(2,4);(5,1)$ 3) $(-8,-7);(-7,-8)$

4) $(5,4);(5,-4);(-5,4);(-5,-4)\ldots$ can also be written as $(\pm5,\pm4)$ 5) $\frac{A}{x-6}+\frac{B}{x+6}$

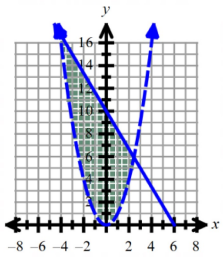
6) $\frac{A}{x+5}+\frac{B}{x+7}+\frac{C}{(x+7)^2}$ 7) $\frac{A}{x+2}+\frac{Bx+C}{x^2+x-4}$ 8) $\frac{6}{x-1}+\frac{9}{x-5}$

9) $\frac{4}{x}-\frac{4}{x-3}+\frac{5}{(x-3)^2}$ 10) $\frac{4}{x-1}+\frac{-4x+2}{x^2+x+1}$ 11) $\frac{1}{x^2+4}+\frac{3x-3}{(x^2+4)^2}$

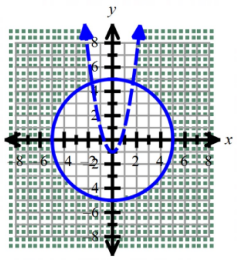
12) $\left(6,i\sqrt{7}\right);\ \left(6,-i\sqrt{7}\right);(-2,5);(-2,-5)$ 13)



14)



15)



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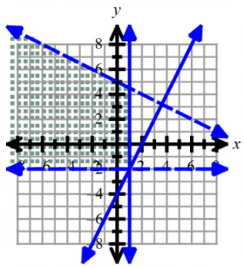
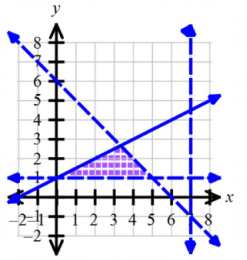


17)



18) $(3,5)$

19) $(-16,9)$ or $(9,-16)$

20) $(-6,1)$ or $(1,-6)$

21) $(6,15)$ or $(15,6)$

22) 4.5 liters of 10% solution

23) 120 ml of 17% solution; 50 ml of 51% solution

24) Solution:

Let x = # of water bottles
Let y = # of medical kits

Objective Function: $10x + 6y$

Constraints:
$20x + 10y \leq 80000$
$x + y \leq 6000$
$x \geq 0$
$y \geq 0$

Solution: (test all 4 vertices)
2000 water bottles and
4000 medical kids