4.1 Notes: Angles in Triangles

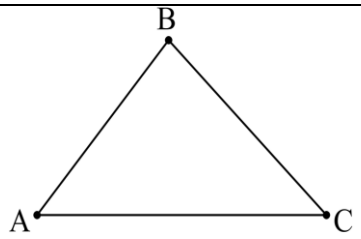
Objectives:

- **Students will be able to find missing interior angles in triangles.**
- **Students will be able to find exterior angles in triangles.**

Exploration #1: Use the following link to explore angles in a triangle:
https://www.geogebra.org/m/FzUM9TeD

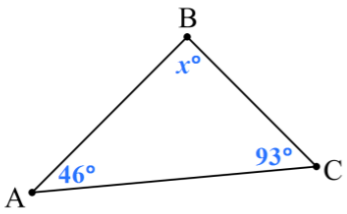
- Click on the vertices of the triangle to change the angle measurements.
- What do you notice?

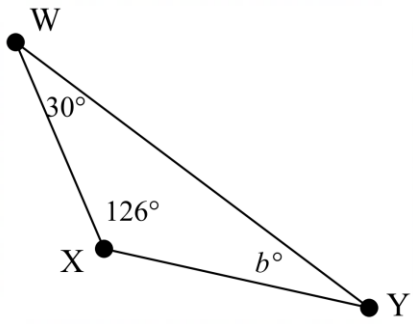
- Make a **conjecture** ("guess") about the angles in a triangle.

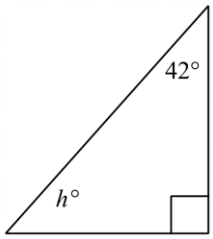
Triangle Sum Theorem	**The sum of the measures of the interior angles of a triangle is _____.** $\angle A + \angle B + \angle C =$ _____°	

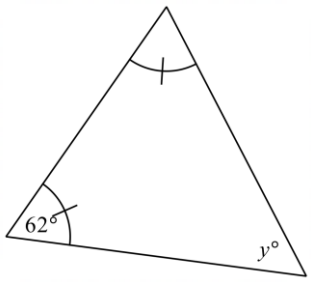
A good animation of the Triangle Sum Theorem: https://www.geogebra.org/m/FAhtKpR5

Examples 1 – 4: Find the measure of each missing angle in the triangle shown.

1) 

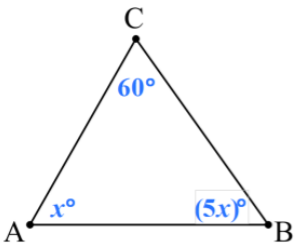
2) **You try!** 

3) **You try!** 

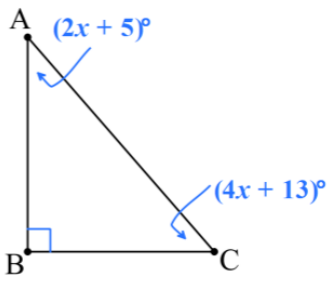
4) **You try!** 

Examples 5 – 7: Find the measure of the variable.

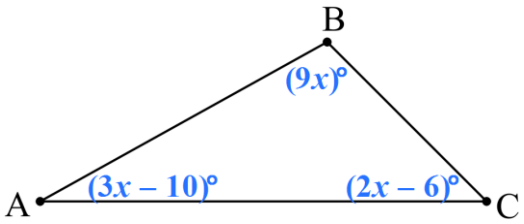
5)



6)

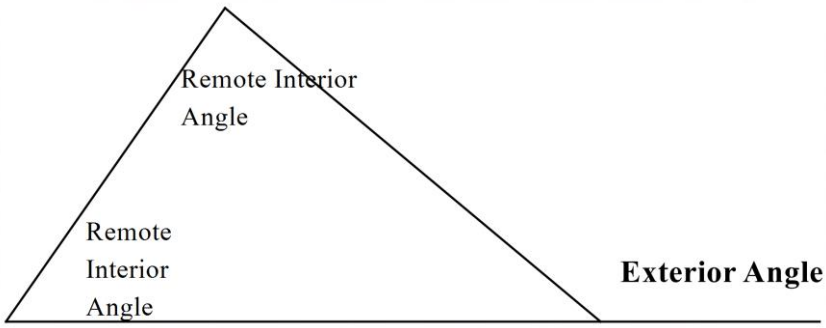


7) **You Try!**



8) Find the measure of $\angle C$ in the triangle from #7.

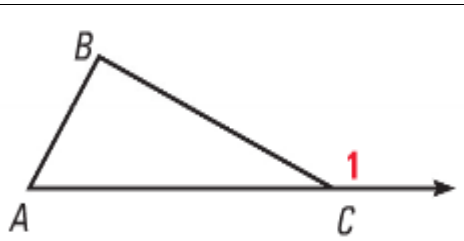
Some new names for special angles in a triangle:



Exploration #2: https://www.geogebra.org/m/sA5Mb4vd

- Click on the link to explore the relationship between an Exterior Angle and its two Remote Interior Angles.
- Make a **conjecture**:

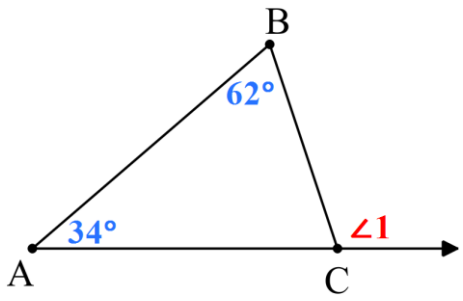
EXTERIOR ANGLE THEOREM

Exterior Angle Theorem	**The measure of an exterior angle of a triangle is equal to the _____ of the two remote interior angles.**	

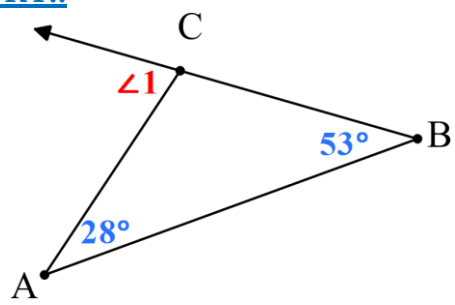
Why is this true?

Example #9: Find the measure of each exterior angle (∠1).

a.

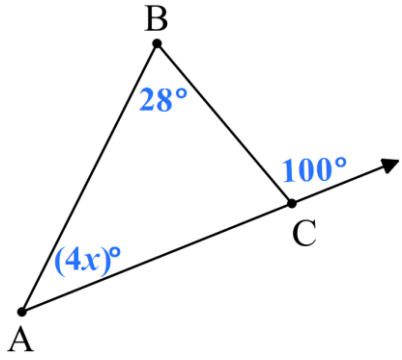


b. YOU TRY!!

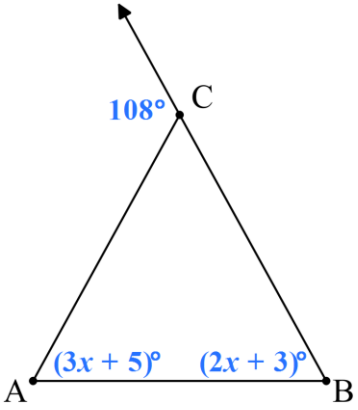


Example #10: Find the value of x in each triangle.

a.



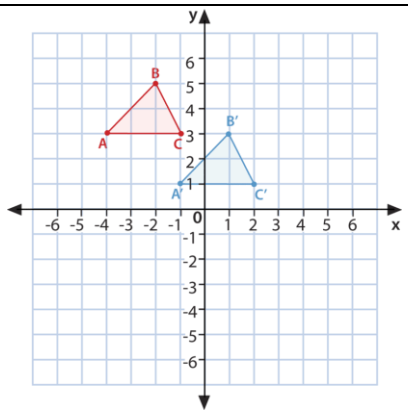
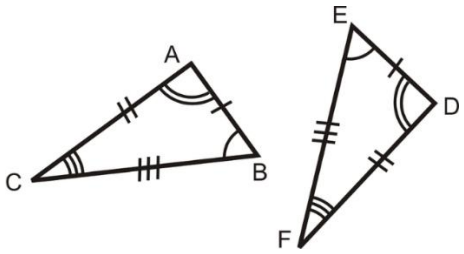
b. YOU TRY!!



4.2 Notes: Intro to Congruent Triangles

Objectives:

- **Students will be able to make a congruency statement for congruent triangles.**
- **Students will be able to list corresponding parts for congruent triangles.**
- **Students will be able to find missing measures in congruent triangles.**

Congruent Triangles	**If two triangles are congruent, then there is a _____ transformation (or more than one) that _____ one shape onto the other.**	
Congruence Statement	**A congruence statement lists two _____ shapes. The order of the letters listed shows which corresponding parts are congruent.**	
CPCTC	Given that two triangles are congruent, then C_____ P_____ of C_____ T_____ are C_____	

Example #1:

Given that $\Delta ABC \cong \Delta FED$. Identify all corresponding sides and corresponding angles.

a. $\overline{AB} \cong$ _____

b. $\overline{BC} \cong$ _____

c. $\overline{AC} \cong$ _____

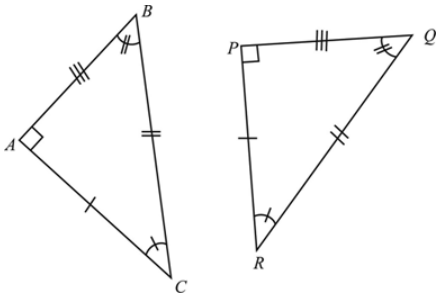
d. $\angle A \cong$ _____

e. $\angle B \cong$ _____

f. $\angle C \cong$ _____

Example #2:

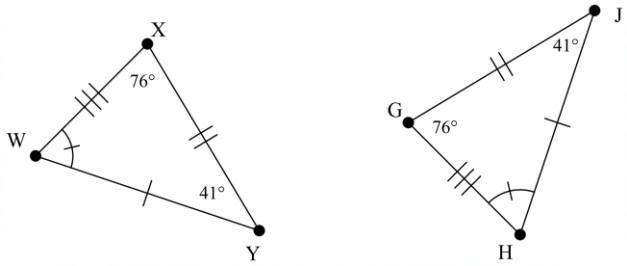
Complete the congruence statement for the pair of triangles shown.



Congruence Statement: $\Delta QRP \cong$ _____

YOU TRY! Example #3:

Complete the congruence statement for the pair of triangles shown. Then identify all corresponding sides and angles.



Congruence Statement:

a. $\overline{WX} \cong$ _____

b. $\overline{WY} \cong$ _____

c. $\overline{XY} \cong$ _____

d. $\angle X \cong$ _____

f. $\angle Y \cong$ _____

g. $\angle W \cong$ _____

YOU TRY! Example #4: Given the corresponding parts that are congruent for two triangles, write the congruence statement for the two triangles.

$\overline{PL} \cong \overline{AB}$ $\quad \angle P \cong \angle A$ $\quad \overline{VP} \cong \overline{CA}$

$\overline{LV} \cong \overline{BC}$ $\quad \angle L \cong \angle B$ $\quad \angle V \cong \angle C$

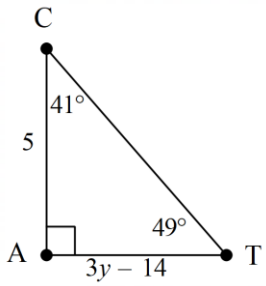
Congruence Statement:

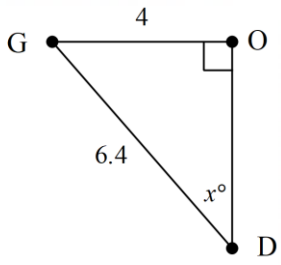
Example #5:

In the diagram, $\Delta DOG \cong \Delta CAT$.

a. Find the value of x.

b. Find the value of y.



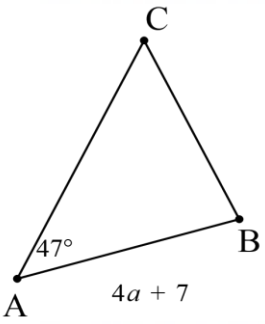


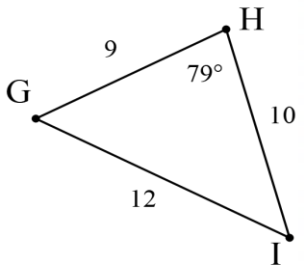
c. Find the perimeter of ΔCAT.

YOU TRY! Example #6:

In the diagram, $\Delta ABC \cong \Delta IHG$.

a. Find the value of a.

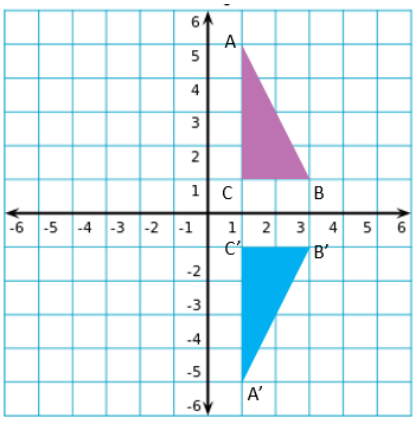




b. Find the measure of $\angle G$.

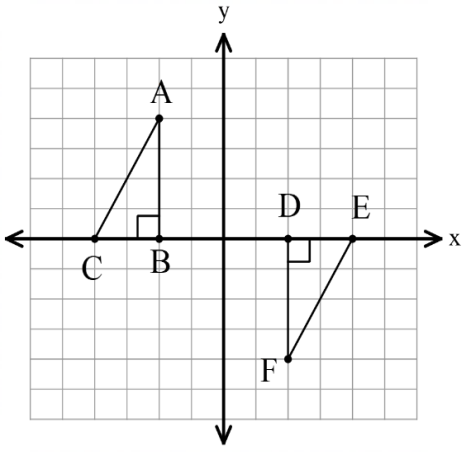
Example #7: Given the two triangles congruent triangles shown. Which statement below lists the rigid transformation that maps the pre-image onto the image?

A) Translation $\langle 0, -2 \rangle$
B) Rotation 180 degrees clockwise about the origin.
C) Reflection in the y-axis.
D) Reflection in the x-axis.
E) Rotation 90 degrees clockwise about the origin.



Example #8: Given the two triangles congruent triangles shown. Which statement below lists the correct congruence statement and also the rigid transformation that maps one triangle onto the other?

A) $\Delta ABC \cong \Delta DEF$; rotation 180 degrees about the origin
B) $\Delta ABC \cong \Delta FDE$; rotation 180 degrees about the origin
C) $\Delta ABC \cong \Delta DEF$; reflection in the y-axis
D) $\Delta ABC \cong \Delta FDE$; reflection in the y-axis



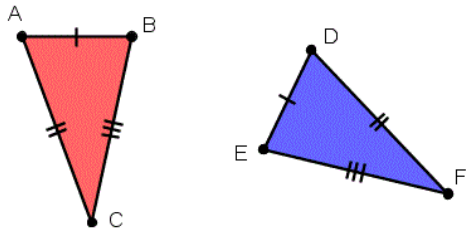
4.3 Notes: SSS and SAS

Objectives:

- **Students will be able to identify the postulate or theorem to prove congruent triangles**
- **Students will be able to prove congruent triangles with SSS or SAS in two-column proofs and use CPCTC.**

Exploration #1: Use the following link to explore triangles with the same side lengths:
https://www.geogebra.org/m/rxsTwHF9

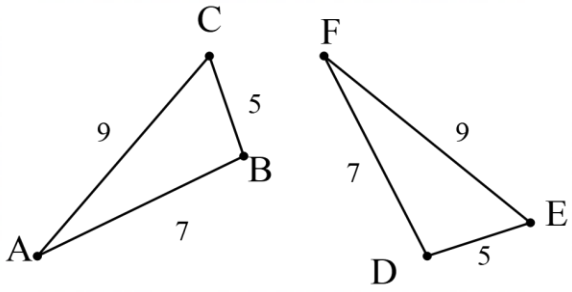
- Use the sliders to change the lengths of the sides of the first triangle.
- Move the white circle to change the location of one vertex of the second triangle.
- Notice that both triangles have the same three side lengths.
- Make a **conjecture** ("guess") about two triangles that have the same three side lengths:

Side-Side-Side Postulate (SSS)	**If 3 sides of one Δ are ≅ to 3 sides of another Δ, then the Δs are _____ by SSS.** A postulate is a statement that is agreed to be true but cannot be proven to be true.	

Example 1: Given the diagram shown, are the triangles congruent?

How do you know?

If they are congruent, then write a congruence statement.

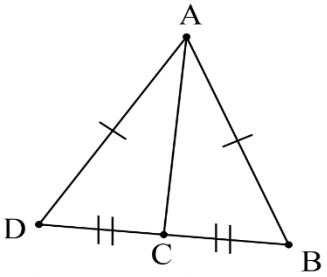


Reflexive Property	**The Reflexive Property states that any side or angle is _____ to _____.**	

Example 2: Given the diagram shown, are the triangles congruent?

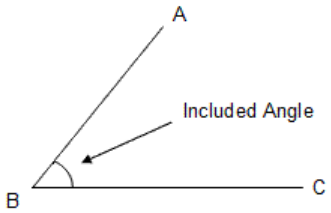
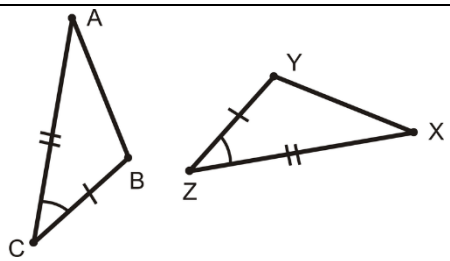
How do you know?

If they are congruent, then write a congruence statement.

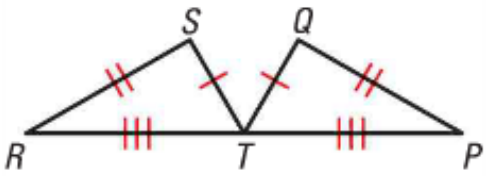


Exploration #2: Use the following link to explore triangles with the two side lengths and one included angle that are congruent: https://www.geogebra.org/m/bM5FkyFK

- Use the sliders on the right to change the lengths of the sides and the measure of the included angle.
- Use the slider on the bottom to copy the first triangle onto the second triangle.
- Make a **conjecture** ("guess") about two triangles that have the two congruent sides and one congruent included angle:

Included Angle	**An Included Angle is the angle _____ two sides of a triangle. The sides of the triangle are the sides of the angle.**	
Side-Angle-Side Postulate (SAS)	**If 2 sides and an included ∠ of one Δ are ≅ to 2 sides and an included ∠ of another Δ, then the Δs are _____ by SAS.**	

Example #4: Which postulate can be used to prove that $\Delta RST \cong \Delta PQT$?



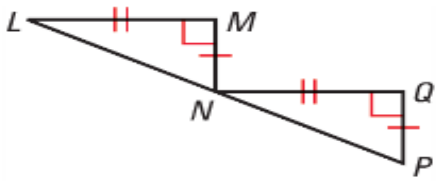
a. SSS

b. SAS

c. Not Possible; the triangles aren't congruent

Example #5:

Which postulate can be used to prove that $\Delta LMN \cong \Delta NQP$.



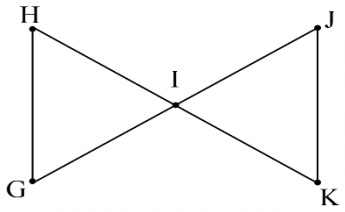
a. SSS

b. SAS

c. Not Possible; the triangles aren't congruent

Two-Column Proof	**A Two-Column Proof is a method of collecting evidence to justify a conclusion**	
Evidence	**Evidence that can be used includes Given information, Definitions, Theorems, Postulates, and Properties.**	**Consider some of the following:** • Given • If two angles are vertical, then they are ≅ • Reflective Property • If a point is a midpoint, then the segment is divided into 2 ≅ segments. • SSS • SAS • CPCTC

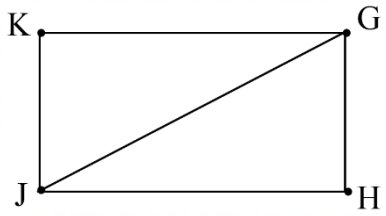
Example #6: Given: $HI \cong IK$ and $GI \cong IJ$
Prove: $\Delta HIG \cong \Delta JIK$.



Statements	Reasons

Example #10: Given: $KG \cong HJ, KJ \cong HG$

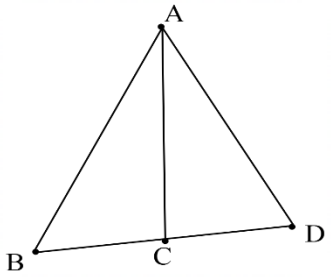
Prove: $\Delta JKG \cong \Delta GHJ$



Statements	Reasons

Example #11: Given: $AB \cong AD, \angle B \cong \angle D$, C is the midpoint of BD.

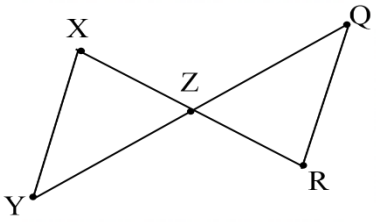
Prove: $\Delta ABC \cong \Delta ADC$



Statements	Reasons

Example #12: Given: $XZ \cong RZ, YZ \cong QZ$

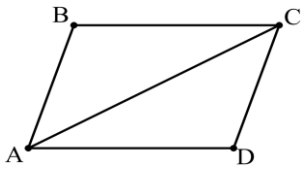
Prove: $\angle Y \cong \angle Q$



Statement	Reason
1.	1. Given
2.	2. If two angles are vertical, then they are congruent.
3.	3. SAS
4.	4.

Example #13: Given: $AB \cong CD, BC \cong DA$

Prove: $\angle B \cong \angle D$



Statement	Reason
1.	1. Given
2.	2.
3. $\Delta ABC \cong \Delta CDA$	3.
4. $\angle B \cong \angle D$	4.

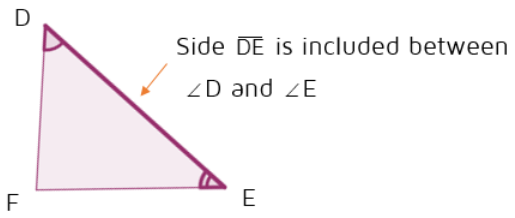
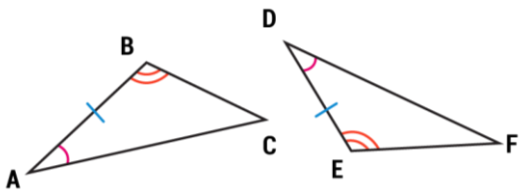
4.4 Notes: ASA, AAS, and HL

Objectives:

- **Students will be able to identify the postulate or theorem to prove congruent triangles**
- **Students will be able to prove congruent triangles in two-column proofs.**

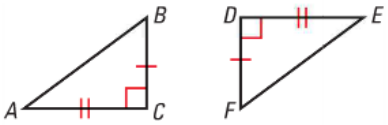
Exploration: Use the following link to explore triangles with two angles and one included side congruent: https://www.geogebra.org/m/WKJJ2uPa

- Use the pink and blue sliders to change the angles of the first triangle.
- Use the black slider. See what happens!
- Make a **conjecture** ("guess") about two triangles with two angles and one included side congruent:

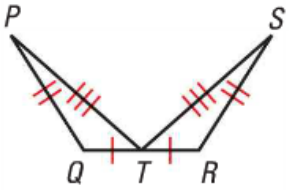
Included Side	**If a side is _____ two angles of a triangle, then that side is the Included Side.** **The included side is the shared side of the two triangles.**	
Angle- Side- Angle Theorem (ASA)	**If 2 ∠s and the included side of one Δ are ≅ to 2 ∠s and the included side of another Δ, then the Δs are _____ by ASA.**	

For #1 – 4: State the theorem or postulate that proves each pair of triangles congruent (SSS, SAS, or ASA).

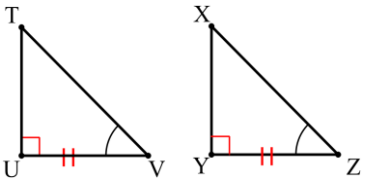
1. $\Delta ABC \cong EFD$



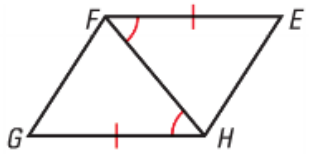
2. $\Delta PQT \cong \Delta SRT$



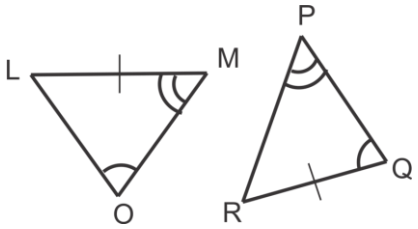
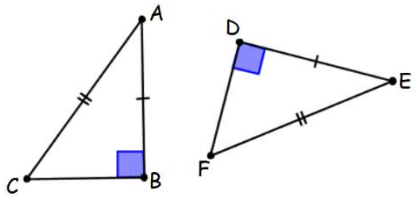
3. $\Delta TUV \cong \Delta XYZ$



4. $\Delta FGH \cong \Delta HEF$

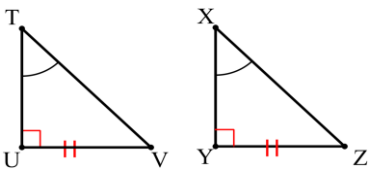


There are two other ways to prove triangles congruent:

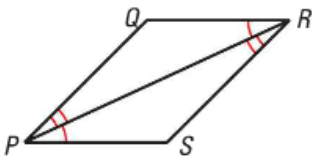
Angle-Angle-Side Theorem (AAS)	**If 2 ∠s and a non-included side of one Δ are ≅ to 2 ∠s and a non-included side of another Δ, then the Δs are _____ by AAS.**	
Hypotenuse-Leg Theorem (HL)	**If the hypotenuse and one leg of one right Δ are ≅ to the hypotenuse and the corresponding leg of another right Δ, then the Δs are _____ by HL.**	

For #5 – 10: State the theorem or postulate that proves each pair of triangles congruent (SSS, SAS, ASA, AAS, or HL).

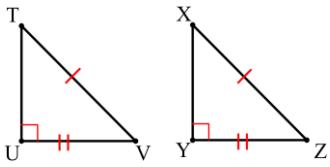
5)



6)



7)



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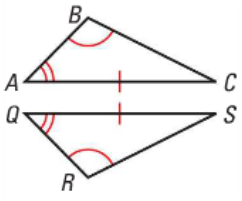
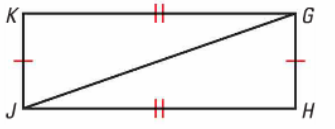
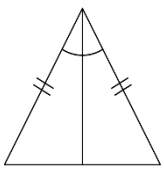


9)

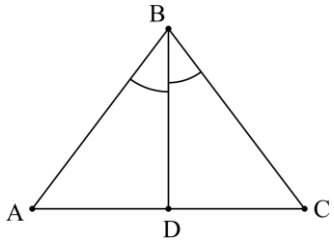


10)



Example #11:

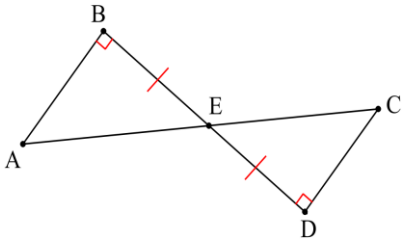
What additional information is needed to prove $\Delta ABC \cong \Delta CDE$ by ASA?



a. ∠ABD ≅∠CBA
b. ∠ADB ≅∠CDB
c. ∠ABD ≅∠CDB
d. ∠ABD ≅∠BCD

You try! Example #12:

What additional information is needed to prove $\Delta ABE \cong \Delta CDE$ by AAS?

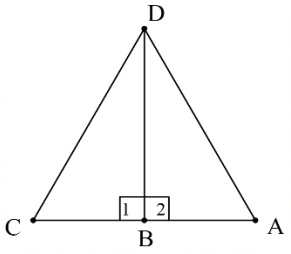


a. ∠A≅∠D
b. ∠D≅∠C
c. ∠B≅∠D
d. ∠C≅∠A

Example #13:

Given: $CB \cong AB$, $\angle 1$ and $\angle 2$ are right angles

Prove: $\Delta BCD \cong \Delta BAD$

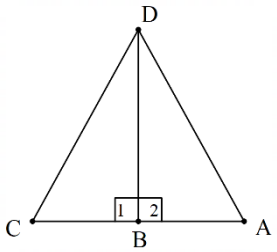


Statement	Reason
1.	1.
2.	2. If two angles are right angles, then they are ≅.
3.	3.
4.	4.

Example #14:

Given: $\angle A \cong \angle C$, $\angle 1$ and $\angle 2$ are right angles

Prove: $\Delta BCD \cong \Delta BAD$



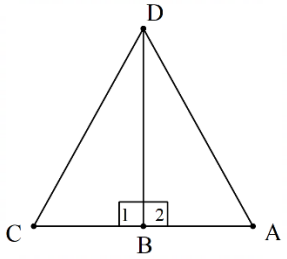
Statement	Reason
1.	1.
2.	2. If two angles are right angles, then they are ≅.
3.	3.
4.	4.

Reminder: If two segments are perpendicular (⊥), then they form right angles.

Example #15:

Given: $CD \cong AD,\ DB \perp AC$

Prove: $\Delta BCD \cong \Delta BAD$

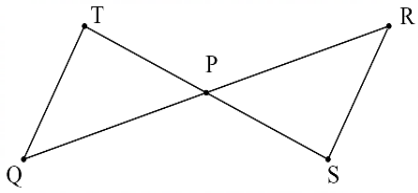


Statement	Reason
1.	1.
2.	2. If segments are perpendicular, then they form right angles.
3.	3.
4.	4.

Example #16:

Given: $QP \cong RP,\ \angle T \cong \angle S$

Prove: $\Delta TQP \cong \Delta SRP$

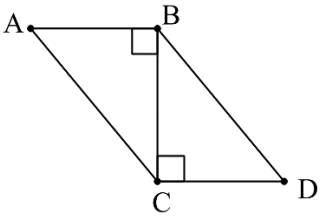


Statement	Reason
1.	1.
2.	2.
3.	3.

Example #17:

Given: $AC \cong BD,\ \angle ABC$ and $\angle DCB$ are right $\angle$s

Prove: $\Delta ABC \cong \Delta DCB$

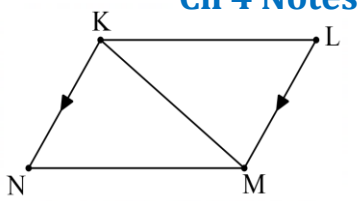


Statement	Reason
1.	1.
2.	2.
3.	3.

Example #18:

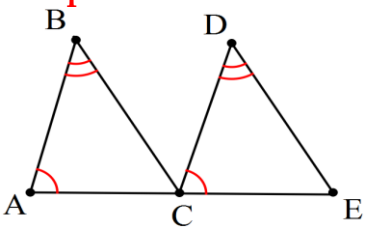
Given: $KN \parallel BD, \angle KMN \cong \angle MKL$

Prove: $\Delta KNM \cong \Delta MLK$



Statement	Reason
1.	1.
2.	2. If $\parallel$ lines, then alternate interior angles are $\cong$.
3.	3.
4.	4.

Example #19: What additional information is needed to prove $\Delta ABC \cong \Delta CDE$ by AAS?



a. $AC \cong DE$

b. $BC \cong DE$

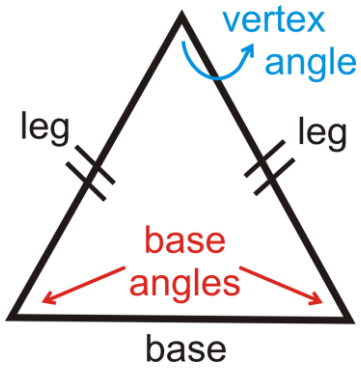
c. $BA \cong CE$

d. $CB \cong CE$

4.5 Notes: Isosceles and Equilateral Triangles

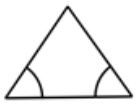
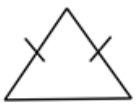
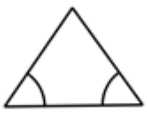
Objectives:

- **Students will be able to find missing angles and sides in equilateral and isosceles triangles.**
- **Students will be able to classify triangles by side lengths when given coordinates of vertices.**

Isosceles Triangle	**An Isosceles Triangle has at least _____ _____ sides.**	
Parts of an Isosceles Triangles	The two congruent sides are called the _____. The other side is called the _____. The angle included by the legs is called the _____. The angles that have one side that is the base are called _____ _____.	

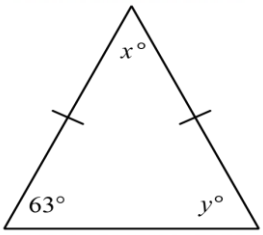
Exploration #1: Use the following link to explore parts of isosceles triangles:
https://www.geogebra.org/m/mXXYSNZG

- Move the vertices around and pay attention to the measures of the sides and angles.
- Make a **conjecture** ("guess") about the measures of the base angles:

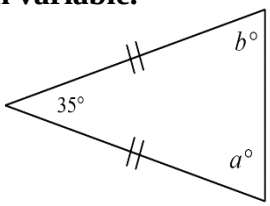
Isosceles Triangle Theorem	**If two sides of a triangle are congruent, then the base angles opposite those sides are _____.**	If  then 
Converse of the Isosceles Triangle Theorem	**If two angles of a triangle are congruent, then the sides opposite those angles are congruent.**	If  then 

For Examples 1 – 6: Find the measure of each variable.

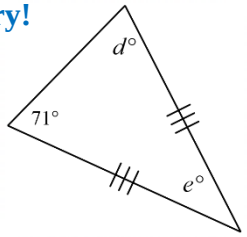
1)



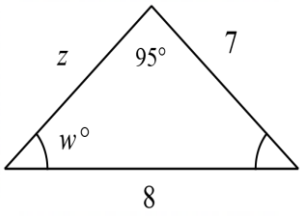
2)



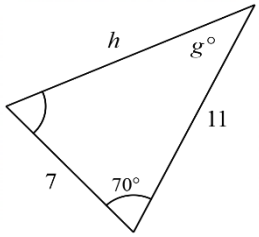
3) **You try!**



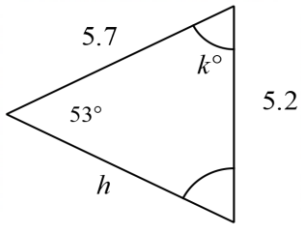
4)



5)



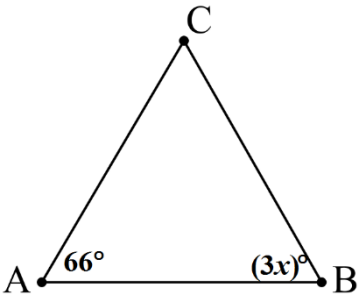
6) **You try!**



For #7 – 9: ΔABC is isosceles, with base AC. AC = 8, and AB = 7.

7) Find the value of x.

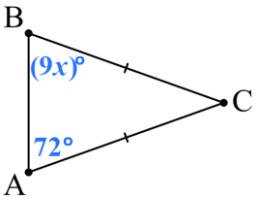
8) Find the measure of ∠B and ∠C.



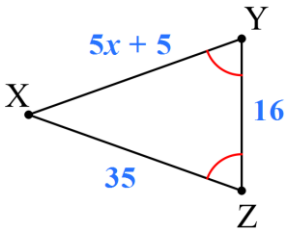
9) Find the perimeter of ΔABC.

For #10 – 12: Find the value of x.

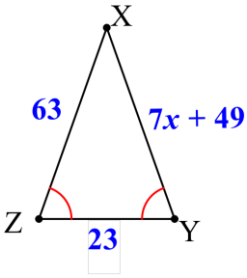
10.

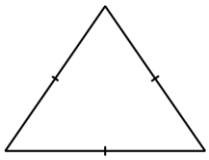


11.



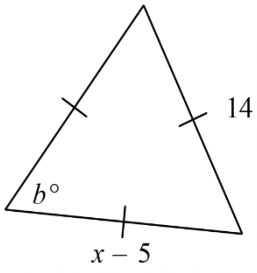
12. **YOU TRY!!**

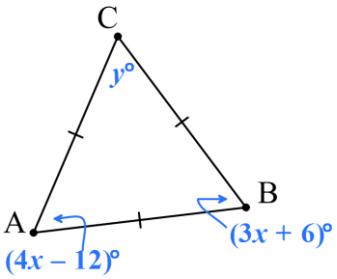


Equilateral Triangle	**A triangle is an Equilateral Triangle if all three sides are _____.**	
Equilateral Triangle Theorem	**If a triangle is equilateral, then it is _____.**	If  then 
Converse	**If a triangle is equiangular, then it is _____.**	If  then 

Explore: What is the measure of each angle of an equilateral triangle?

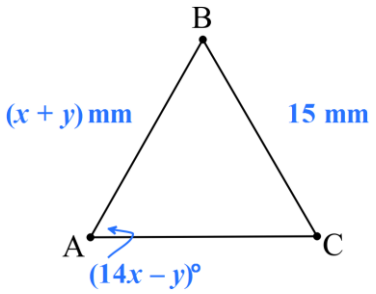
For #13 – 14: Find the value of the variables for each equilateral triangle.

13) 

14) You try!


Also, what is the perimeter for #13?

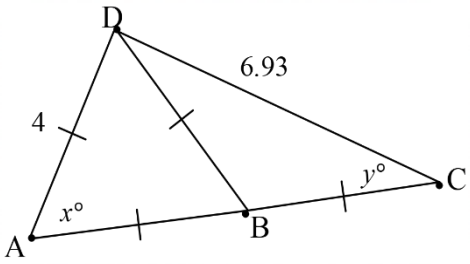
15) **Challenge!** Find x and y, if ΔABC is an equilateral triangle.



For #16 – 18: Use the diagram shown below.

16) Find x.

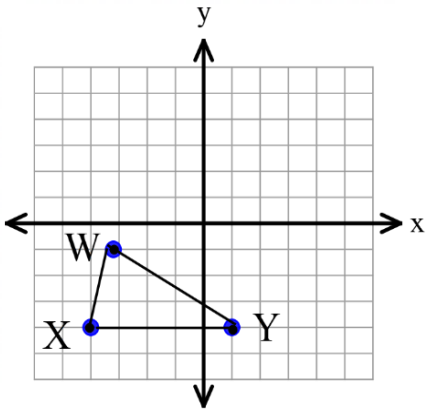
17) Find y.



18) Find the perimeter of ΔACD.

Example #19: Determine if ΔWXY is isosceles or not. Use the distance formula as needed. Show your work!

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Ch 4 Study Guide

- Triangle Sum Theorem: all three angles of a triangle will add to exactly 180°.
- Exterior Angle Theorem: the exterior angle of a triangle is equal to the sum of the two remote interior angles.
- CPCTC: If two triangles are congruent, then corresponding parts are congruent.
- Ways to prove triangles are congruent: SSS, SAS, ASA, AAS, or HL
- Reflexive Property: Any side (or angle) is congruent to itself.
- Vertical Angles: If two angles are vertical, then they are congruent.
- Midpoint: If a point is a midpoint, then it divides the segment into two congruent segments.
- Isosceles Triangles
    - Two sides (legs) are congruent.
    - The non-congruent side is called the base.
    - If two sides of a triangle are congruent, then the base angles are congruent.
    - If two angles of a triangle are congruent, then the sides opposite those triangles are congruent.
- Equilateral Triangles
    - All three sides are congruent.
    - All three angles are congruent (called "equiangular".)
    - Each angle has a measure of 60°.