Lesson 8.3 Part 1: What Shape Is the Oval Office?

p. 1: Exploration

> What happens when you change the dimensions of a circle? How would the shape change if we made it wider and shorter? Narrower and taller? Is it still a circle?

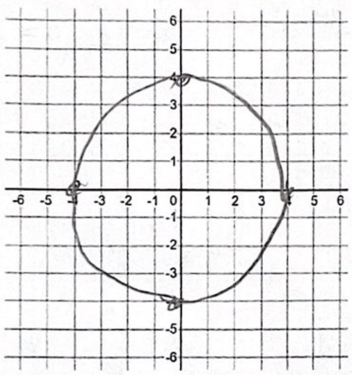
$(x-h)^2+(y-k)^2=r^2$

Students can use Desmos.com to see it!

1. Fill in the table below:

	Center	Radius
$x^2+y^2=9$	$(0,0)$	3
$25\cdot\frac{1}{25}(x^2+y^2)=1\cdot 25 \Rightarrow \boxed{x^2+y^2=25}$	$(0,0)$	5
$16\left(\frac{x^2}{16}+\frac{y^2}{16}\right)=1\cdot 16 \Rightarrow \boxed{x^2+y^2=16}$	$(0,0)$	4
$16\left(\frac{(x-2)^2}{16}+\frac{(y+3)^2}{16}\right)=1\cdot 16 \Rightarrow (x-2)^2+(y+3)^2=16$	center: $(2,-3)$	4

2. Sketch the circle given by $\frac{x^2}{16}+\frac{y^2}{16}=1$.



a. What is the distance from the center of the circle to the top of the circle?

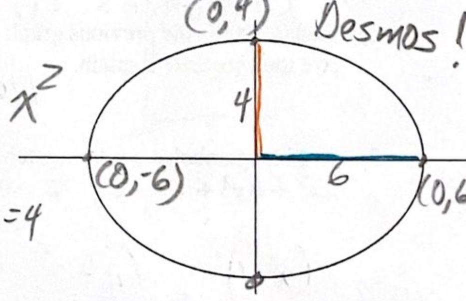
4 units

b. What is the distance from the center of the circle to the side of the circle?

4 units

Let's see what happens when we change the equation to $\frac{x^2}{36}+\frac{y^2}{16}=1$. A sketch of the equation is shown.

Use Desmos!



3. Does the circle get wider or taller?

wider

4. How does this match the equation?

There's a bigger number under x^2

5. What do you think is the distance from the center of the shape to the top of the shape? What about from the center to the side of the shape?

$\to \sqrt{16}=4$

$\to \sqrt{36}=6$

6. Label the ordered pairs at the four points shown on the graph. →

7. What do you think would be different if the equation was $\frac{x^2}{16}+\frac{y^2}{36}=1$?

taller instead

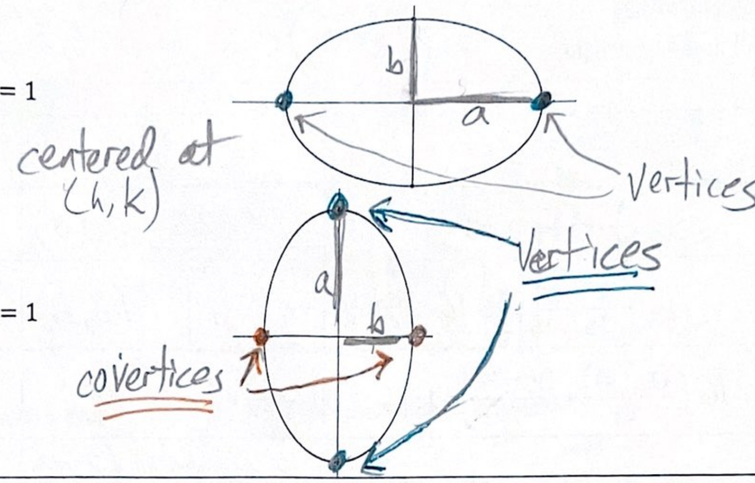
Lesson 8.3 Part1—Ellipses

In the equations below, $\boldsymbol{a}$ always represents the longer dimension (length of the major axis is 2a)

and $\boldsymbol{b}$ represents the shorter dimension (length of the minor axis is 2b).

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$



Examples:

1) Using what you've explored so far, let's look at a new equation $\frac{(x-1)^2}{25}+\frac{(y+2)^2}{9}=\mathbf{1}$ $a=$ 5 $b=$ 3

   a. Where is this ellipse centered?

   (1, -2)

   b. What's the length of the major axis?

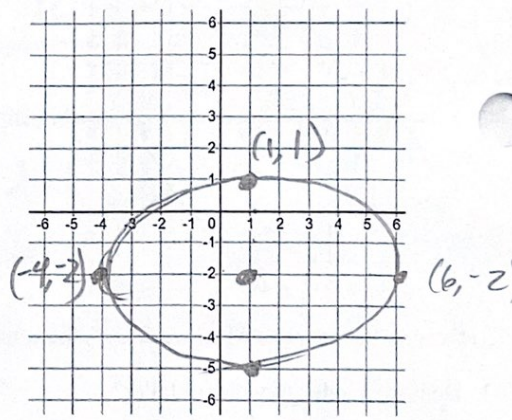
   2·5 = 10 units

   c. What's the length of the minor axis?

   2·3 = 6 units

   d. Sketch a graph, and label the vertices and co-vertices.

   vertices: (6,-2), (-4,-2)
   covertices (1,1) (1,-5)



2) Jordan claims the previous graph can also be represented by the equation $(\boldsymbol{x}-\mathbf{1})^2+(\boldsymbol{y}+\mathbf{2})^2=\mathbf{225}$. Are they correct? Explain.

   No - that would be a circle

3) Just like parabolas can be written in different forms, so can ellipses! The **general form of an ellipse** is given by $\boldsymbol{Ax^2+By^2+Cx+Dy+E=0}$. Re-write the equation from Example 1 in this way:

$$25\cdot 9\cdot\left(\frac{(x-1)^2}{25}+\frac{(y+2)^2}{9}\right)=1\cdot 25\cdot 9$$

$$9(x-1)^2+25(y+2)^2=225$$

$$9(x^2-2x+1)+25(y^2+4y+4)=225$$

$$9x^2-18x+9+25y^2+100y+100=225$$

$$9x^2+25y^2-18x+100y-116=0$$

You try:

$5^2 \quad 7^2 \Rightarrow \boxed{a=7}$ (bigger) $\quad \boxed{b=5}$ (smaller)

I) Given $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{49} = 1$ identify the following:

a. Center: $(-2,3)$ → make graph

b. Vertices: $(-2,10)$ & $(-2,-4)$

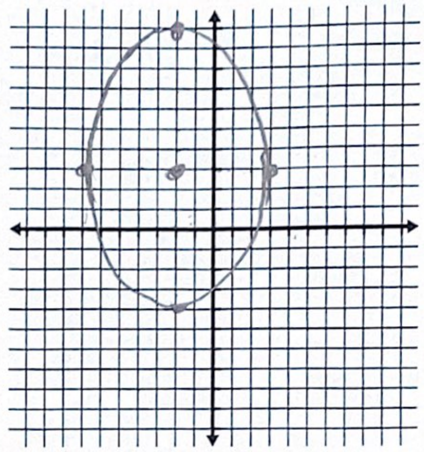
c. Co-Vertices: $(-7,3)$ & $(3,3)$

d. Domain: $[-7,3]$

e. Range: $[-4,10]$

from graph

f. Graph:



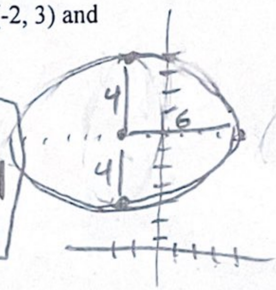
II) Write the equation of an ellipse with a major axis length of 12 and co-vertices (-2, 3) and (-2, 11).

center: $(-2,7)$ $\quad \hookrightarrow a=6$

Always:
- added
- $=1$

$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

$$\boxed{\frac{(x+2)^2}{36} + \frac{(y-7)^2}{16} = 1}$$



More Examples:

4) Rewrite $4x^2 + 9y^2 + 16x + 90y + 205 = 0$ in standard form, then identify the center and dimensions.

$$4x^2 + 16x + \underline{16} + 9y^2 + 90y + \underline{225} = -205 + \underline{16} + \underline{225}$$

$$4(x^2 + 4x + \underline{4}) + 9(y^2 + 10y + \underline{25}) = 36$$

$$\frac{4(x+2)^2}{36} + \frac{9(y+5)^2}{36} = \frac{36}{36} \Rightarrow \boxed{\frac{(x+2)^2}{9} + \frac{(y+5)^2}{4} = 1}$$

center: $(-2,-5)$
$a=3, \ b=2$

5) Find the center of $4y^2 + x^2 - 6x - 8y - 3 = 0$.

$$x^2 - 6x + \underline{9} + 4(y^2 + 2y + \underline{1}) = 3 + \underline{9} + \underline{4}$$

$$\frac{(x-3)^2}{16} + \frac{4(y+1)^2}{16} = \frac{16}{16}$$

$$\boxed{\frac{(x-3)^2}{16} + \frac{(y+1)^2}{4} = 1}$$

center: $(3,-1)$
$a=4, \ b=2$

6) Which of the following is an equation of an ellipse?

a. ~~$x^2 = \frac{1}{2}(y-3)^2 + 4$~~

b. $y^2 - 6y + 2x^2 + 17 = 0$

c. ~~$2x^2 + 4 = (y-3)^2$~~

d. $x - 8 = (y-3)^2$ → parabola