Consider the following altitude-on-hypotenuse triangle. Identify any similar triangles.



#### **Geometric Mean Theorems:**

### **Examples:** Find the missing variable(s). Exact answers only



2)





4) Given that BC = 6 and AB = 9, find CD.



Formal Geometry 8.2 Day 1 Notes: Pythagorean Triples Name\_\_

**Pythagorean Triples:** 

**Scale Factor:** 

For #1-6: Use Pythagorean Triples to find each variable. Use exact answers only for your final answer. Also, what is the scale factor for each dilation from the original triple?



7) Create a right triangle with two known sides that are fractions, and where the unknown side can be found by using a triple. Then find the missing side without using the Pythagorean Theorem.

8) Silvia drives 14 miles north, 58 miles west, 4 miles north, and then 22 miles west. How far is she from her starting location?

9) Garrett wants to drive from point A to point B on Interstate 25, but due to a detour, he must first drive south for 7.5 miles on Colorado Avenue and then drive east on 6<sup>th</sup> Avenue for 4 miles. How much further did he have to drive due to the detour than if he could have stayed on Interstate 25?



**Challenge:** 3, 4,5 and 7, 24, 25 are special triples in that the hypotenuse is one unit longer than the larger leg. Find a Pythagorean Triple where one leg is 11 units and the hypotenuse is 1 unit more than the larger leg. Justify your conclusion with mathematical reasoning.

#### Formal Geometry 8.2 Day 2 Notes: The Pythagorean Theorem

Name\_

#### **Proof of the Pythagorean Theorem:**

Given:  $\triangle ABC$  is a right triangle with hypotenuse *c*, legs *a* and *b*, and altitude *h*, as shown. Prove:  $a^2 + b^2 = c^2$ 

Step 1: Consider the following similar triangles:

Why is  $\Delta BXC \sim \Delta BCA$ ?

Why is  $\triangle AXC \sim \triangle ACB$ ?



Step 2: Complete the following proportions based on the similar triangles.

Proportion 1 (use  $\Delta BXC \sim \Delta BCA$ ):  $\frac{a}{c} = \frac{?}{a}$ 

Proportion 2 (use  $\triangle AXC \sim \triangle ACB$ ):  $\frac{b}{c} = \frac{?}{b}$ 

Step 3: Cross-multiply each proportion to get two equations.

Step 4: Add your equations together.

Step 5: Factor out a GCF from the right side of your equation. Use substitution (look at the diagram to help with this!)

1) Find the height of the roof shown, if the base of the roof is 30 feet. Write your answer as a simplified radical.



2) Is  $\triangle ABC$  is a right triangle? Justify your conclusion.

3) Given a right triangle with hypotenuse  $c = 2\sqrt{14}$  and one leg of 6. Find the length of the other leg. Exact answers only (no decimals.)





Converse of the Pythagorean Theorem: If the sum of the squares of the shorter sides of a triangle are equal to the square of the longest side, then the triangle is a right triangle.

If  $a^2 + b^2 = c^2$ , then \_\_\_\_\_

If  $a^2 + b^2 < c^2$ , then \_\_\_\_\_

If  $a^2 + b^2 > c^2$ , then \_\_\_\_\_

For #6 – 7, classify each triangle as acute, right, or obtuse when given the 3 sides.

6) 7, 15, 21 7) 4.5, 20.5, 20

For #8 - 9: Solve for *x*. Leave your answer in radical form, if needed.



10) A radio tower has a shadow that is 40.2 meters long. At the same time of day, a mailbox that is 1.2 meters tall has a shadow that is 0.9 meters long. Find the distance from the end of the radio tower's shadow to the top of the radio tower, rounded to the nearest tenth of a meter.



## **Formal Geometry**

**Consider the following equilateral triangle:** 



30-60-90 Triangles:

**Examples:** 

## **Additional Examples:**

- Find the area of an equilateral triangle with a height of 15cm.
- Find the perimeter of a 30-60-90 triangle with the shortest leg of  $2\sqrt{3}$  in.

**Consider the following square:** 

1

45-45-90 triangles:

## **Additional Examples:**

- Find the area of a square with a diagonal of 12 mm.
- Find the diagonal of a square with a perimeter of 30 in. Round your answer to the nearest tenth.

#### **8.4 Notes: Right Triangle Trig**

**Trigonometry:** The study of the relationship between sides and angles in triangles. Today we will go over **right triangle trigonometry.** 

Soh Cah Toa

Label the sides in the triangle below, with respect to angle A, as opposite, adjacent, or hypotenuse.



Which sides change labels if you change the angle of reference to angle B?

Which side is still labeled the same way?

**Trig ratios:** 

Sine  $(\sin) = \frac{\text{opposite}}{\text{hypotenuse}}$ 

 $Cosine (cos) = \frac{adjacent}{hypotenuse}$ 

Tangent (tan) = 
$$\frac{\text{opposite}}{\text{adjacent}}$$

**Example 1:** Use the triangle below to find the following trig functions. Simplify all answers.

$\sin \angle A$	$\cos \angle A$	tan∠A	A
$\sin \angle B$	$\cos \angle B$	$tan \angle B$	16 34
Note: we only will only find trig functions for <i>acute angles</i> this year. Try to find the tangent of angle C to see why.			C 30 B

**Example 2: Calculator practice:** Make sure your calculator is in degrees. Find the following values to the nearest hundredth.

sin 33 tan 16 cos 79



**Example 5:** Find *x* and *y* to the nearest hundredth.



Angle of elevation: The angle made from a horizontal line going up.

In the diagram shown, \_\_\_\_\_ is the angle of elevation.



**Example 6:** A 15-foot long ladder is leaning against a building. Find the height that the ladder hits the building at if the angle of elevation is 39 degrees. Round to the nearest hundredth.

Inverse Trig Functions: Functions used to "undo" a trig function and find a missing angle.

Inverse sine:  $\sin^{-1} x$  Inverse cosine:  $\cos^{-1} x$  Inverse tangent:  $\tan^{-1} x$ 

Note: these buttons are "above" the sin, cos, tan buttons on your calculator. Hit SHIFT in order to use them.

**Example 7:** Find *x*. Round to the nearest tenth.

- a)  $\sin x = 0.35$  b)  $\tan x = 1.72$  c)  $\cos x = 3$
- d)  $\tan 25 = x$  e)  $\sin x = 0.5$

f)  $\cos x = 0.882$ 

**Example 9:** Find *x*. Round to the nearest hundredth.



Example 10: SOLVE the right triangle. (This means find all angles and all sides.) Round to the nearest tenth.



Example 11: SOLVE the right triangle. Round to the nearest tenth.



# Formal Geometry 8.5 Guided Notes: Angles of Elevation and Depression

#### **Angle of Elevation**

**Angle of Depression** 

**Example 1:** Find the angle of elevation to the peak of a mountain for an observer who is 155 meters from the base of the mountain (directly below the peak of the mountain) if the observer's eye is 1.5 meters above the ground and the mountain is 350 meters tall.



**Example 2:** The Unzen Volcano in Japan has a magma reservoir that is located beneath the Chijiwa Bay. The magma rises eastward at an angle of elevation of 40° for 25 km. How far below sea level is the magma reservoir?



**Example 3**) Austin is standing on the high dive at the local pool. Two of his friends are in the water at the opposite side of the pool. If the angle of depression to the closest friend is  $40^{\circ}$ , and  $30^{\circ}$  to his other friend, and the diving platform is 9.3 feet tall, then how far apart are Austin's friends?

**Example 4)** In the figure below, *A* and *B* represent the top and the bottom of a large balloon floating directly above the street. Arnold is standing 30 feet from a point on the street directly beneath the balloon. Find the height *h* of the balloon, rounded to 2 decimal places.



The Law of Sines



**Examples:** Find the variable(s).







## **Formal Geometry**



**Examples:** Find the variable(s).





6) Find all angles of a triangle with sides of 6, 7, and 8.

7) Two ships leave the port at 8 a.m. One is headed at a bearing of N 38 E and is traveling at 13.5 miles per hour. The other ship is traveling at 16 miles per hour at a bearing of S 47 E.

- A. Each ship travels 2 hours from the port. After 2 hours, how far apart are the ships?
- B. After travelling for 2 hours, the ships turn toward each other to rendezvous for lunch. What time is lunch (to the nearest minute)?

N 38° a distance d= between ships



S

Ν

8) Two ships leave the port at 8 a.m. One is headed at a bearing of N 38 E and is traveling at 13.5 miles per hour. The other ship is traveling at 16 miles per hour at a bearing of S 47 E.

- A. Each ship travels 4 hours from the port. After 4 hours, how far apart are the ships?
- B. After travelling for 4 hours, the ships turn toward each other to rendezvous for dinner. What time is dinner (to the nearest minute)?

Compare the hypothetical lunch and dinner times. Did the problem "scale" the way that you anticipated?