Plot the following (x, y) coordinates to reveal a picture:

Connect the following points in order:

(1, 10)

→ (-1, _____) if -3x = y - 7 → (-1, 10) (x, y) Plug in for x: $-3x = y - 7$, $-3(-1) = y - 7$, $3 = y - 7$, $+7 \; +7$, $10 = y$

(-10, -8)

(-9, -10)

(-1, _____) if y = 2x+8 → (-1, 6) (x, y) $y = 2(-1) + 8$, $y = -2 + 8 = 6$

(0, 4)

STOP

Connect the following points in order: → Does this combination of x & y make it true?

Plot the point (10, -8) only if it is a solution to x – 2y = 26

(-6, -8)

(_____, -6) if 2y = 3x + 3 (x, y) $2(-6) = 3x + 3$

(5, -6)

STOP

$10 - 2(-8) = 26$?

$10 + 16 = 26$

$26 = 26$ ✓ Yes, it's a solution!

Connect the following points in order:

Use 2x + y = 12 to find the missing value, then plot the points

x	1	4	8	10
y	10	4	-4	-8

x=1: $2 \cdot 1 + y = 12$, $2 + y = 12$, $-2 \; -2$, $y = 10$

y=4: $2x + 4 = 12$, $-4 \; -4$, $2x = 8$, $\frac{2x}{2} = \frac{8}{2}$, $x = 4$

y=-4: $2x - 4 = 12$, $+4 \; +4$, $2x = 16$, $\frac{2x}{2} = \frac{16}{2}$, $x = 8$

x=10: $2 \cdot 10 + y = 12$, $20 + y = 12$, $-20 \; -20$, $y = -8$

Use y = -10 to find the missing value, then plot the points

x	9	5	-3	-9
y	-10	-10	-10	-10

← y doesn't depend on x! It's just constantly -10 in this one.

STOP

Connect the following points in order:

Plot the point (0, 7) only if it is a solution to 5x - y = 9

(-5, -6) x y

(0, 4)

$5 \cdot 0 - 7 = 9$?

$-7 \neq 9$ No, it's not a solution

Plot the point (5, -6) only if it is a solution to x = 5

(7, -6) x y

(-1, 10)

Yes, the point has an x-coordinate of 5.

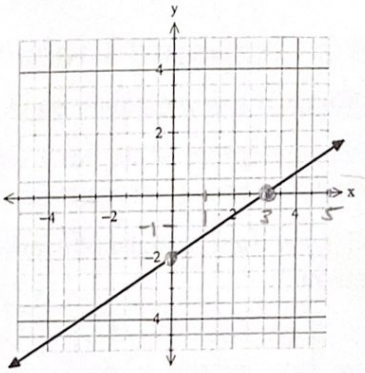
STOP

Now color your picture ☺

3.2 Notes

Pre-College Math Unit 3 Guided Notes

Two useful points on a graph: the *x*-intercept and *y*-intercept

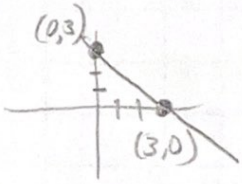


x-intercept: (3, 0) where the line crosses the x-axis.

y-intercept: (0, -2) where it crosses the y-axis

5) Complete the following ordered pairs for the line y = -x + 3:

(0, 3) $y = -x+3$, $y = -(0)+3 = 3$ (3, 0) $0 = -x+3$, $-3 \quad -3$, $-3 = -1x$, $\frac{-3}{-1} = \frac{-1x}{-1}$, $3 = x$



What do these ordered pairs represent on a graph?

y-int x-int

How can these help us graph the line?

2 points are enough to graph a line

6) What is true about every x-intercept? What is true about every y-intercept?

Ex. (3,0) The y-coordinate is 0.

Ex. (0,-2) The x-coord. is 0.

Graph the following equations using x and y-intercepts:

7) 4x - 5y = 20

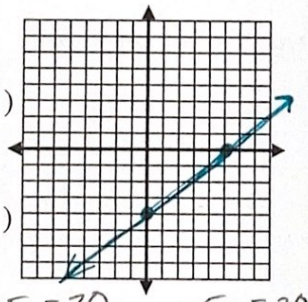
$4x - 5\cdot 0 = 20$, $\frac{4x}{4} = \frac{20}{4}$

x-intercept: (5, 0)

Find x when y is 0

y-intercept: (0, -4)

Find y when x is 0



$4x - 5y = 20$, $4\cdot 0 - 5y = 20$, $\frac{-5y}{-5} = \frac{20}{-5}$, $y = -4$

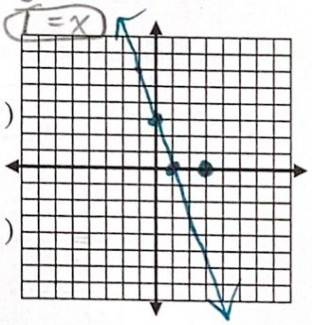
8) y = -3x + 3

$0 = -3x + 3$, $-3 \quad -3$, $\frac{-3}{-3} = \frac{-3x}{-3}$, $1 = x$

x-intercept: (1, 0)

y-intercept: (0, 3)

$y = -3x + 3$, $y = -3\cdot 0 + 3$, $y = 3$



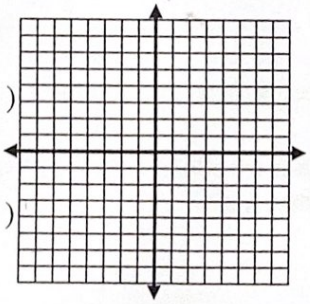
10) 2x - y = -4

Put 0 in for y: $2x - 0 = -4$, $x = -2$

x-intercept: (-2, 0)

y-intercept: (0, 4)

$2\cdot 0 - y = -4$, $-y = -4$, $y = 4$ (Put 0 in for x)



11) x – 3y = 0

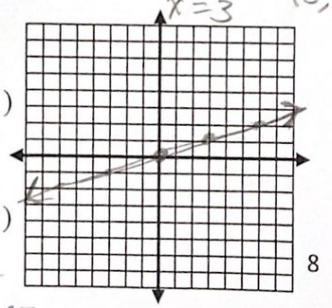
Try y=1: $x - 3\cdot 1 = 0$, $x = 3$ (3, 1)

$x - 3\cdot 0 = 0$, $x = 0$

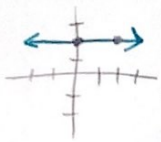
x-intercept: (0, 0)

y-intercept: (0, 0)

Both are the same point! Need another...



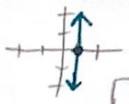
Horizontal lines



$y = 2$ Cuts through y-axis at 2

$y =$ a number

Vertical lines



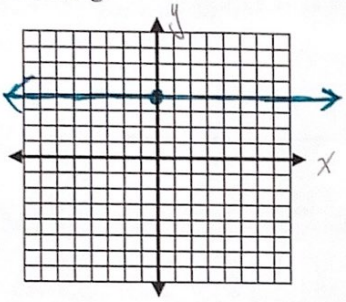
$x = 1$ Cuts through x-axis at 1

$x =$ a number

Graph the following:

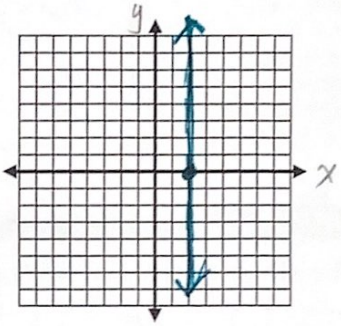
12) $y = 4$

horizontal



13) $x = 2$

vertical



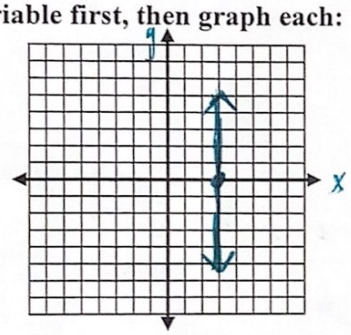
Solve for the variable first, then graph each:

14) $x - 3 = 0$

$+3 \quad +3$

$x = 3$

vertical

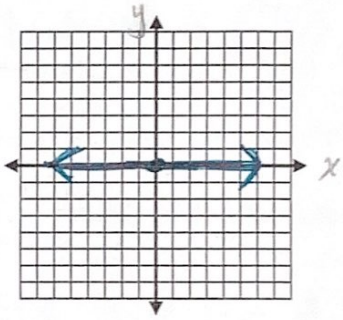


15) $y + 5 = 5$

$-5 \quad -5$

$y = 0$

horizontal



16) Credit card debt in the United States increased steadily from 2000 through 2008. The amount of debt y, in billions of dollars, can be modeled by the linear equation $y = 32x + 684$, where $x = 0$ represents 2000, $x = 1$ represents 2001, and so on. (source: The Nilson Report)

a) Use the equation to approximate credit card debt in the years 2000, 2004, and 2008.

$x=0$ $x=4$ $x=8$

2000: $x = 0 \Rightarrow y = 32 \cdot 0 + 684 = 684$

2004: $x = 4 \Rightarrow y = 32 \cdot 4 + 684 = 812$

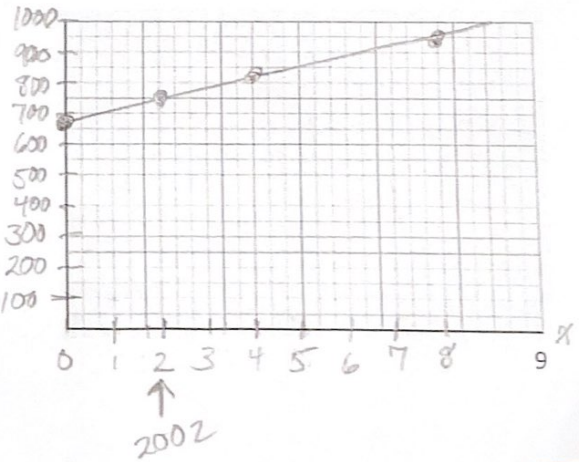
2008: $x = 8 \Rightarrow y = 32 \cdot 8 + 684 = 940$

b) Write the information from part (a) as three ordered pairs, then use them to graph the equation.

(0, 684)

(4, 812)

(8, 940)



c) Use the graph and then the equation to approximate the credit card debt in 2002.

(2, 750) on the graph

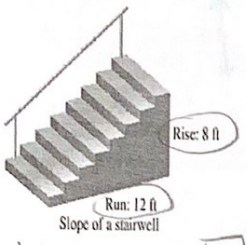
x y

$y = 32x + 684$

$y = 32 \cdot 2 + 684 =$ \$748 billion

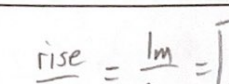
3.3 Slope of a Line

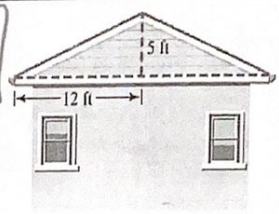
1. Find the slope of a line, given two points.
2. Find the slope from the equation of a line.
3. Use slopes to determine whether two lines are parallel, perpendicular, or neither.



$\frac{\text{rise}}{\text{run}} = \frac{8\text{ft}}{12\text{ft}} = \frac{2}{3}$

$\frac{\text{rise}}{\text{run}} = \frac{1\text{m}}{10\text{m}} = \frac{1}{10}$





$\frac{\text{rise}}{\text{run}}$ $\frac{5}{12}$ roof pitch

EXAMPLE 1 Finding the Slope of a Line

Find the slope of the line in **FIGURE 19**.

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-4}{4} = -1$

Negative slope: goes downward from left to right

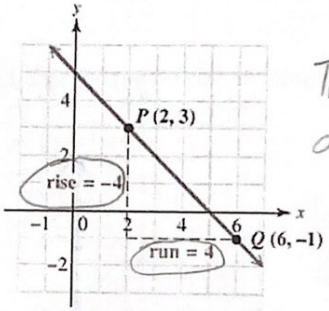
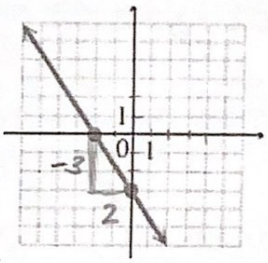


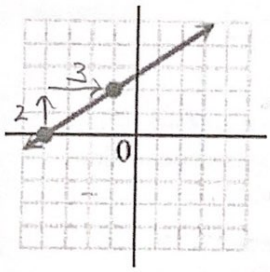
FIGURE 19

The "rise" is actually a drop here!

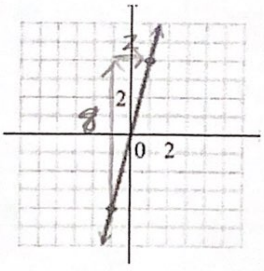
Find the slope of each line below.

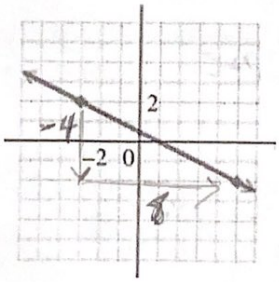
a. 

slope = $\frac{-3}{2}$

b. 

slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{3}$

c. 

d. 

slope = $\frac{-4}{8} = \frac{-1}{2}$

slope = $\frac{\text{rise}}{\text{run}} = \frac{8}{2} = 4$

Slope formula

$\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = m$

Positive Slope

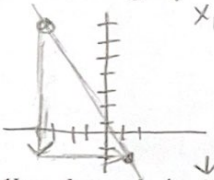
Negative Slope

EXAMPLE 2 Finding Slopes of Lines

Find the slope of each line.

(a) The line through $(-4, 7)$ and $(1, -2)$

x_1 y_1 x_2 y_2



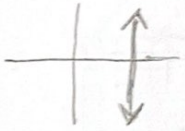
$m = \frac{\text{rise}}{\text{run}} = \frac{\text{drops } [?]}{\text{right } [?]} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{1-(-4)} = \frac{-9}{1+4} = \boxed{\frac{-9}{5}}$

(b) The line through $(-9, -2)$ and $(12, 5)$

x_1 y_1 x_2 y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{12-(-9)} = \frac{5+2}{12+9} = \frac{7}{21} = \boxed{\frac{1}{3}}$

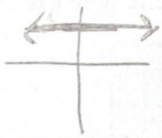
Slope of Vertical Line



$\frac{\text{rise}}{\text{run}} = \frac{\text{rise}}{0}$

Undefined!

Slope of Horizontal Line



$\frac{\text{rise}}{\text{run}} = \frac{0}{\text{run}} = \boxed{0}$

EXAMPLE 3 Finding the Slope of a Horizontal Line

Find the slope of the line through $(-5, 4)$ and $(2, 4)$.

x_1 y_1 x_2 y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{2-(-5)} = \frac{0}{2+5} = \frac{0}{7} = \boxed{0}$

EXAMPLE 4 Finding the Slope of a Vertical Line

Find the slope of the line through $(6, 2)$ and $(6, -4)$.

x_1 y_1 x_2 y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{6-6} = \frac{-6}{0} \Rightarrow \boxed{\text{Undefined}}$

Slope-intercept form

$y = mx + b$

m = slope

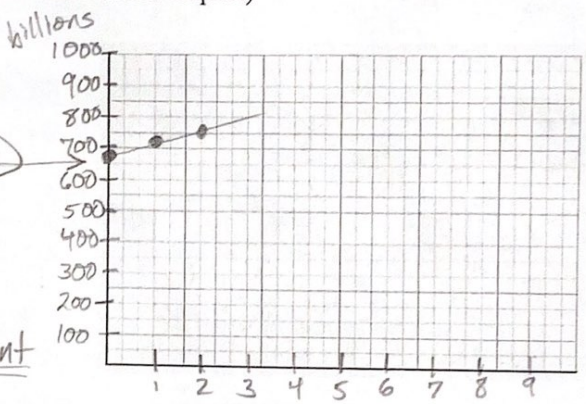
The point $(0, b)$ is the y-intercept

Recall the credit card debt example and graph:

Credit card debt in the United States increased steadily from 2000 through 2008. The amount of debt y, in billions of dollars, can be modeled by the linear equation $y = 32x + 684$, where $x = 0$ represents 2000, $x = 1$ represents 2001, and so on. (source: The Nilson Report)

x	y
0	684
1	716
2	748

y-int



Identify the slope and y-intercept:

slope: $\frac{\text{rise}}{\text{run}} = \frac{32}{1}$

y-intercept: 684 billion, starting point

Example 5:

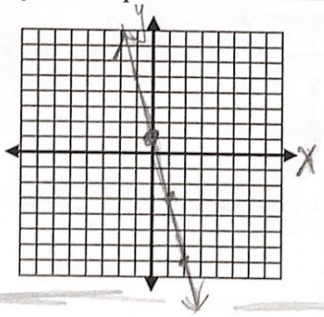
Identify the slope and the y-intercept of the line with each equation. Then graph each.

a) $y = -4x + 1$

$y = mx + b$

slope: -4

y-int: $(0, 1)$ START

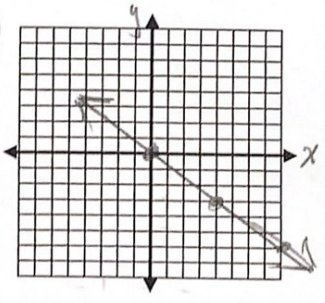


b) $y = -\frac{3}{4}x + 0$

$y = mx + b$

slope $= -\frac{3}{4}$ $\frac{\text{rise}}{\text{run}}$

y-int $= (0, 0)$



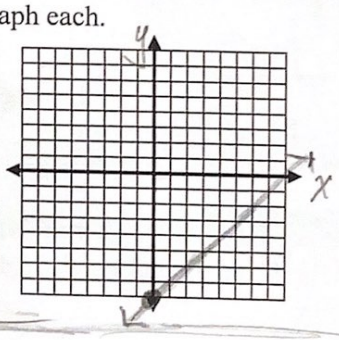
c) $y = x - 8$

$y = 1x - 8$

$y = mx + b$

slope: 1

y-int: $(0, -8)$



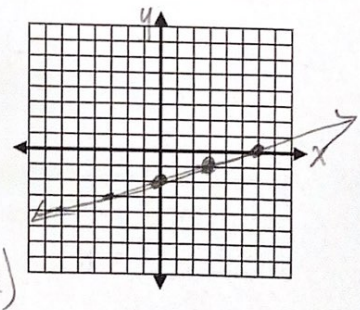
d) $y = \frac{1}{3}x - 2$

$y = \frac{1}{3}x - 2$

$y = mx + b$

slope $= \frac{1}{3}$

y-int $= (0, -2)$



Example 6: Write an equation of the line with slope $\frac{2}{3}$ and y-intercept (0, 1).

$m =$ $b =$

$y = mx + b$

$y = \frac{2}{3}x + 1$

3.4 Writing and Graphing Equations of Lines

Example 1: Find the slope of each line.

Hint: Solve for y so that you can see the equation in y=mx+b form!

a) $2x - 5y = 4$

$-2x \quad -2x$ move x away

$-5y = -2x + 4$ ← Not like terms!

$\frac{-5y}{-5} = \frac{-2x+4}{-5}$ Divide everything by -5

$y = \frac{2}{5}x - \frac{4}{5}$

slope $= \frac{2}{5}$

b) $8x + 4y = 1$

$-8x \quad -8x$

$4y = -8x + 1$

$\frac{4y}{4} = \frac{-8x+1}{4}$

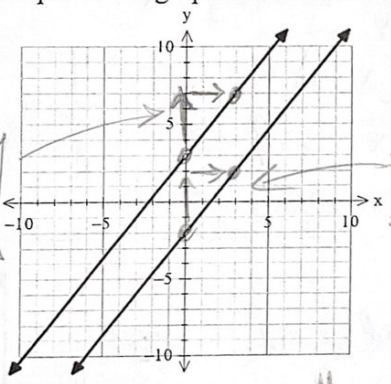
$y = -2x + \frac{1}{4}$

slope $= -2$

Example 2: Special pairs of lines

Find the slopes of the graphed lines below: Count from any place the lines cross the grid neatly

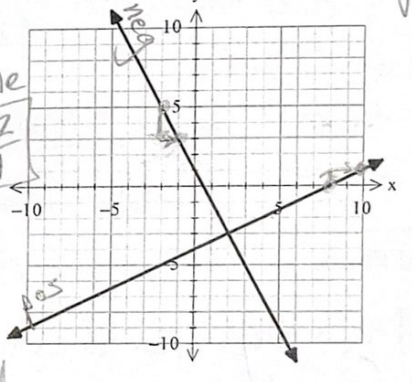
a)



slope: rise/run $= \frac{4}{3}$

slope $= \frac{4}{3}$

b)



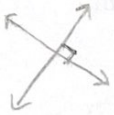
slope $= \frac{-2}{1}$

slope $= \frac{1}{2}$

Slope of parallel lines

same slope

Slope of perpendicular lines



- one neg., one pos. (opposites)
- flipped (reciprocal)

Example 3: Decide whether each pair of lines is parallel, perpendicular, or neither. Solve for y to see slopes

a) $x + 3y = 7$

$-3x + y = 3$

$+3x \quad +3x$

$y = 3x + 3$

$y = mx + b$

$m = \frac{3}{1}$

$x + 3y = 7$

$-x \quad -x$

$\frac{3y}{3} = \frac{-x+7}{3}$

$y = \frac{-1}{3}x + \frac{7}{3}$

$m = \frac{-1}{3}$

$y = mx + b$

✓ one pos, one neg

✓ flipped

⇒ Perpendicular

b) $2x - 3y = 1$

$4x + 6y = 5$

$-4x \quad -4x$

$\frac{6y}{6} = \frac{-4x+5}{6}$

$y = \frac{-4}{6}x + \frac{5}{6}$

$y = \frac{-2}{3}x + \frac{5}{6}$

$m = \frac{-2}{3}$

$2x - 3y = 1$

$-2x \quad -2x$

$\frac{-3y}{-3} = \frac{-2x+1}{-3}$

$m = \frac{2}{3}$ $y = \frac{2}{3}x - \frac{1}{3}$

X Not same

X Not opposite & reciprocal

Point-slope form

$y - y_1 = m(x - x_1)$

slope, point (x_1, y_1)

h,k form for a line

$y = m(x - h) + k$

slope, point (h, k)

Example 4: Write an equation of the line having slope of 4 and passes through the point (2, 5).
Put your final answer into slope-intercept form. →

Start with h,k form: $y = m(x-h)+k$

$y = 4(x-2)+5$

$y = 4(x-2)+5$

$y = 4x - 8 + 5$

$y = 4x - 3$ slope-int. form

Example 5: Write an equation of each line. Give the final answer in slope-intercept form.

a) Through (-2, 4), with slope -3

$y = -3(x - -2) + 4$

$y = -3(x+2)+4$

$y = -3x - 6 + 4$

$y = -3x - 2$

b) Through (4, 2), with slope $\frac{3}{5} = m$

$y = \frac{3}{5}(x-4)+2$

$y = \frac{3}{5}x - \frac{3}{5}\cdot\frac{4}{1} + 2$

$y = \frac{3}{5}x - \frac{12}{5} + \frac{2\cdot 5}{1\cdot 5}$

$y = \frac{3}{5}x - \frac{12}{5} + \frac{10}{5}$

$y = \frac{3}{5}x - \frac{2}{5}$

Standard Form: (x & y on same side)

$Ax + By = C$ where A, B, and C are integers (not fractions or decimals)

Example 6: Write the equations in Standard form.

a. $y = 2x - 4$

$-2x \quad -2x$

$y - 2x = -4 \rightarrow -2x + y = -4$

b. $y = \frac{2}{3}x - 2$ Clear fractions: mult. by 3

$3y = \frac{3}{1}\cdot\frac{2}{3}x - 3\cdot 2$

$3y = 2x - 6$

$-2x \quad -2x$

$-2x + 3y = -6$

Writing equations for horizontal and vertical lines

Horizontal ↔ $y =$ a number

Vertical ↕ $x =$ a number

Example 7: Write the equation for the line described:

a. horizontal through (2, -3) $y = ?$ (x, y) $y = -3$

b. vertical through (3, 4) $x = ?$ (x, y) $x = 3$

c. horizontal through (0, 5) $y = ?$ (x, y) $y = 5$

d. vertical through (-6, 7) $x = ?$ (x, y) $x = -6$