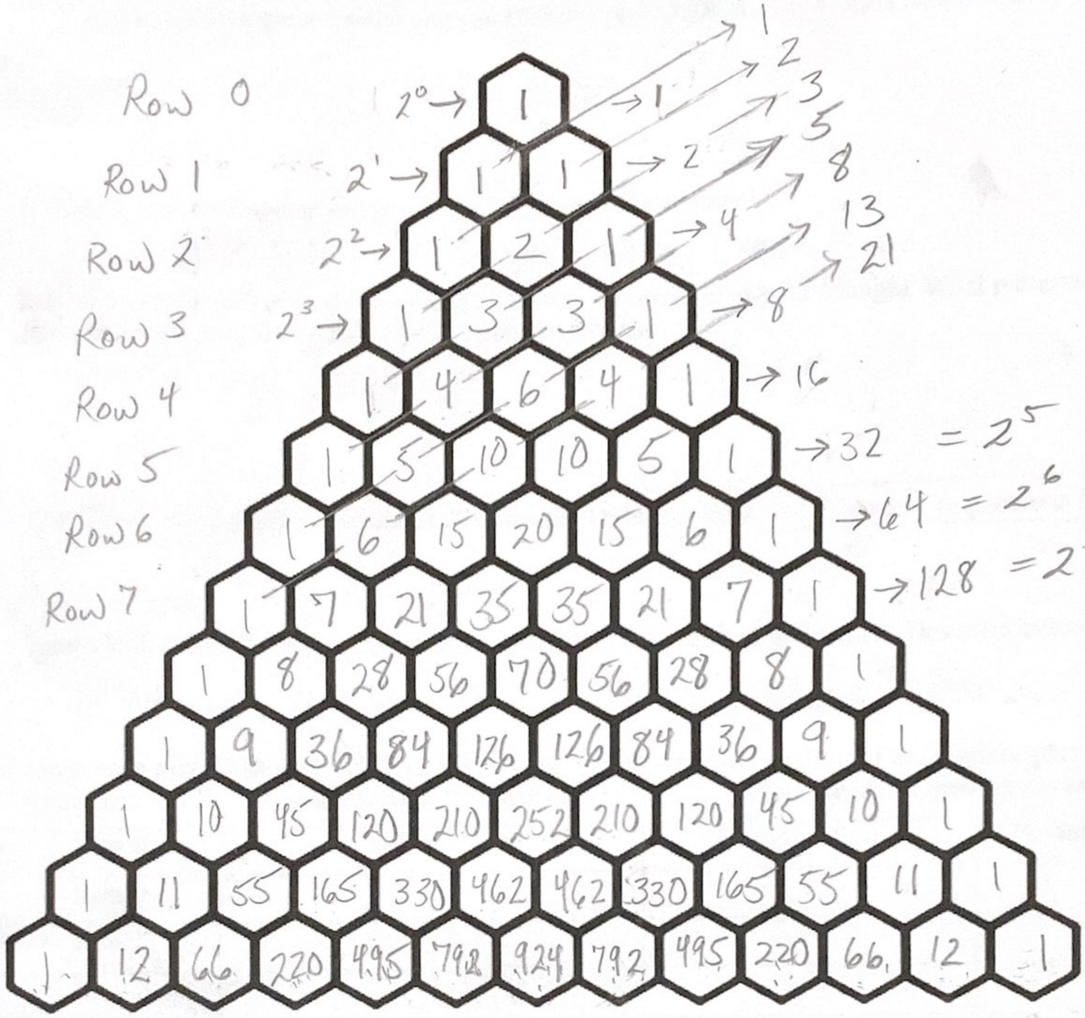
Day 1
p.1-4

Pascal's Triangle



Pascal's Patterns

1. There are over 100 patterns and mathematical ideas that come from Pascal's Triangle. Take a good look at Pascal's Triangle and see if you can find any patterns in it. List at least two below.

2. If you look at the diagonal just inside the ones, what do you see?

   1, 2, 3, 4, 5, 6... counts up by 1 each time

3. Find the sum of each row of Pascal's and list them on one side of your triangle. What patterns are emerging from these sums. Describe the patterns below.

   It doubles each time

   Powers of 2

4. The Fibonacci sequence is present in this triangle. Describe where the Fibonacci sequence is found.

   Each term is sum of previous two 1, 1, 2, 3, 5, 8, 13, 21, ...   Sum of

5. Take a look at row 7, what do you notice about all of the numbers in that row. Describe below.

   All are divisible by 7   (Any others? Row 5...)

6. Make each row into a single digit number by using each element as a digit of the number (carrying over when an element itself has more than one digit). What do you notice about these new numbers?

Row #	Actual Row	Single Digit	Pattern
Row 0	1	1	11^0
Row 1	1 1	11	11^1
Row 2	1 2 1	121	11^2
Row 3	1 3 3 1	1331	11^3
Row 4	1 4 6 4 1	14641	etc
Row 5	1 5 10 10 5 1	161051	
Row 6	1 6 15 20 15 6 1	1771561	
Row 7	1 7 21 35 35 21 7 1	19487171	
Row 8	1 8 28 56 70 56 28 8 1	214358881	

7. Color the odd numbers in the triangle on page 4, what patterns or formations are emerging?

small Δs with open center . . .

8. Color the even numbers in a different triangle on page 4, what patterns or formations are emerging?

upside-down Δs . . .

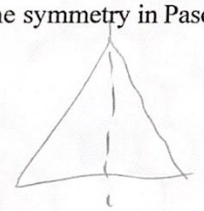
9. Color the numbers in Pascal's Triangle that are divisible by 3. What patterns are forming?

2 small Δs then large . . . all upside down

10. What pattern do you think will form if you color in all numbers divisible by 5? Divisible by 7? Now choose one pattern to color in the last triangle on page 4. Was your guess correct?

More Δs . . .

11. Describe the symmetry in Pascal's Triangle.



Fold it down the center: the left side is a reflection of the right.

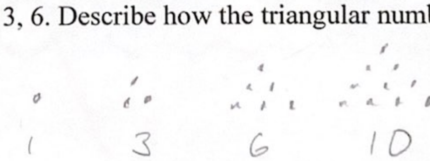
Each row increases then comes back down with the same numbers

12. There is a pattern known as a hockey stick in Pascal's Triangle, can you find it and describe the pattern?

Follow a diagonal from any 1 on the edge, then turn the other way to go 1 more. That will be the sum of the others

13. Find the triangular numbers. The first three numbers are 1, 3, 6. Describe how the triangular numbers are found.

2 diagonals from edge



14. Last but not least, try to find the square numbers in this triangle. These are numbers like 4, 9, 16, 25. Hint they are found in the same place the triangular numbers are found. Describe how you found these.

Sum of pairs of #s in the triangular # diagonal

When get to #7: put steps on sticky note! Day 2

Exploring the Binomial Expansion Theorem and Pascal's Triangle Relationships

2 colored pencils EACH person

1. Multiply the following binomials.

   a. $(x + 2)^2$ $(x+2)(x+2)$
   $= x^2 + 2x + 2x + 4 = \boxed{x^2 + 4x + 4}$

   b. $(x - 5)^2 = (x-5)(x-5) = x^2 - 5x - 5x + 25 = \boxed{x^2 - 10x + 25}$

   c. $(4m + 3n)^2 = (4m+3n)(4m+3n) = 16m^2 + 12mn + 12mn + 9n^2$
   $= 16m^2 + 24mn + 9n^2$

   d. $(r + 2)^3 = (r+2)(r+2)(r+2)$ above
   $= (r+2)(r^2+4r+4) = r^3 + 4r^2 + 4r + 2r^2 + 8r + 8$
   $= r^3 + 6r^2 + 12r + 8$

2. Exploration. Fill in the following table.
   *Hint: to expand $(x + y)^3$, you can multiply $(x + y)^2$ by $(x + y)^1$

Product	Expansion
$(x + y)^0$	1
$(x + y)^1$	$1x + 1y$
$(x + y)^2$	$1x^2 + 2xy + 1y^2$
$(x + y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$
$(x + y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$
$(x + y)^5$	$1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$
$(x+y)^6$	$1x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + 1y^6$

3. Write conjectures about the number of terms and about symmetry in the terms of the expansion in any row of the table. Verify your conjectures by filling in the row that would follow $(x + y)^5$.

   # of terms: 1 more than power
   { start $x^{\square}$ / end $y^{\square}$ }
   in between: $x^{\square}y^{\square}$ with x-powers decreasing & y's increasing

4. Do you notice any relationships between the expansion and Pascal's Triangle? Write these below.

   coefficients match the row for that power

5. Use the pattern you saw to try to expand $(x + y)^{11}$.

$1x^{11} + 11x^{10}y + 55x^9y^2 + 165x^8y^3 + 330x^7y^4 + 462x^6y^5 + 462x^5y^6 + 330x^4y^7 + 165x^3y^8 + 55x^2y^9 + 11xy^{10}$
$+ 1y^{11}$

Notice: exponents of any term add up to row#

6. Expand the following binomials.

a. $(a+b)^6$ $= 1a^6 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6a^1b^5 + 1b^6$

b. $(p+q)^4$ $= 1p^4 + 4p^3q^1 + 6p^2q^2 + 4p^1q^3 + 1q^4$

Expand the binomials below.

c. $(x-y)^3$ *hint: think of this one as $(x+(-y))^3$

$1x^3 + 3x^2(-y)^1 + 3x(-y)^2 + 1(-y)^3$ → $(-y)\cdot(-y) = y^2$ positive

neg × neg

$= 1x^3 - 3x^2y^1 + 3x^1y^2 - 1y^3$

alternating between + & −

d. $(y+3)^4$ Now complete the multiplication below.

$1y^4 + 4\cdot y^3\cdot 3^1 + 6\cdot y^2 3^2 + 4\cdot y^1\cdot 3^3 + 1\cdot 3^4$

$= 1y^4 + 12y^3 + 54y^2 + 108y + 81$

$27 \times 4 = 108$

7. Fill in the blanks below to expand $(2x-3y)^4$. Remember think of this one as $(2x+(-3y))^4$

① 4th row coefficients
② $(2x)(-3y)$ for each 4
③ + between terms
④ exp for $(2x)$: start at 4↓
⑤ exp for $(-3y)$: start at 0↑
⑥ simplify

$1(2x)^4(-3y)^0 + 4(2x)^3(-3y)^1 + 6(2x)^2(-3y)^2 + 4(2x)^1(-3y)^3 + 1(2x)^0(-3y)^4$

$= 16x^4 + 4\cdot 8\cdot x^3\cdot -3\cdot y + 6\cdot 4\cdot x^2\cdot 9\cdot y^2 + 4\cdot 2\cdot x\cdot -27\cdot y^3 + 81y^4$

$\boxed{= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4}$

8. Try to expand the binomials below using the pattern above.

a. $(a+2b)^4$ $1(a)^4(2b)^0 + 4(a)^3(2b)^1 + 6(a)^2(2b)^2 + 4(a)^1(2b)^3 + 1(a)^0(2b)^4$

$\boxed{1a^4 + 8a^3b^1 + 24a^2b^2 + 32a^1b^3 + 16b^4}$

b. $(2x+3)^3$ $1(2x)^3(3)^0 + 3(2x)^2(3)^1 + 3(2x)^1(3)^2 + 1(2x)^0(3)^3$

$= 1\cdot 2^3\cdot x^3\cdot 1 + 3\cdot 2^2\cdot x^2\cdot 3 + 3\cdot 2\cdot x\cdot 9 + 1\cdot 1\cdot 27$

$= \boxed{8x^3 + 36x^2 + 54x + 27}$

c. $(3x+2y)^5$

$1(3x)^5(2y)^0 + 5(3x)^4(2y)^1 + 10(3x)^3(2y)^2 + 10(3x)^2(2y)^3 + 5(3x)^1(2y)^4 + 1(3x)^0(2y)^5$

$= 3^5x^5 + 5\cdot 3^4 x^4\cdot 2\cdot y + 10\cdot 3^3\cdot x^3\cdot 2^2\cdot y^2 + 10\cdot 3^2\cdot x^2\cdot 2^3\cdot y^3 + 5\cdot 3\cdot x\cdot 2^4 y^4 + 2^5y^5$

$= 243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

d. $(2x-2y)^4$ (be careful on this one! Remember think of the binomial as $(2x+(-2y))^4$

$1(2x)^4(-2y)^0 + 4(2x)^3(-2y)^1 + 6(2x)^2(-2y)^2 + 4(2x)^1(-2y)^3 + 1(2x)^0(-2y)^4$

$= 2^4x^4 + 4\cdot 2^3 x^3 (-2) y^1 + 6\cdot 2^2\cdot x^2\cdot (-2)^2\cdot y^2 + 4\cdot 2\cdot x\cdot (-2)^3\cdot y^3 + (-2)^4\cdot y^4$

$= 16x^4 - 64x^3y^1 + 96x^2y^2 - 64xy^3 + 16y^4$