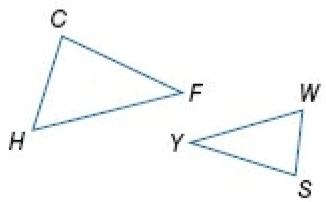
List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.

12. $\Delta CHF \sim \Delta YWS$



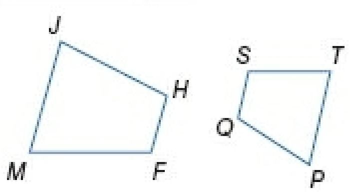
SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $\Delta CHF \sim \Delta YWS$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar triangles are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$\angle C \cong \angle Y, \angle H \cong \angle W, \angle F \cong \angle S;$

$\frac{CH}{YW} = \frac{HF}{WS} = \frac{FC}{SY}$

13. $JHFM \sim PQST$



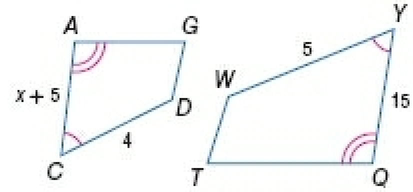
SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $JHFM \sim PQST$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar polygons are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$\angle J \cong \angle P, \angle H \cong \angle Q, \angle F \cong \angle S, \angle M \cong \angle T;$

$\frac{PQ}{JH} = \frac{TS}{MF} = \frac{SQ}{FH} = \frac{TP}{MJ}$

REGULARITY Each pair of polygons is similar. Find the value of *x*.

18. 

SOLUTION:

Use the corresponding side lengths to write a proportion.

$\frac{x+5}{15} = \frac{4}{5}$

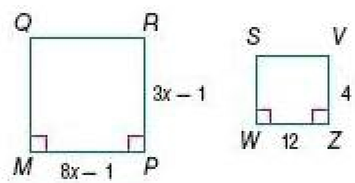
Solve for *x*.

$5(x+5) = 60$

$5x + 25 = 60$

$5x = 35$

$x = 7$

20. 

SOLUTION:

Use the corresponding side lengths to write a proportion.

$\frac{3x-1}{4} = \frac{8x-1}{12}$

Solve for *x*.

$12(3x-1) = 4(8x-1)$

$36x - 12 = 32x - 4$

$4x = 8$

$x = 2$

22. Rectangle *ABCD* has a width of 8 yards and a length of 20 yards. Rectangle *QRST*, which is similar to rectangle *ABCD*, has a length of 40 yards. Find the scale factor of rectangle *ABCD* to rectangle *QRST* and the perimeter of each rectangle.

SOLUTION:

Let x be the width of rectangle *QRST*.
Use the corresponding side lengths to write a proportion.

$$\frac{x}{40} = \frac{8}{20}$$

Solve for x.

$$20x = 320$$
$$x = 16$$

$$\text{Scale factor} = \frac{\text{Length of rectangle } ABCD}{\text{Length of rectangle } QRST} = \frac{20}{40} = \frac{1}{2}$$

Therefore, the scale factor is 1:2.

$$\text{Perimeter of rectangle } ABCD = 2(l+b) = 2(20+8) = 56$$

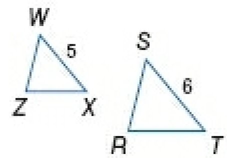
Therefore, the perimeter of rectangle *ABCD* is 56 yards.

$$\text{Perimeter of rectangle } QRST = 2(l+b) = 2(40+16) = 112$$

Therefore, the perimeter of rectangle *QRST* is 112 yards.

Find the perimeter of the given triangle.

24. ΔWZX, if $\Delta WZX \sim \Delta SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\Delta SRT = 15$



SOLUTION:

The scale factor of triangle *SRT* to triangle *WZX* is $\frac{ST}{WX}$ or $\frac{6}{5}$.

Use the perimeter of triangle *SRT* and the scale factor to write a proportion and then substitute in the value of the perimeter of triangle *SRT* and solve for the perimeter of triangle *WZX*.

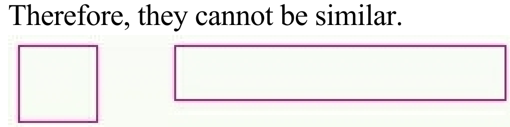
$$\frac{6}{5} = \frac{\text{Perimeter of triangle } SRT}{\text{Perimeter of triangle } WZX}$$
$$\frac{6}{5} = \frac{15}{\text{Perimeter of triangle } WZX}$$

$$\text{Perimeter of triangle } WZX = \frac{75}{6} = 12.5$$

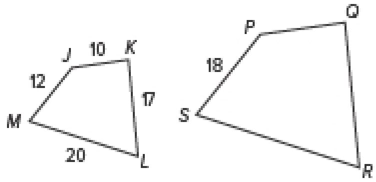
35. **OPEN-ENDED** Find a counterexample for the following statement.
All rectangles are similar.

SOLUTION:

Not all rectangles are similar.
Shown are two rectangles. However, the ratios of their corresponding sides are not the same. Therefore, they cannot be similar.

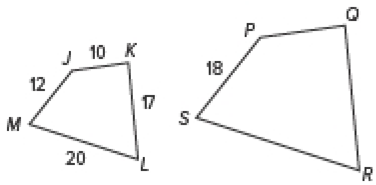


38. In the figure, *JKLM* ~ *PQRS*. What is the perimeter of *PQRS*?



A 18
B 39.3
C 50
D 88.5

SOLUTION:



The perimeter of *JKLM* is 12 + 10 + 17 + 20 = 59.

Because *PS* = 1.5(*JM*), the perimeter of *PQRS* is (1.5)59 = 88.5

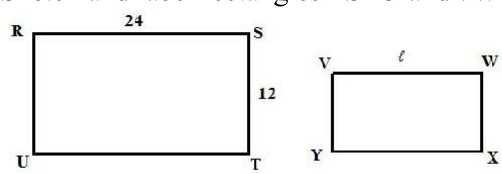
39. Braden drew two rectangles, *RSTU* and *VWXY*, so that *RSTU* ~ *VWXY*. The ratio of the perimeter of *RSTU* to the perimeter of *VWXY* is $\frac{3}{4}$. Given that the length of *RSTU* is 24 and the width of *RSTU* is 12, what is the length of *VWXY*?

A 9
B 16
C 18
D 32

SOLUTION:

According to Theorem 7.1 Perimeters of Similar Polygons, if two polygons are similar, then their perimeters are proportional to the scale factor between them. Therefore, if *RSTU* ~ *VWXY,* the corresponding sides have the same ratio as the ratio of the perimeters, which is $\frac{3}{4}$.

Sketch and label rectangles *RSTU* and *VWXY*.

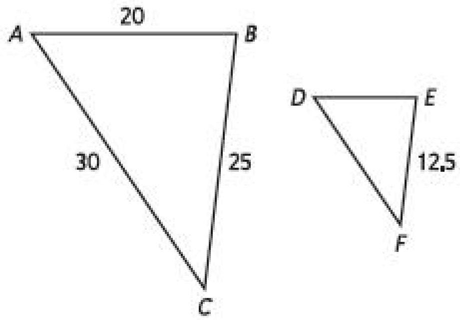


The length of *RSTU* ($\overline{RS}$) corresponds to the length of *VWXY* ($\overline{VW}$). Set up a proportion of the corresponding side lengths to the ratio of the perimeters.

$$\frac{RS}{VW} = \frac{3}{4}$$
$$\frac{24}{\ell} = \frac{3}{4}$$
$$3\ell = 96$$

$\ell = 32$ The correct choice is D.

40. In the figure, $\triangle ABC \sim \triangle DEF$.



What is the perimeter of $\triangle DEF$?

SOLUTION:

If the triangles are similar, then the corresponding sides are proportional to each other.

Set up a proportion of corresponding sides to find *DE* and *DF*.

$$\begin{aligned} \frac{25}{12.5} &= \frac{20}{DE} \\ \frac{2}{1} &= \frac{20}{DE} \\ 2DE &= 20 \\ DE &= 10 \end{aligned} \qquad \begin{aligned} \frac{25}{12.5} &= \frac{30}{DF} \\ \frac{2}{1} &= \frac{30}{DF} \\ 2DF &= 30 \\ DF &= 15 \end{aligned}$$

The perimeter of $\triangle DEF$ is $10 + 12.5 + 15 = 37.5$.

41. Two similar rectangles have a scale factor of 3 : 5. The perimeter of the larger rectangle is 65 meters. What is the perimeter in meters of the smaller rectangle?

SOLUTION:

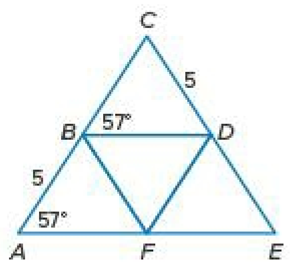
If two rectangles are similar and their scale factor is 3:5, then their perimeters have the same scale factor. Set up a proportion comparing the perimeters of the two rectangles to the scale factor.

$$\begin{aligned} \frac{P_s}{P_L} &= \frac{3}{5} \\ \frac{P_s}{65} &= \frac{3}{5} \\ 5(P_s) &= 195 \\ P_s &= 39 \end{aligned}$$

The perimeter of the smaller rectangle, in meters, is 39.

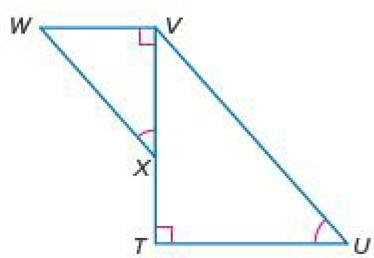
Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

9. $\triangle ACE$, $\triangle BCD$



SOLUTION:

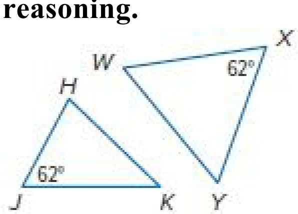
$\triangle ACE \sim \triangle BCD$ by AA Similarity. It is given that one angle is 57°, which can be used to find that the other angles are 57° and 66°, respectively. These measure hold true for each triangle, and thus the larger triangle as well.

11. 

SOLUTION:

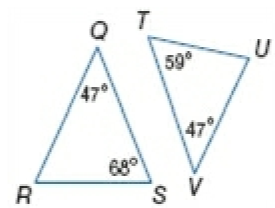
It is given that two of the angles are congruent, so the remaining angles must also be congruent. So, $\triangle TUV \sim \triangle VXW$ by AA Similarity.

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

13. 

SOLUTION:

There is not enough information given to determine that the triangles are similar.

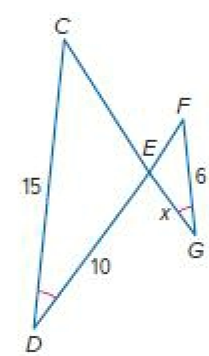
14. 

SOLUTION:

No; the angles of ΔTUV are 59, 47, and 74 degrees and the angles of ΔQRS are 47, 68, and 65 degrees. Since the angles of these triangles won't ever be congruent, so the triangles can never be similar.

ALGEBRA Identify the similar triangles. Then find each measure.

16. *EG*



SOLUTION:

We are given one pair of congruent angles, and we know that $\angle CED \cong \angle FEG$ by the Vertical Angles Theorem. So, we know that $\triangle DEC \sim \triangle GEF$ by AA Similarity.

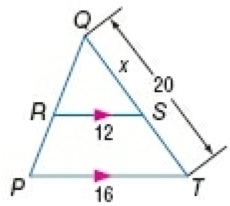
Use the corresponding side lengths to write a proportion.

$$\frac{ED}{CD} = \frac{EG}{FG}$$
$$\frac{10}{15} = \frac{x}{6}$$

Solve for *x*.

$$15x = 60$$
$$x = 4$$

17. *ST*



SOLUTION:

By the Reflexive Property, we know that $\angle Q \cong \angle Q$

.

.

Also, since $\overline{RS} \parallel \overline{PT}$, we know that $\angle QRS \cong \angle QPT$ (Corresponding Angle Postulate).

Therefore, by AA Similarity, $\Delta QRS \sim \Delta QPT$.

Use the corresponding side lengths to write a proportion.

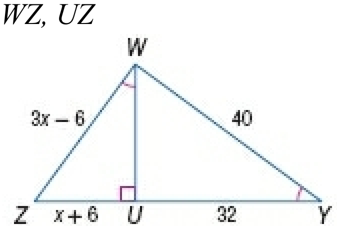
$$\frac{QS}{QT} = \frac{RS}{PT}$$
$$\frac{x}{20} = \frac{12}{16}$$

Solve for x.

$$16x = 240$$
$$x = 15$$

$$ST = 20 - x$$
$$= 20 - 15$$
$$= 5$$

18. *WZ, UZ*



SOLUTION:

We are given that $\angle ZWU \cong \angle WYU$ and we also know that $\angle WUZ \cong \angle YUW$ (All right angles are congruent.)

Therefore, by AA Similarity, $\Delta WUZ \sim \Delta YUW$.

Use the Pythagorean Theorem to find *WU*.

$$WU^2 + 32^2 = 40^2$$
$$WU^2 + 1024 = 1600$$
$$WU^2 = 576$$
$$WU = \sqrt{576} = \pm 24$$

Since the length must be positive, *WU* = 24.

Use the corresponding side lengths to write a proportion.

$$\frac{WZ}{WY} = \frac{WU}{UY}$$
$$\frac{3x - 6}{40} = \frac{24}{32}$$

Solve for x.

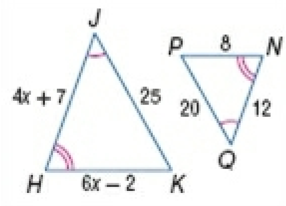
$$32(3x - 6) = 40 \cdot 24$$
$$96x - 192 = 960$$
$$96x = 1152$$
$$x = 12$$

Substitute $x = 12$ in *WZ* and *UZ*.

$WZ = 3x - 6$
$= 3(12) - 6$
$= 30$

$UZ = x + 6$
$= 12 + 6$
$= 18$

19. *HJ, HK*



SOLUTION:

Since we are given two pairs of congruent angles, we know that $\triangle HJK \sim \triangle NQP$, by AA Similarity.

Use the corresponding side lengths to write a proportion.

$$\frac{HJ}{NQ} = \frac{JK}{QP}$$

$$\frac{4x+7}{12} = \frac{25}{20}$$

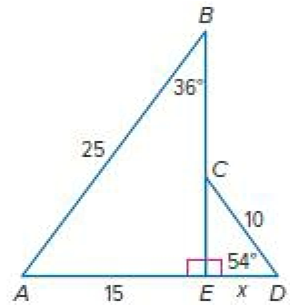
Solve for *x*.

$$20(4x+7) = 12 \cdot 25$$
$$80x + 140 = 300$$
$$80x = 160$$
$$x = 2$$

Substitute *x* = 2 in *HJ* and *HK*.

HJ = 4*(2)* + 7
=15
HK = 6*(2)* – 2
= 10

20. *EB*



SOLUTION:

We are given that each triangle has a right angle, and using the Triangle Sum Theorem we can determine that $\angle A = 54°$. Because both triangles have angle measures of 90°, 36°, and 54°, we know that $\triangle ABE \sim \triangle DCE$ by AA Similarity.

Use the corresponding side lengths to write a proportion.

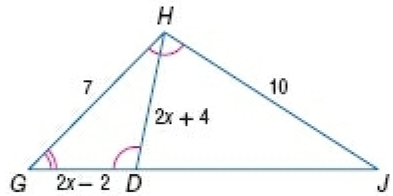
$$\frac{ED}{CD} = \frac{EA}{BA}$$

$$\frac{x}{10} = \frac{15}{25}$$

Solve for *x*.

$$25x = 150$$
$$x = 6$$

21. *GD, DH*



SOLUTION:

We know that $\angle G \cong \angle G$ (Reflexive Property) and are given $\angle GDH \cong \angle GHJ$.
Therefore, $\Delta GHJ \sim \Delta GDH$ by AA Similarity.

Use the corresponding side lengths to write a proportion:

$$\frac{DH}{HJ} = \frac{GD}{GH}$$
$$\frac{2x+4}{10} = \frac{2x-2}{7}$$

Solve for *x*.

$$7(2x+4) = 10(2x-2)$$
$$14x + 28 = 20x - 20$$
$$-6x = -48$$
$$x = 8$$

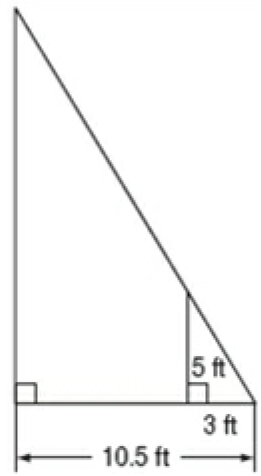
Substitute *x* = 8 in *GD* and *DH*.

GD = 2 *(8)* – 2
= 14
DH = 2 (8) + 4
= 20

22. **STATUES** Mei is standing next to a statue in the park. If Mei is 5 feet tall, her shadow is 3 feet long, and the statue's shadow is $10\frac{1}{2}$ feet long, how tall is the statue?

SOLUTION:

Make a sketch of the situation. 4 feet 6 inches is equivalent to 4.5 feet.



In shadow problems, you can assume that the angles formed by the Sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate.

So, the following proportion can be written:

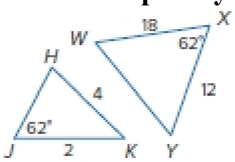
$$\frac{\text{Statue's height}}{\text{Mei's height}} = \frac{\text{Statue's shadow length}}{\text{Mei's shadow length}}$$

Let *x* be the statue's height and substitute given values into the proportion:

$$\frac{x}{5} = \frac{10.5}{3}$$
$$3x = 52.5$$
$$x = 17.5$$

So, the statue's height is 17.5 feet tall.

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

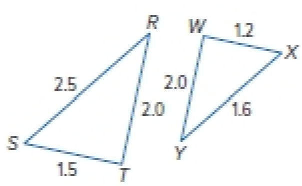
13. 

SOLUTION:

There is one known pair of congruent angles, but another pair of angles or two pairs of congruent sides are needed to prove the triangles similar. In triangle *JHK* the given sides are not the sides adjacent to the angle, so they cannot be used to prove the triangles similar.

No; not enough information is given to determine that the triangles are similar.

If JH = 3 and WY = 24. then $\Delta JHK \sim \Delta XWY$ by SSS Similarity.

14. 

SOLUTION:

For these two triangles to be similar all corresponding sides must be proportional. Write ratios comparing the sides in order from least to greatest.

$$\frac{RS}{YW} = \frac{2.5}{2.0} = 1.25$$
$$\frac{RT}{YX} = \frac{2.0}{1.6} = 1.25$$
$$\frac{ST}{WX} = \frac{1.5}{1.2} = 1.25$$

Since all three pairs of sides are proportional, $\Delta RST \sim \Delta YWX$ by SSS Similarity.

Determine whether the triangles are similar.

18. ΔLMN with $L(-6, -2)$, $M(2, 4)$, and $N(8, -4)$ and ΔPQR with $P(3, 1)$, $Q(-1, -2)$, and $R(-2, 2)$

SOLUTION:

Find the side lengths for each triangle and compare ratios of the side lengths to see if they are proportional.

$$LM = \sqrt{(-6-2)^2 + (-2-4)^2}$$
$$LM = \sqrt{64+36}$$
$$LM = \sqrt{100}$$
$$LM = 10$$

$$LN = \sqrt{(-6-8)^2 + (-2-(-4))^2}$$
$$LN = \sqrt{196+4}$$
$$LN = \sqrt{200}$$
$$LN = 10\sqrt{2}$$

$$NM = \sqrt{(8-2)^2 + (-4-4)^2}$$
$$NM = \sqrt{36+64}$$
$$NM = \sqrt{100}$$
$$NM = 10$$

$$PQ = \sqrt{(3-(-1)^2 + (1-(-2))^2}$$
$$PQ = \sqrt{16+9}$$
$$PQ = \sqrt{25}$$
$$PQ = 5$$

$$PR = \sqrt{(3-(-2))^2 + (1-2)^2}$$
$$PR = \sqrt{25+1}$$
$$PR = \sqrt{26}$$

$$QR = \sqrt{(-2-(-1))^2 + (2-(-2))^2}$$
$$QR = \sqrt{1+16}$$
$$QR = \sqrt{17}$$

$$\frac{LM}{PQ} = \frac{10}{5} = 2$$
$$\frac{MN}{QR} = \frac{10}{\sqrt{17}} \neq 2$$
$$\frac{LN}{PR} = \frac{10\sqrt{2}}{\sqrt{26}} \neq 2$$

The corresponding sides are not proportional, so the triangles are not similar.

19. $\triangle DEF$ with $D(3, -1)$, $E(-1, 4)$, and $F(2, -3)$ and $\triangle GHI$ with $G(-3, 9)$, $H(12, -3)$, and $I(-9, 6)$

SOLUTION:

Find the length of each side. Then compare ratios of corresponding sides.

$$DE = \sqrt{(3-(-1))^2 + (-1-4)^2}$$
$$DE = \sqrt{16+25}$$
$$DE = \sqrt{41}$$

$$DF = \sqrt{(3-2)^2 + (-1-(-3))^2}$$
$$DF = \sqrt{1+4}$$
$$DF = \sqrt{5}$$

$$FE = \sqrt{(2-(-1))^2 + (-3-4)^2}$$
$$FE = \sqrt{9+49}$$
$$FE = \sqrt{58}$$

$$GH = \sqrt{(-3-12)^2 + (9-(-3))^2}$$
$$GH = \sqrt{225+144}$$
$$GH = \sqrt{369}$$
$$GH = 3\sqrt{41}$$

$$GI = \sqrt{(-3-(-9))^2 + (9-6)^2}$$
$$GI = \sqrt{36+9}$$
$$GI = \sqrt{45}$$
$$GI = 3\sqrt{5}$$

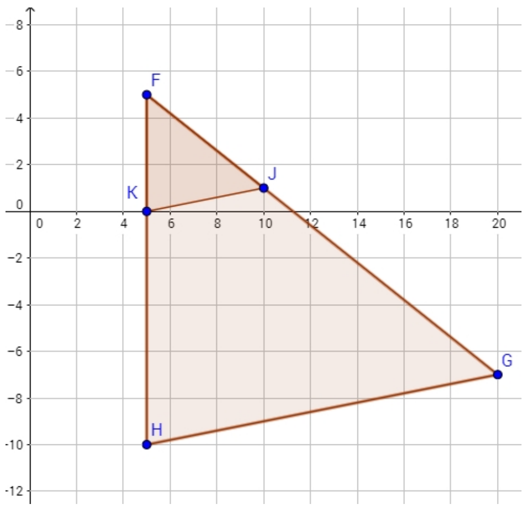
$$IH = \sqrt{(-9-12)^2 + (6-(-3))^2}$$
$$IH = \sqrt{441+81}$$
$$IH = \sqrt{522}$$
$$IH = 3\sqrt{58}$$

$$\frac{DE}{GH} = \frac{\sqrt{41}}{3\sqrt{41}} = \frac{1}{3}$$
$$\frac{DF}{GI} = \frac{\sqrt{5}}{3\sqrt{5}} = \frac{1}{3}$$
$$\frac{FE}{IH} = \frac{\sqrt{58}}{3\sqrt{58}} = \frac{1}{3}$$

By SSS $\triangle DEF \sim \triangle GHI$.

23. $\triangle FGH$ with $F(5, 5)$, $G(20, -7)$, and $H(5, -10)$ and $\triangle FJK$ with $J(10, 1)$, and $K(5, 0)$

SOLUTION:
First, make a sketch of the triangles.



In this figure, $\angle GFH \cong \angle JFK$ by symmetry.

$$FG = \sqrt{(5-20)^2 + (5-(-7))^2} = \sqrt{225 + 144} = 3\sqrt{41}$$

$$FJ = \sqrt{(5-10)^2 + (5-1)^2} = \sqrt{25 + 16} = \sqrt{41}$$

$$FH = \sqrt{(5-5)^2 + (5-(-10))^2} = \sqrt{0 + 225} = 15$$

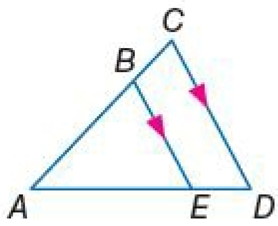
$$FK = \sqrt{(5-5)^2 + (5-0)^2} = \sqrt{0 + 25} = 5$$

$$\frac{FG}{FJ} = \frac{3\sqrt{41}}{\sqrt{41}} = 3$$

$$\frac{FH}{FK} = \frac{15}{5} = 3$$

$\triangle FGH \sim \triangle FJK$ by SAS Similarity since $\angle GFH \cong \angle JFK$ and $\dfrac{FG}{FJ} = \dfrac{FH}{FK} = 3$.

12. If $AC = 14$, $BC = 8$, and $AD = 21$, find ED.



SOLUTION:

Triangle Proportionality Theorem:
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Here, $BC = 8$. So, $AB = 14 - 8 = 6$. Let x be the length of the segment AE. So, $ED = 21 - x$.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute.

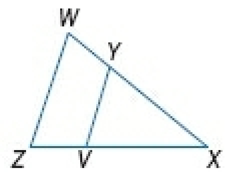
$$\frac{6}{8} = \frac{x}{21 - x}$$

Solve for x.

$$\begin{aligned} 6(21 - x) &= 8x \\ 126 - 6x &= 8x \\ -14x &= -126 \\ x &= 9 \end{aligned}$$

So, $AE = 9$ and $ED = 21 - 9 = 12$.

Determine whether $\overline{VY} \parallel \overline{ZW}$. Justify your answer.



14. $ZX = 18$, $ZV = 6$, $WX = 24$, and $YX = 16$

SOLUTION:

$ZV = 6$ and $YX = 16$. Therefore, $VX = 18 - 6 = 12$ and $WY = 24 - 16 = 8$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{WY}{YX} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{1}{2}$, then $\overline{VY} \parallel \overline{ZW}$.

16. $ZV = 8$, $VX = 2$, and $YX = \frac{1}{2}WY$

SOLUTION:

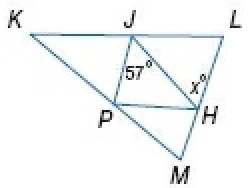
Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{8}{2} = \frac{4}{1}$$

$$\frac{WY}{YX} = \frac{WY}{\frac{WY}{2}} = WY \cdot \frac{2}{WY} = \frac{2}{1}$$

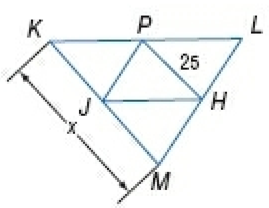
Because $\frac{ZV}{VX} \neq \frac{WY}{YX}$, $\overline{VY}$ and $\overline{ZW}$ are not parallel.

$\overline{JH}$, $\overline{JP}$, and $\overline{PH}$ are midsegments of $\triangle KLM$. Find the value of x.

18. 

SOLUTION:

By the Triangle Midsegment Theorem, $\overline{JP} \parallel \overline{LM}$.
By the Alternate Interior Angles Theorem, $x = 57$.

20. 

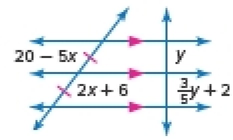
SOLUTION:

By the Triangle Midsegment Theorem, $PH = \frac{1}{2}KM$.

Substitute.

$$25 = \frac{1}{2}(KM)$$
$$KM = 50$$

ALGEBRA Find x and y.

24. 

SOLUTION:

We are given three parallel lines, so we know that $y = \frac{3}{5}y + 2$ and $2x + 6 = 20 - 5x$.

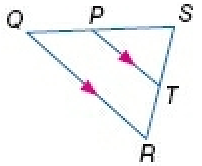
Solve for x.

$$2x + 6 = 20 - 5x$$
$$7x = 14$$
$$x = 2$$

Solve for y.

$$y = \frac{3}{5}y + 2$$
$$y - \frac{3}{5}y = 2$$
$$5\left(y - \frac{3}{5}y\right) = 5(2)$$
$$5y - 3y = 10$$
$$2y = 10$$
$$y = 5$$

Refer to ΔQRS.



34. If $SP = 4$, $PT = 6$, and $QR = 12$, find SQ.

SOLUTION:
Since $\overline{PT} \parallel \overline{QR}$, we know that $\angle SPT \cong \angle SQR$ and $\angle STP \cong \angle SRQ$. Therefore, by AA Similarity, $\Delta SPT \sim \Delta SQR$.

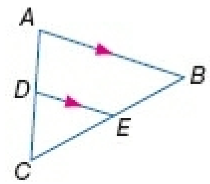
Use the definition of similar polygons to set up a proportion:

$$\frac{QR}{PT} = \frac{SQ}{SP}$$

Substitute and solve for SQ:

$$\frac{12}{6} = \frac{SQ}{4}$$
$$2 = \frac{SQ}{4}$$
$$SQ = 8$$

35. If $CE = t - 2$, $EB = t + 1$, $CD = 2$, and $CA = 10$, find t and CE.



SOLUTION:
Triangle Proportionality Theorem:
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{CE}{EB} = \frac{CD}{DA}$$

Since $CA = 10$ and $CD = 2$, then $DA = 10-2= 8$.

Substitute and solve for t.

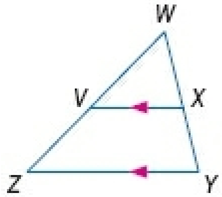
$$\frac{t-2}{t+1} = \frac{2}{8}$$

$$8(t-2) = 2(t+1)$$
$$8t - 16 = 2t + 2$$
$$6t = 18$$
$$t = 3$$

Find CE.

$$CE = t - 2$$
$$= 3 - 2$$
$$= 1$$

36. If $WX = 7$, $WY = a$, $WV = 6$, and $VZ = a - 9$, find WY.



SOLUTION:

Triangle Proportionality Theorem: If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{WX}{XY} = \frac{WV}{VZ}$$

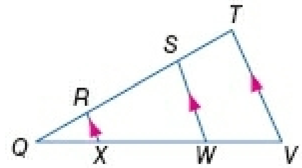
Since $WY = a$ and $WX = 7$, $XY = a - 7$.

Substitute and solve for a.

$$\frac{7}{a-7} = \frac{6}{a-9}$$
$$7(a-9) = 6(a-7)$$
$$7a - 63 = 6a - 42$$
$$a = 21$$

So, $a = WY = 21$.

37. If $QR = 2$, $XW = 12$, $QW = 15$, and $ST = 5$, find RS and WV.



SOLUTION:

Triangle Proportionality Theorem:
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{QR}{RS} = \frac{QX}{XW}$$

Since $QW = 15$ and $WX = 12$, then $QX = 3$.

Substitute and solve for RS.

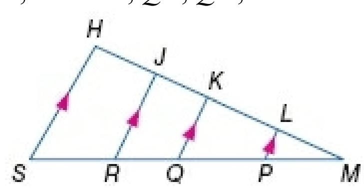
$$\frac{2}{RS} = \frac{3}{12}$$
$$3RS = 24$$
$$RS = 8$$

Additionally, we know that $\frac{QS}{ST} = \frac{QW}{WV}$.

Substitute and solve for WV.

$$\frac{10}{5} = \frac{15}{WV}$$
$$10WV = 75$$
$$WV = 7.5$$

38. If $LK = 4$, $MP = 3$, $PQ = 6$, $KJ = 2$, $RS = 6$, and $LP = 2$, find ML, QR, QK, and JH.



SOLUTION:

Triangle Proportionality Theorem: If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{ML}{LK}=\frac{MP}{PQ}$$

Substitute and solve for *ML*.

$$\frac{ML}{4}=\frac{3}{6}$$
$$6ML=12$$
$$ML=2$$

Also, we know that $\frac{LK}{JK}=\frac{PQ}{QR}$.

Substitute and solve for

$$\frac{4}{2}=\frac{6}{QR}$$
$$4QR=12$$
$$QR=3$$

Because $\Delta MLP \sim \Delta MKQ$, by AA Similarity, we know that $\frac{MP}{PL}=\frac{MQ}{QK}$.

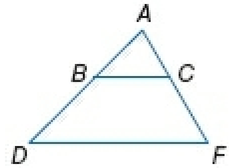
Substitute and solve for *QK*.

$$\frac{MP}{PL}=\frac{MQ}{QK}$$
$$\frac{3}{2}=\frac{9}{QK}$$
$$3QK=18$$
$$QK=6$$

Finally, by Triangle Proportionality Theorem, $\frac{KJ}{JH}=\frac{QR}{RS}$. Substitute and solve for *JH*.

$$\frac{2}{JH}=\frac{3}{6}$$
$$3JH=12$$
$$JH=4$$

Determine the value of *x* so that $\overline{BC} \parallel \overline{DF}$.



40. $AB = x + 5$, $BD = 12$, $AC = 3x + 1$, and $CF = 15$

SOLUTION:

Triangle Proportionality Theorem: If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

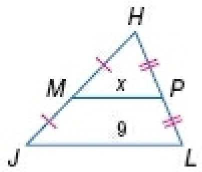
Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC}=\frac{AC}{CF}$$

Substitute.

$$\frac{x+5}{12}=\frac{3x+1}{15}$$
$$15x+75=36x+12$$
$$21x=63$$
$$x=3$$

48. **CRITIQUE ARGUMENTS** Jacob and Sebastian are finding the value of x in ΔJHL. Jacob says that MP is one half of JL, so x is 4.5. Sebastian says that JL is one half of MP, so x is 18. Is either of them correct? Explain.



SOLUTION:

Jacob is correct.

Since M is the midpoint of $\overline{JH}$ and P is the midpoint of $\overline{HL}$, then $\overline{MP}$ is the midsegment of ΔJHL.

Therefore,

$MP = \frac{1}{2}JL$

Substitute:

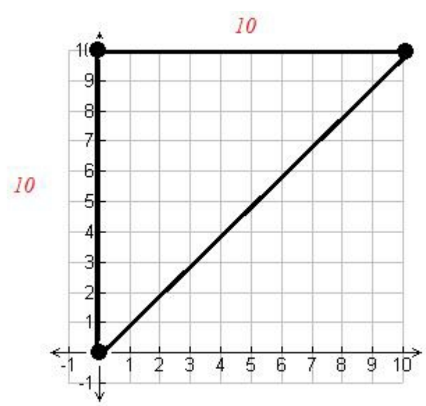
$x = \frac{1}{2}(9)$

$x = 4.5$

57. If the vertices of $\triangle JKL$ are (0, 0), (0, 10), and (10, 10), what is the area of $\triangle JKL$ in square units?

SOLUTION:

First, graph the points and make the triangle. Then, calculate the base and height.

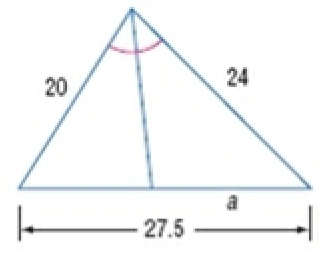


$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(10)$$

$$A = 50 \text{ square units}$$

SENSE-MAKING Find the value of each variable.

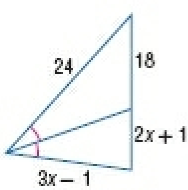
13. 

SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{20}{24} = \frac{27.5 - a}{a}$$
$$20a = 24(27.5 - a)$$
$$20a = 660 - 24a$$
$$44a = 660$$
$$a = 15$$

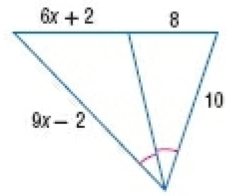
ALGEBRA Find *x*.

20. 

SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

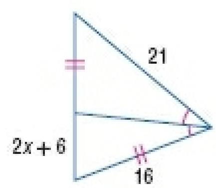
$$\frac{18}{2x+1} = \frac{24}{3x-1}$$
$$18(3x-1) = 24(2x+1)$$
$$54x - 18 = 48x + 24$$
$$6x = 42$$
$$x = 7$$

21. 

SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

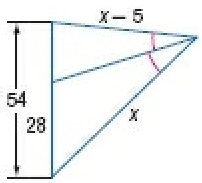
$$\frac{6x+2}{8} = \frac{9x-2}{10}$$
$$10(6x+2) = 8(9x-2)$$
$$60x + 20 = 72x - 16$$
$$12x = 36$$
$$x = 3$$

22. 

SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{2x+6}{16} = \frac{16}{21}$$
$$21(2x+6) = 256$$
$$42x + 126 = 256$$
$$42x = 130$$
$$x \approx 3.1$$

23. 

SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{x-5}{x} = \frac{54-28}{28}$$
$$\frac{x-5}{x} = \frac{26}{28}$$
$$\frac{x-5}{x} = \frac{13}{14}$$
$$14(x-5) = 13x$$
$$14x - 70 = 13x$$
$$x = 70$$