Math 127 **Final Review** **Name_______________________________**

$\frac{n}{2}(a_1 + a_n)$	$a_1 r^{n-1}$	$\frac{a_1}{1-r}$
$a_1 + d(n-1)$ or $dn + a_0$	$\frac{a_1(1-r^n)}{1-r}$	$\frac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$

For 1 – 2, write the partial fraction decomposition of the rational expression.

1) $\frac{9x+8}{(x-6)^2}$

2) $\frac{32-5x}{x(x-4)^2}$

3) Write the **form** of the partial fraction decomposition of the rational expression. $\frac{6x+2}{(x-7)(x^2+x+3)^2}$

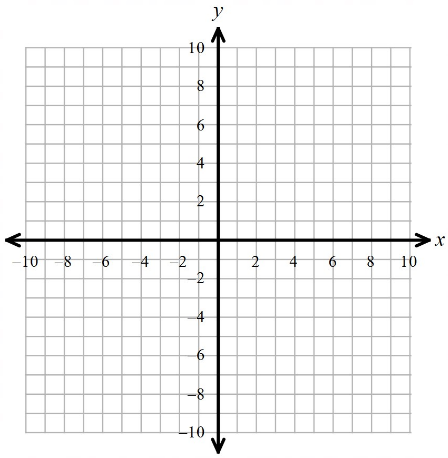
For 4 – 5, solve the system by any method.

4) $\begin{cases} x^2 + y^2 = 113 \\ x + y = 15 \end{cases}$

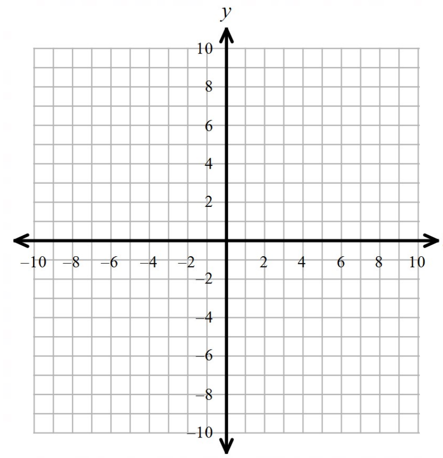
5) $\begin{cases} xy = 12 \\ x^2 + y^2 = 40 \end{cases}$

For 6 – 7, graph of the system of inequalities (use the provided graphs for your answers).

6) $\begin{cases} -8x + 3y \leq -24 \\ x^2 + y^2 \leq 36 \end{cases}$



7) $\begin{cases} y - x^2 > 0 \\ x^2 + y^2 \leq 49 \end{cases}$



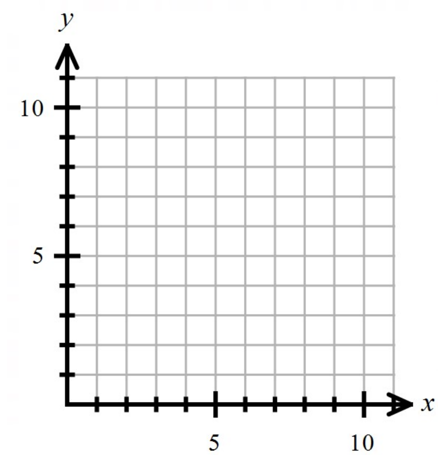
8) Graph the constraints and use the objective function to maximize the function.

Objective function: $z = 23x + 8y$

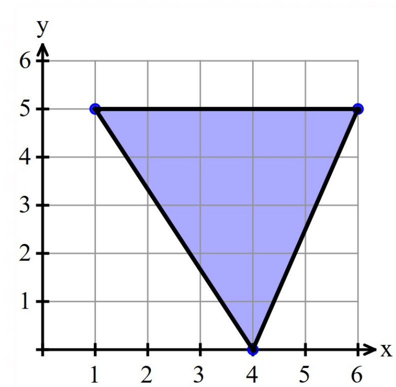
Constraints: $0 \leq x \leq 10$

$0 \leq y \leq 5$

$3x + 2y \geq 6$



9) Given the graph shown on the right, which shows the feasible region of a linear program problem. If the objective function is $z = 3x + 4y$, then what is the maximum value?

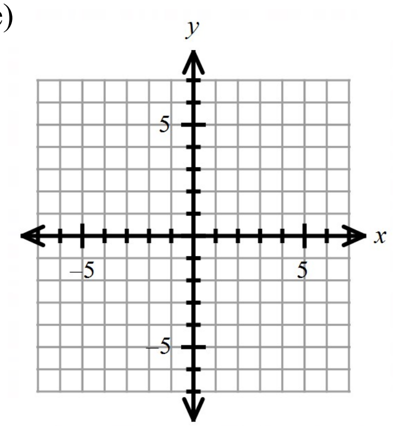


10) Write the equation of an ellipse in standard form that meets the requirements below:
foci: $(0, -2), (0, 2)$; y-intercepts: -5 and 5.

11) Write the equation of a hyperbola in standard form that meets the requirements below:
foci: $(0, -4), (0, 4)$; vertices: $(0, -3), (0, 3)$.

For #12 – 14, graph the conic and find the requested information. If needed, round to 3 decimal places.

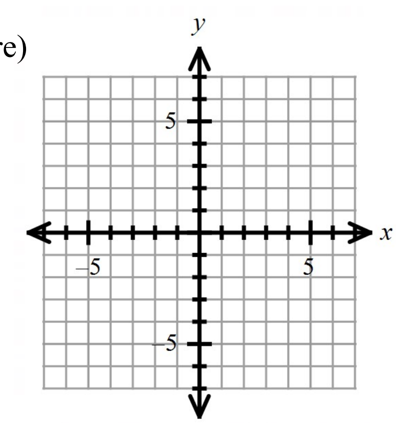
12) $4x^2 + 8x + 9y^2 = 32$ (Convert to standard form by completing the square)



Center:

Foci:

13) $x^2 - 2x + 7y - 34 = 0$ (Convert to standard form by completing the square)



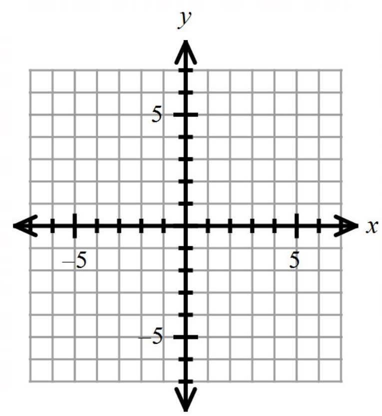
Vertex:

Focus:

Directrix:

Length of
Latus Rectum:

14) $\frac{(x-2)^2}{16} - \frac{(y+2)^2}{4} = 1$

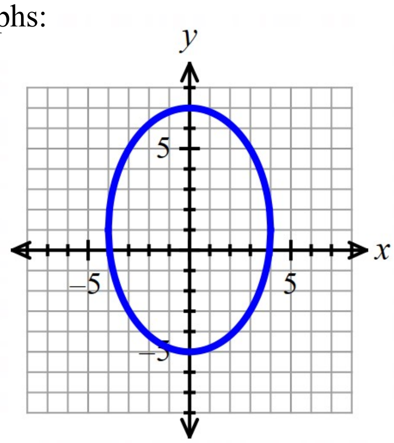


Center:

Foci:

For #15 – 20, match each graph to its equation. No item will be used more than once. Not all equations will be used.

Graphs:

15)


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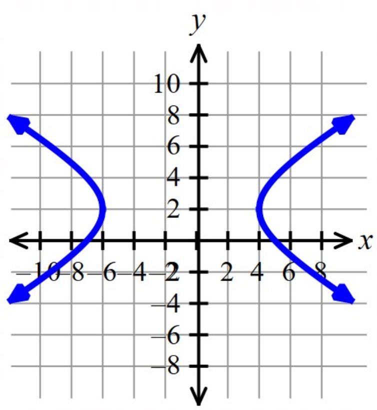
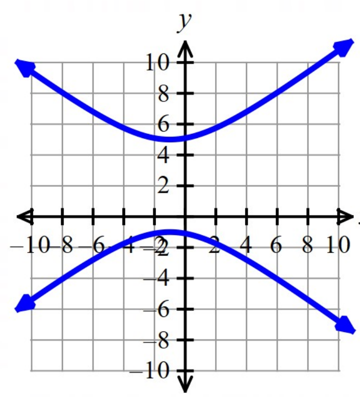


17)


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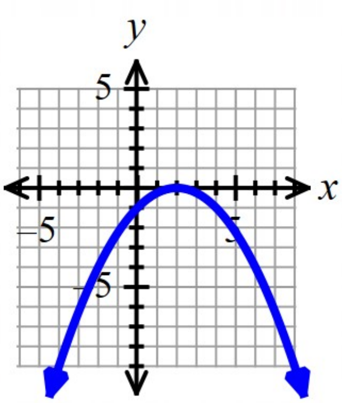

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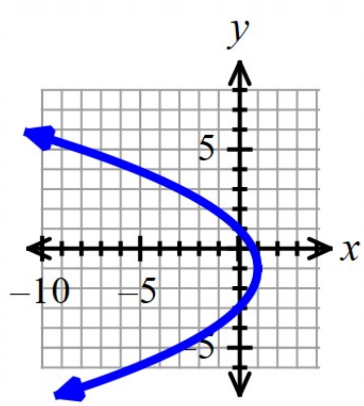

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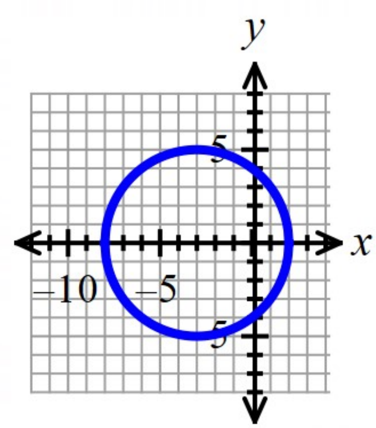


Equations:

A) $\frac{(y-2)^2}{9} - \frac{(x+1)^2}{16} = 1$

B) $\frac{(x+1)^2}{25} - \frac{(y-2)^2}{9} = 1$

C) $(y+1)^2 = -4(x-1)$

D) $\frac{x^2}{16} + \frac{(y-1)^2}{36} = 1$

E) $\frac{x^2}{36} + \frac{(y+1)^2}{16} = 1$

F) $(x-2)^2 = -4y$

G) $(x-3)^2 + y^2 = 25$

H) $(x+3)^2 + y^2 = 25$

21) Find a_{20} (the 20th term) for the arithmetic sequence: $11, 4, -3, -10, \ldots$

22) Find the sum of the first 20 terms of the arithmetic sequence: -12, -6, 0, 6,…

23) Find a_{12} (the 12th term) for the geometric sequence: $a_1 = -5 \; and \; r = 2$

24) Write a formula for the general term (the n^{th} term) of the geometric sequence: $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, \ldots$

25) Find the sum of the geometric sequence: $\sum_{n=1}^{8}(-4)^n$

26) To save for retirement, you decide to deposit $2250 into an IRA at the end of each month for the next 35 years, with an interest rate of 5% compounded monthly. Find the value of the IRA after 35 years. Round to the nearest dollar.

For 27 – 28: Find the sum of the infinite geometric series, or state that it does not exist:

27) $96 + 24 + 6 + \frac{3}{2} + \cdots$

28) $\frac{1}{3} - 1 + 3 - \cdots$

29) Express the decimal as a fraction in lowest terms, $0.\overline{58}$

30) Write the equation of a parabola in standard form that meets the requirements below:
Focus at $(-4, 5)$ and directrix at $x = -2$.

For 31 – 35 , verify the trig identity:

31) $\tan x(\cot x - \cos x) = 1 - \sin x$

32) $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

33) $\frac{1+\cos}{\sin 2x} = \cot x$

34) $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

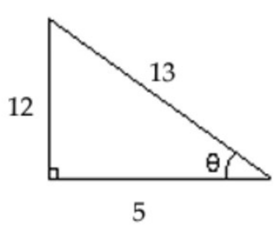
35) Verify the identity: $\tan^2\theta + 4 = \sec^2\theta + 3$

For 36 – 37: Find the exact value by using a sum or difference identity.

36) $\tan\frac{7\pi}{12}$

37) $\cos\frac{11\pi}{12}$

38) Use the figure to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.



39) Use the given information to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.
$\sin\theta = \frac{4}{5}$, θ lies in quadrant I

For 40 – 41, find all solutions of the following equations.

40) $2\sin x - \sqrt{3} = 0$

41) $\tan x \sec x = -2\tan x$

For 42 – 45, solve the equation on the interval $[0, 2\pi)$.

42) $\cos 2x = \frac{\sqrt{3}}{2}$

43) $\cos^2 x + 2\cos x + 1 = 0$

44) $\cos x + 2\cos x \sin x = 0$

45) $\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 1$

46) Evaluate the given binomial coefficient: $\binom{10}{5}$

47) Use the Binomial Theorem to expand the binomial and express the result in simplified terms: $(2x-1)^5$

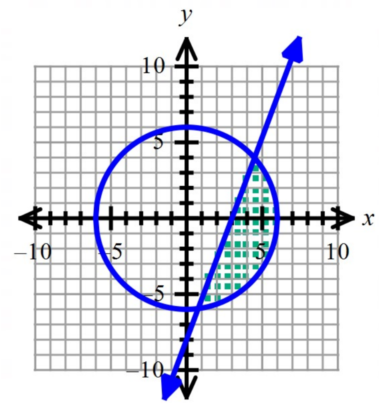
48) Find the 8^{th} term in the following binomial expansion: $(x-3y)^{11}$

Final Review Key:

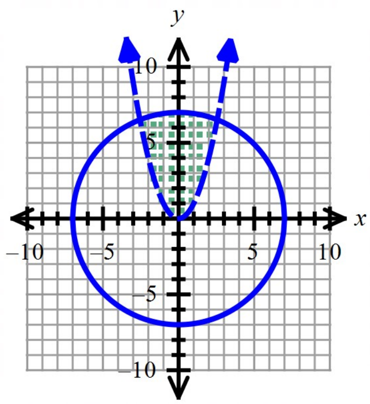
1) $\frac{9}{x-6} + \frac{62}{(x-6)^2}$ 2) $\frac{2}{x} + \frac{-2}{x-4} + \frac{3}{(x-4)^2}$ 3) $\frac{A}{x-7} + \frac{Bx+C}{x^2+x+3} + \frac{Dx+E}{(x^2+x+3)^2}$ 4) $\{(8,7), (7,8)\}$

5) $\{(2,6), (-2,-6), (6,2), (-6,-2)\}$

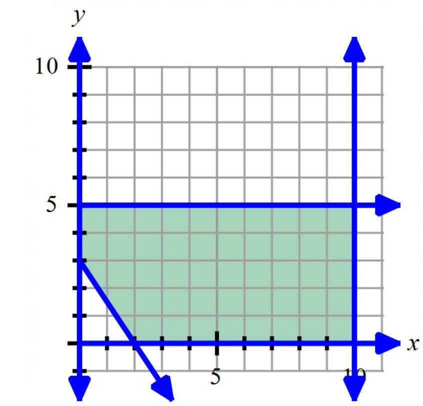
6)



7)

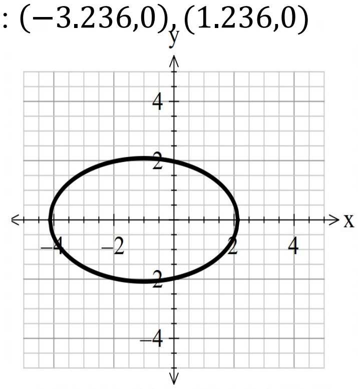


8) Max value: 270

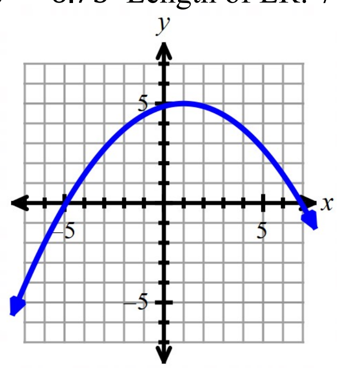


9) 38 10) $\frac{x^2}{21} + \frac{y^2}{25} = 1$ 11) $\frac{y^2}{9} - \frac{x^2}{7} = 1$

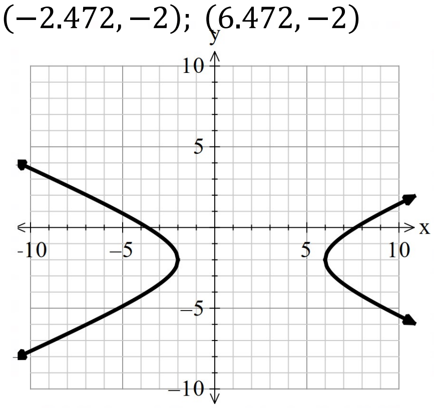
12) C: (-1, 0)
F: $(-3.236,0), (1.236,0)$



13) V: (1, 5) F: (1,3.25)
D: $y = 6.75$ Length of LR: 7



14) C: (2, -2);
F: $(-2.472, -2)$; $(6.472, -2)$



15) D 16) B 17) A 18) F 19) C 20) H 21) $a_n = 18 - 7n;\ a_{20} = -122$

22) 900 23) -10,240 24) $a_n = 3\left(-\frac{1}{2}\right)^{n-1}$ 25) 52,428 26) $2,556,208

27) 128 28) DNE 29) $\frac{58}{99}$ 30) $(y-5)^2 = -4(x+3)$ 31 – 35) Possible solutions on the next page

36) $-2-\sqrt{3}$ 37) $\frac{-\sqrt{2}-\sqrt{6}}{4}$ 38) $\frac{120}{169}; -\frac{119}{169}; -\frac{120}{119}$ 39) $\frac{24}{25}; -\frac{7}{25}; -\frac{24}{7}$ 40) $x = \frac{\pi}{3} + 2\pi n$ or $x = \frac{2\pi}{3} + 2\pi n$

41) $x = \frac{2\pi}{3} + 2\pi n$ or $x = \frac{4\pi}{3} + 2\pi n$ or $x = \pi n$ 42) $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$ 43) π

44) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ 45) 0 46) 252 47) $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$

48) $-721710x^4y^7$

Math 127 Final Review Name______________________________

Solutions for 31 – 35:

31) $\tan x\,(\cot x - \cos x) = 1 - \sin x$

$\tan x \cdot \cot x - \tan x \cos x$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \cdot \cos x$

$1 - \sin x = 1 - \sin x$ ✓

32) $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$\tan\alpha + \tan\beta = \tan\alpha + \tan\beta$ ✓

33) $\frac{1+\cos}{\sin 2} = \cot x$

$1 - \sin^2 x = \cos^2 x$

$\frac{1 + \cos^2 x - \sin^2 x}{2\sin x\cos x}$

$\frac{\cos^2 x + \cos^2 x}{2\sin x\cos x}$

$\frac{2\cos^2 x}{2\sin x\cos x}$

$\frac{\cos x}{\sin x}$

$\cot x = \cot x$ ✓

34) $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2}$

$\sin x \cdot 0 + \cos x \cdot 1$

$\cos x = \cos x$ ✓

35) Verify the identity: $\tan^2\theta + 4 = \sec^2\theta + 3$

$\sec^2\theta - 1 + 4$

$\sec^2\theta + 3 = \sec^2\theta + 3$

✓