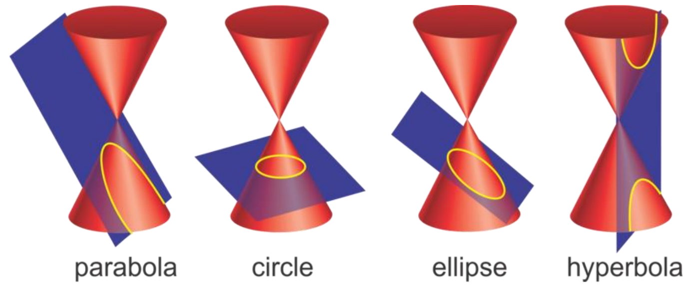
9.0 Notes: Introduction to Conics

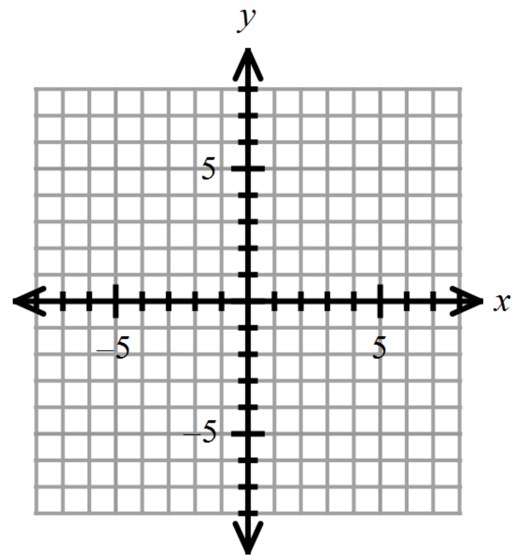


Type of Conic and Standard Form	Key Features and Basic Hints (We will learn more details about graphing conics this unit)
Circle $(x-h)^2+(y-k)^2=r^2$	Center = (h,k) Radius = r The circle is *r* units from the center in all directions
Ellipse $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$	Center = (h,k) Axes length = $2a$ and $2b$ Major axis: the longer axis Minor axis: the shorter axis The endpoints of the horizontal axis are $\pm a$ units left and right from the center The endpoints of the vertical axis are $\pm b$ units up and down from the center
Hyperbola (opens horizontally) $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$	Center = (h,k) the transverse axis is parallel to the *x*-axis the coordinates of the vertices are $(h\pm a,k)$ the coordinates of the co-vertices are $(h,k\pm b)$ Draw a rectangle that includes the vertices and co-vertices; draw asymptotes as diagonals of the rectangle. Use the vertices and the asymptotes to draw the horizontal hyperbola.
Hyperbola (opens vertically) $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$	Center = (h,k) the transverse axis is parallel to the *x*-axis the coordinates of the vertices are $(h\pm a,k)$ the coordinates of the co-vertices are $(h,k\pm b)$ Draw a rectangle that includes the vertices and co-vertices; draw asymptotes as diagonals of the rectangle. Use the vertices and the asymptotes to draw the horizontal hyperbola.
Parabola: *vertex form* **(opens vertically)** $y=a(x-h)^2+k$	Vertex (h,k) Use $\frac{a}{1}$ as $\frac{rise}{run}$ to plot one point on either side of the vertex.
Parabola: *vertex form* **(opens horizontally)** $x=a(y-k)^2+h$	Vertex (h,k) Use $\frac{a}{1}$ as $\frac{run}{rise}$ to plot one point on either side of the vertex.

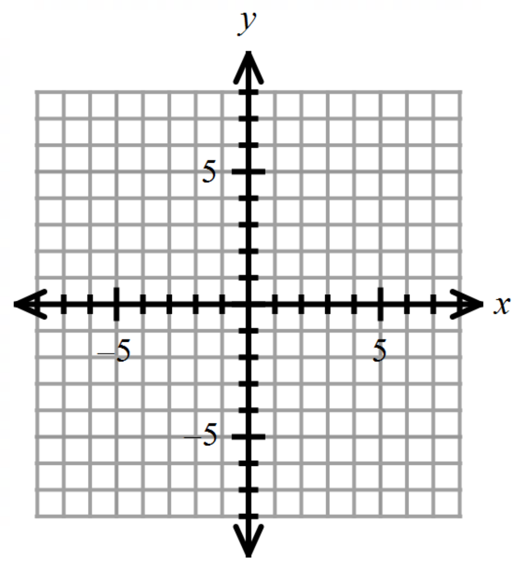
9.0 Notes, continued.

For Examples 1 – 4, identify the type of conic section represented, convert to Standard Form, and then graph each conic.

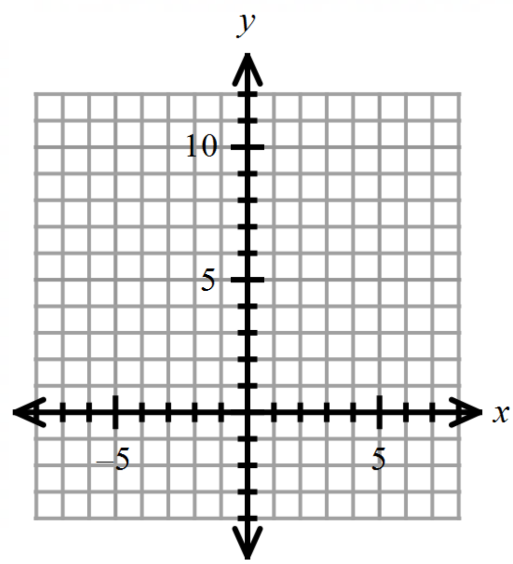
1) $4x^2 + 4y^2 = 64$



2) $x^2 + 9y^2 - 2x - 8 = 0$

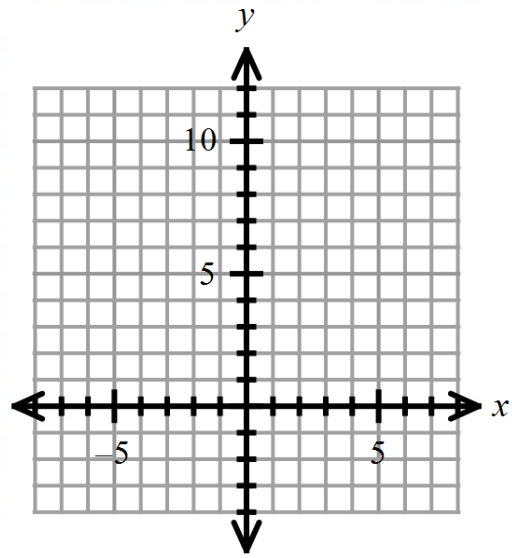


3) $16x^2 - 4y^2 - 64x + 40y = 100$

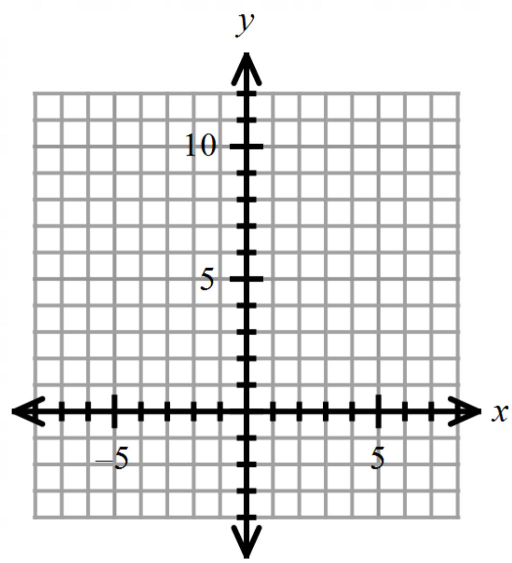


9.0 Notes, continued.

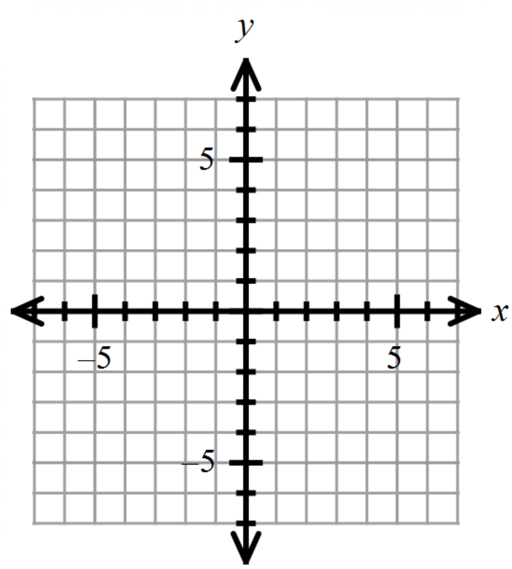
4) $y - x^2 + 8x = 13$



5) $25y^2 - 100y + 100 - x^2 = 25$

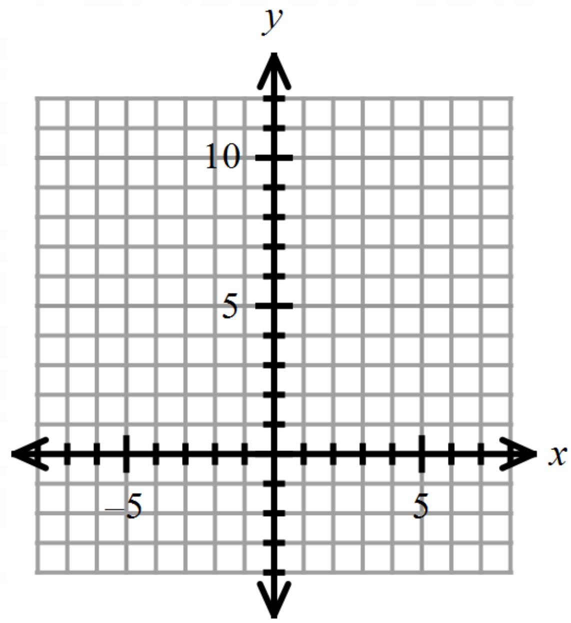


6) $y^2 + 4y - 3x = -16$

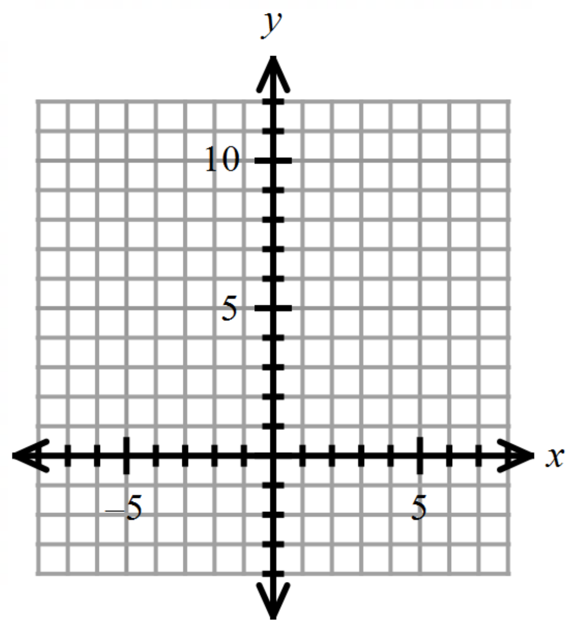


9.0 Notes, continued.

7) $x^2 - 6x - 10y = 2 - y^2$

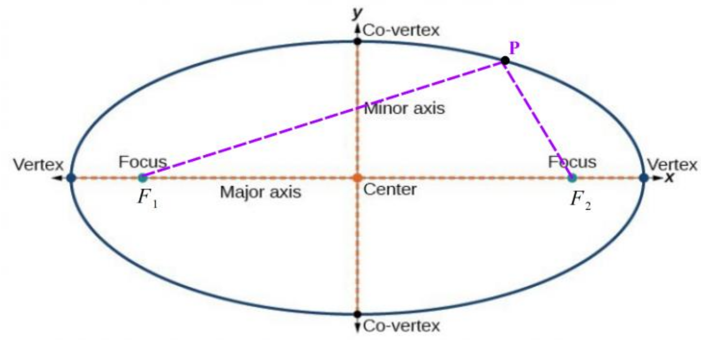


8) $4x^2 + 25y^2 + 16x + 50y - 59 = 0$



9.1 Notes: Ellipses

An ellipse is the set of all points P in a plane such that the sum of the distances from two fixed points F_1 and F_2 is constant.



- These two fixed points are called the foci.
- The midpoint of the segment connecting the foci is the center of the ellipse.

Standard Form of an Ellipse

Centered at the origin	Centered at (h, k)
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ **Center:**	$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ **Center:**

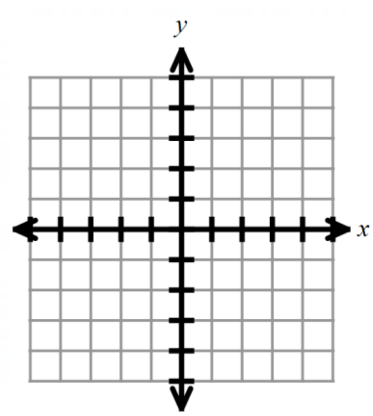
*If $a^2 > b^2$, then the major axis is horizontal. *If $b^2 > a^2$, then the major axis is vertical.

Foci of an Ellipse

Location	Foci Formula
• on the major axis • c units from the center in 2 directions	$b^2 = a^2 - c^2$ or $c^2 = a^2 - b^2$

Example 1: Graph the following and locate the foci:

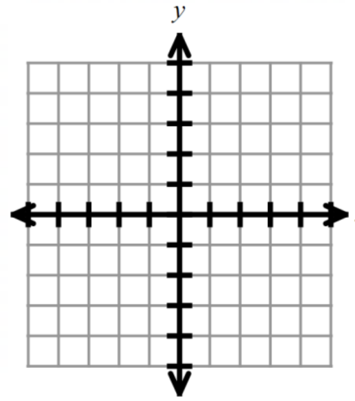
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$



9.1 Notes, continued.

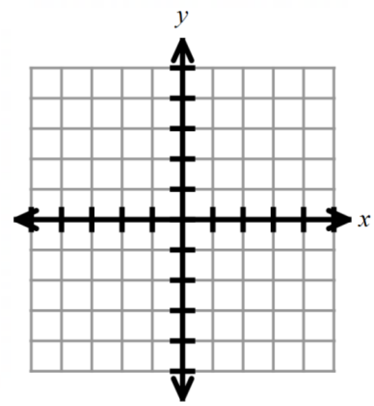
Example 2: Graph the following and locate the foci:

$$16x^2 + 9y^2 = 144$$

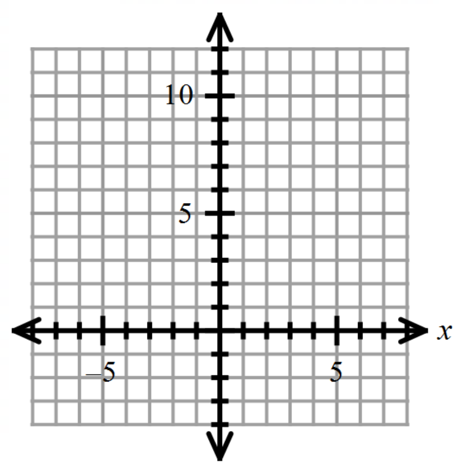


For #3 – 5, find the standard form of the equation of each ellipse and sketch the graph.

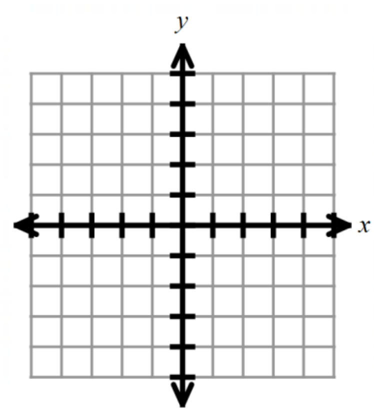
3) foci at (-2, 0) & (2, 0) and vertices at (-3,0) and (3, 0)



4) foci at (0, -4) & (0, 4) and vertices at (0, -7) and (0, 7)



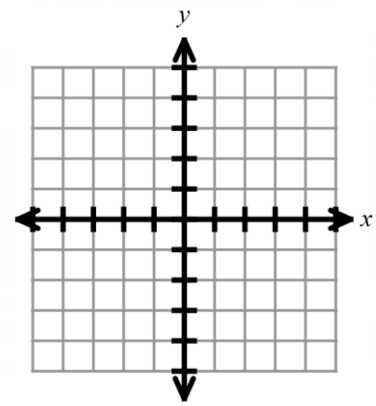
5) endpoints of the major axis at (2, 2) and (8, 2); endpoints of the minor axis at (5, 3) and (5, 1)



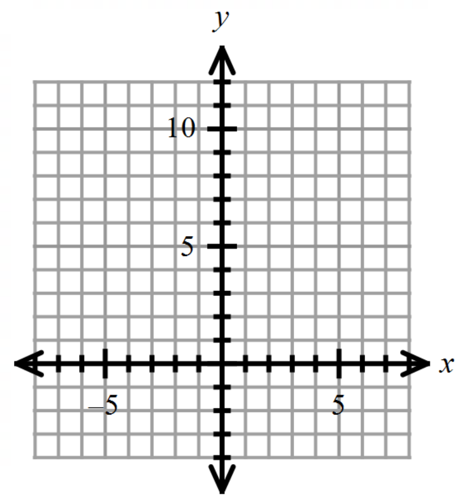
9.1 Notes, continued.

For #6 – 7: Graph each ellipse and locate the foci.

6) $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$



7) $9x^2 + 25y^2 - 36x + 50y - 164 = 0$



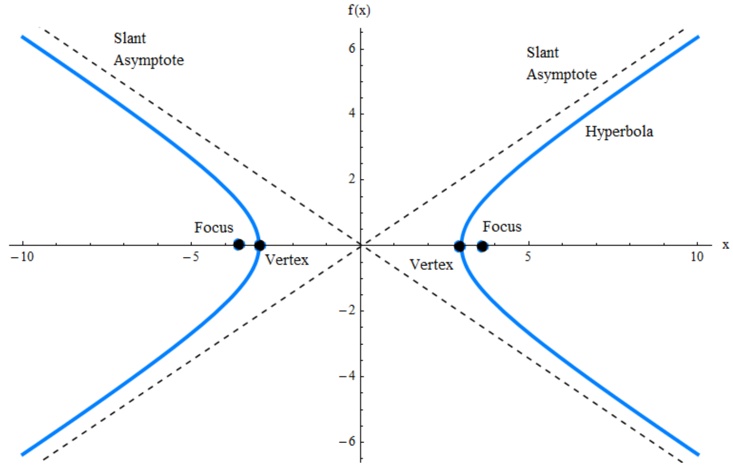
8) A semi-elliptical archway over a one-way road has a height of 10 feet and a width of 40 feet. Your truck has a width of 12 feet and a height of 9 feet. Will your truck clear the opening of the archway?

An exploration with ellipses: https://www.geogebra.org/m/CrVpc8pP

9.2 Notes: Hyperbolas

A hyperbola is the set of all points in a plane whose *difference* from two fixed points, called foci, is constant.

- **Vertices:** Two points that are intersected by a line segment that joins the foci
- **Transverse axis:** The line segment that joins the vertices



Standard Form of a Hyperbola

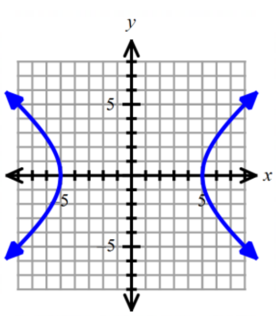
	Centered at the origin	Centered at (h, k)	Transverse Axis	Asymptotes
Opens horizontally	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **Center:**	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ **Center:**		Slope $m = \pm\frac{\text{rise}}{\text{run}}$ Centered at origin: $y = \pm mx$ Centered at (h, k): $y = \pm m(x-h) + k$
Opens vertically	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ **Center:**	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ **Center:**		

Foci of a Hyperbola

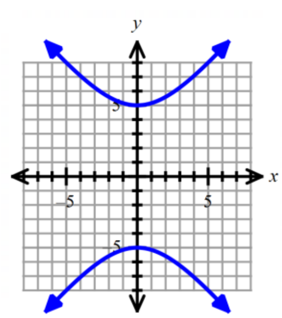
Location	Foci Formula
• on the transverse axis • c units from the center in two directions	$c^2 = a^2 + b^2$

Examples 1 – 2: Find the vertices and locate the foci:

1) $\frac{x^2}{25} - \frac{y^2}{16} = 1$



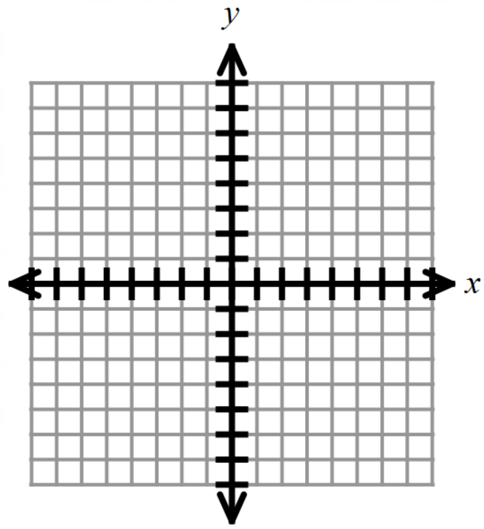
2) $\frac{y^2}{25} - \frac{x^2}{16} = 1$



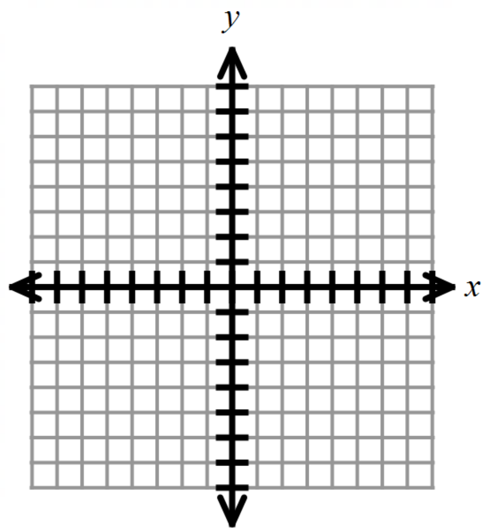
9.2 Notes, continued.

For #3 – 5, write the standard form of the equation of each hyperbola, and sketch each graph.

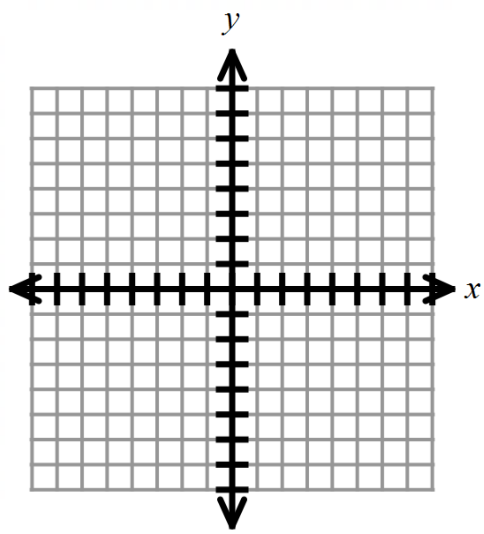
3) foci at (0, -5) and (0, 5) and vertices (0, -3) and (0, 3)



4) foci at (-4, 0) and (4, 0) and vertices (-3, 0) and (3, 0)



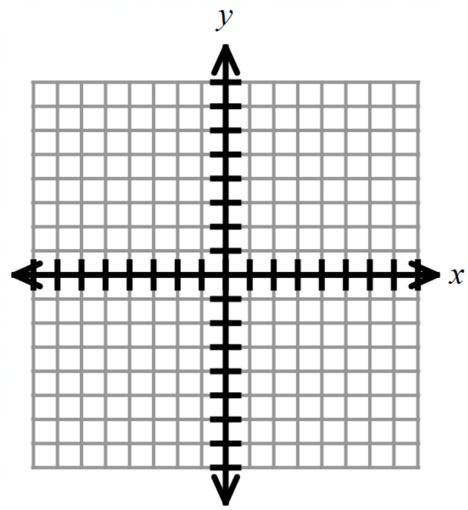
5) center at (4, -2), focus at (7, -2) and vertex at (6, -2)



9.2 Notes, continued.

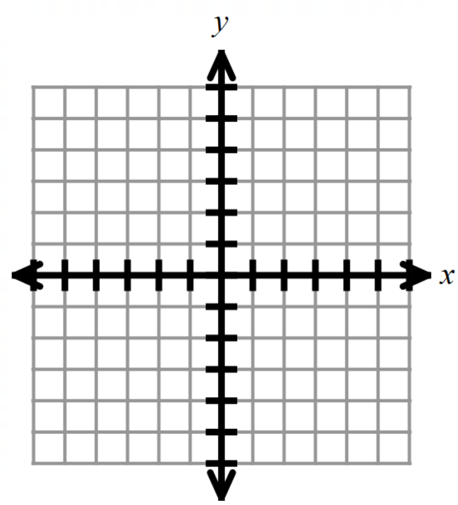
For #6 – 8: Graph each hyperbola and find the requested information.

6) $\frac{x^2}{36} - \frac{y^2}{9} = 1$



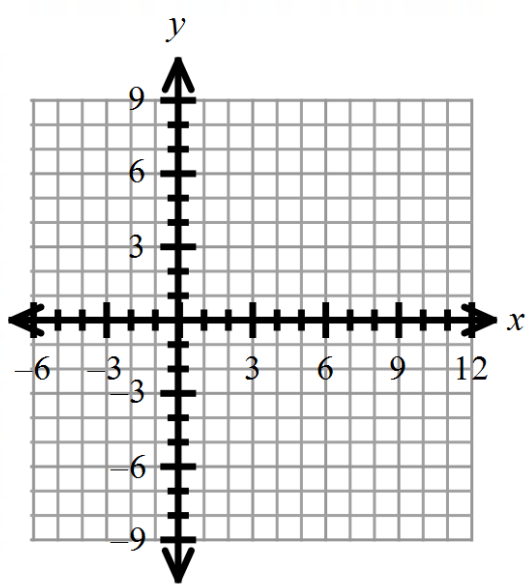
Coordinates of foci	
Equations of asymptotes	

7) $y^2 - 4x^2 = 4$



Coordinates of foci	
Equations of asymptotes	

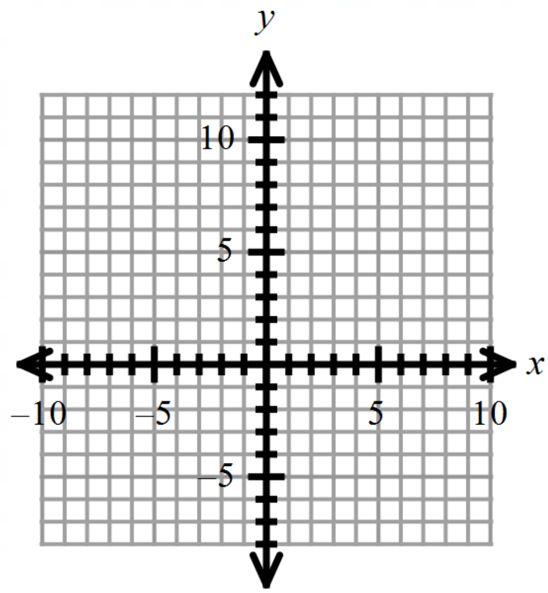
8) $\frac{(x-3)^2}{49} - \frac{(y-1)^2}{25} = 1$



Coordinates of foci	
Equations of asymptotes	

For #9: Graph each hyperbola and find the requested information.

9) $4x^2 - 24x - 25y^2 + 250y - 489 = 0$

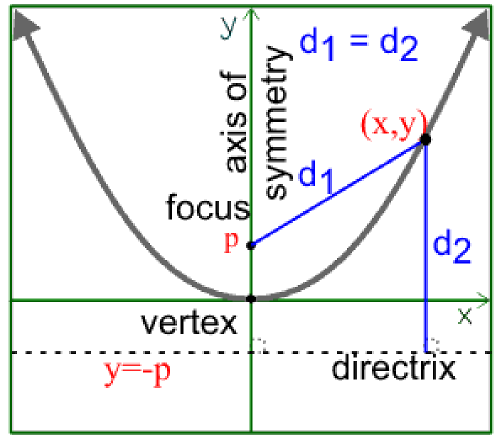


Coordinates of foci	
Equations of asymptotes	

Visualization of Hyperbolas and Foci: https://www.geogebra.org/m/cD5ZyfbC

9.3 Notes: Parabolas

A *parabola* is the set of all points in a plane that are equidistant from a fixed line (the directrix) and a fixed point (the focus).



In previous classes, you have explored the vertex form of a parabola:

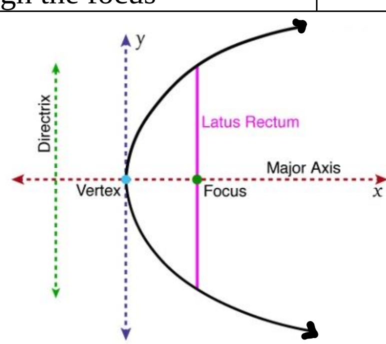
Parabola: *vertex form* **(opens vertically)**	**Parabola:** *vertex form* **(opens horizontally)**
$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$

Standard Form of a Parabola

	Centered at the origin	**Centered at (h, k)**	**Focus**	**Directrix**
Opens horizontally *right:* *left:*	$y^2 = 4px$ **Center:**	$(y-k)^2 = 4p(x-h)$ **Center:**		
Opens vertically *up:* *down:*	$x^2 = 4py$ **Center:**	$(x-h)^2 = 4p(y-k)$ **Center:**		

Latus Rectum of a Parabola

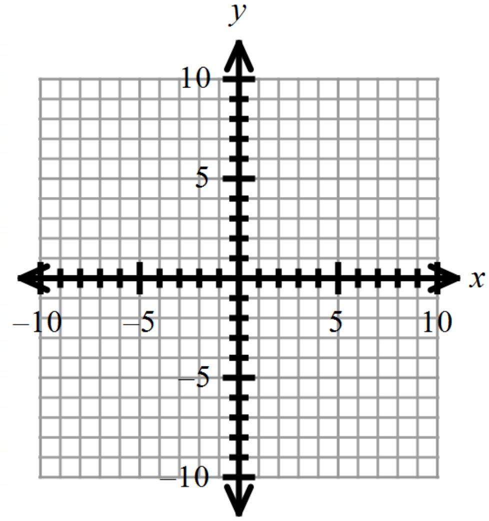
Description	**Length Formula**
• Line segment that connects two points on the parabola • Is parallel to the directrix • Passes through the focus	The length of the latus rectum is $\lvert 4p \rvert$. This is helpful to determine the width of the parabola.



9.3 Notes, continued.

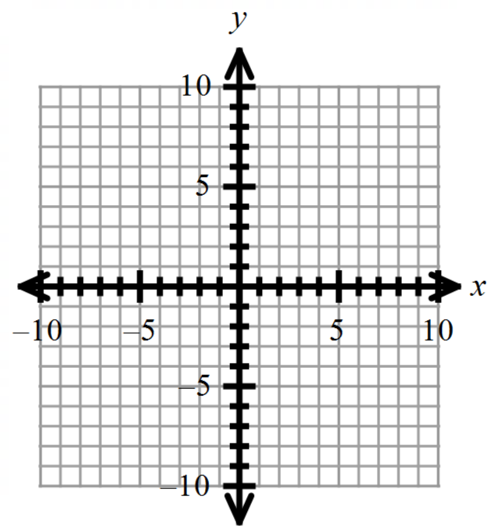
For #1 – 3: Find the requested information and graph each parabola. Include the latus rectum and directrix.

1) $y^2 = 8x$



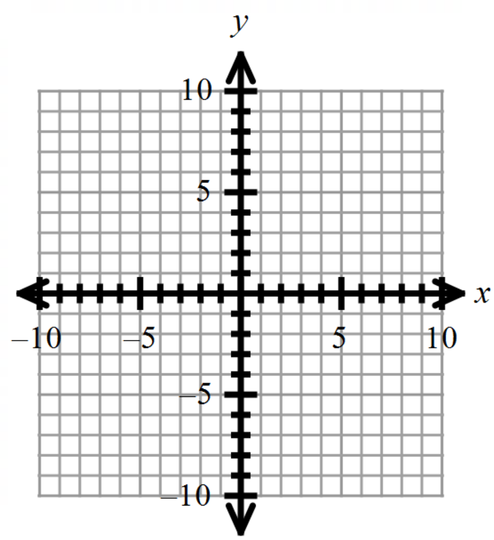
Vertex	Focus
Equation of directrix	**Length of latus rectum**

2) $x^2 = -12y$



Vertex	Focus
Equation of directrix	**Length of latus rectum**

3) $(x-2)^2 = 4(y+1)$

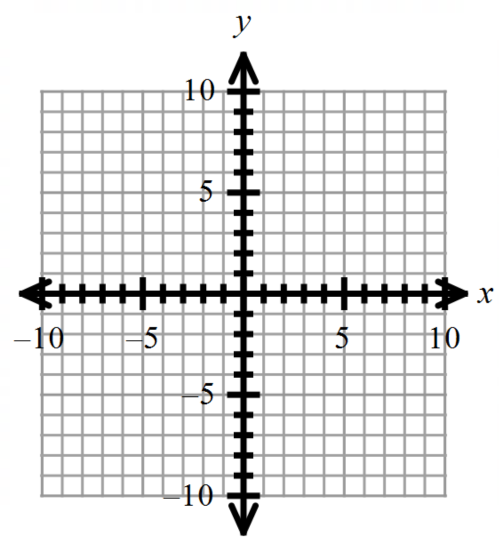


Vertex	Focus
Equation of directrix	**Length of latus rectum**

9.3 Notes, continued.

4) Find the requested information and graph the parabola. Include the latus rectum and directrix.

$y^2 + 2y + 4x - 7 = 0$



Vertex	Focus
Equation of directrix	**Length of latus rectum**

For #5 – 6, write each parabola in standard form with the given information.

5) focus (8, 0) and directrix x = -8

6) focus (0, 20) and directrix $y = -20$

9.3 Notes, continued.

For #7 – 8, write each parabola in standard form with the given information.

7) focus (3, 2) and directrix $x = 5$

8) focus (-3, 4) and directrix $y = 6$

9) An engineer is designing a flashlight using a parabolic reflecting mirror and a light source. The casting has a diameter of 4 inches and a depth of 3 inches. Assume that the parabola is centered at the vertex and opens upward. What is the equation of the parabola used to shape the mirror? At what point should the light source be placed relative to the mirror's vertex?

9.5 Parametric Equations

*Khan Academy Video:
http://www.khanacademy.org/math/trigonometry/parametric_equations/parametric/v/parametric-equations-1

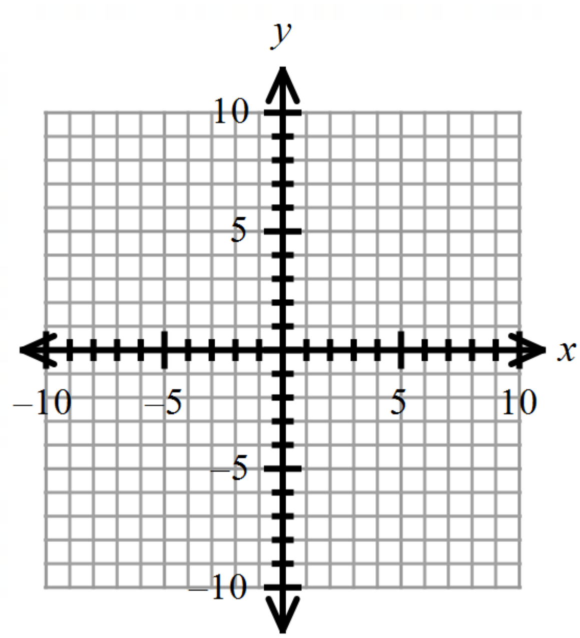
Key Terms

Parametric Equations	
Parameter	
Plane Curve	The set of ordered pairs (x, y) where $x = f(t)$ and $y = g(t)$ for an interval I.

Examples:

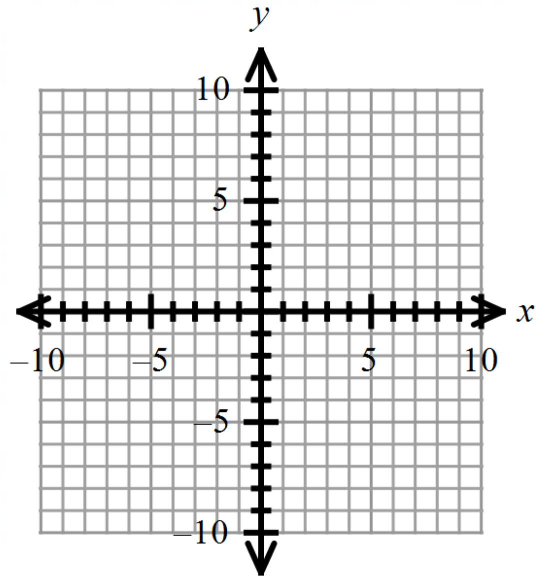
1) Graph the plane curve defined by the parametric equations below on the interval $-2 \le t \le 2$.

$x = t^2 - 1; y = 3t$



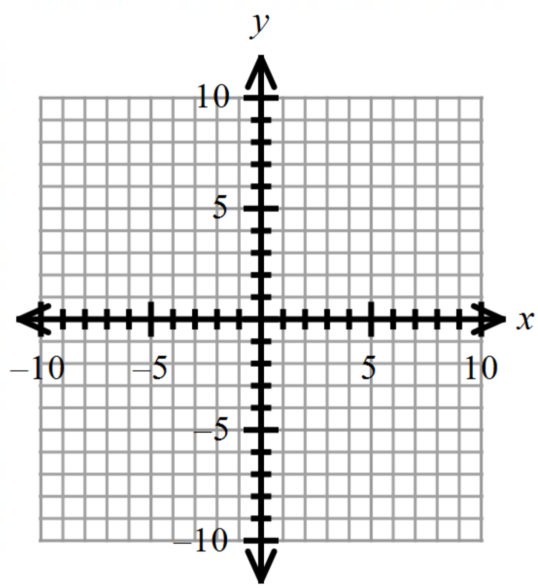
9.5 Notes, continued.

2) Sketch the plane curve represented by the parametric equations $x = \sqrt{t}$ and $y = 2t - 1$ by eliminating the parameter.
Use a table to verify your work.



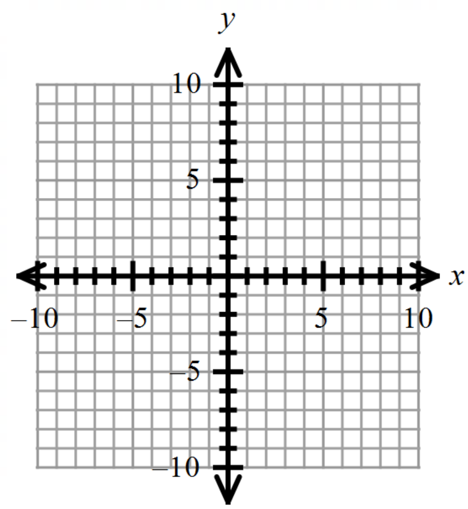
3) Sketch the plane curve represented by the parametric equations below by eliminating the parameter. Use a table to verify your work.

$x = 6\cos t\,;\, y = 4\sin t$ on the interval $\pi \le t \le 2\pi$



For #4 – 5: Find the set of parametric equations for each parabola.

4) $y = 9 - x^2$



5) $y = x^2 - 25$