Unit 10 Assignments for Prob/Stat/Discrete
Chapter 6/7: Equations and Linear Systems

Day	Date	**Assignment** (Due the next class meeting)
		10.1 Worksheet
		10.2 Worksheet
		10.3 Worksheet
		10.4 Worksheet
		10.5 Worksheet
		10.6 Worksheet
		Unit 10 Practice Test
		Unit 10 Test

NOTE: You should be prepared for daily quizzes.

HW reminders:

- If you cannot solve a problem, get help **before** the assignment is due.
- Help is available before school, during lunch, or during IC.
- For extra practice, visit www.interactmath.com
- Don't forget that you can get 24-hour math help from www.smarthinking.com!

Section 10.1: Applications of Linear Equations

Objective

1. Use linear equations to solve problems.

Strategy for Solving Word Problems

Step 1. Read the problem carefully at least __________. Attempt to state the problem in your own words and state what the problem is looking for. Let *x* (or any variable) represent one of the quantities in the problem.

Step 2. If necessary, write expressions for any other unknown quantities in the problem in terms of ____.

Step 3. Write an equation in *x* that describes the verbal conditions of the problem.

Step 4. Solve the equation and answer the problem's question.

Step 5. _____ the solution *in the original wording* of the problem, not in the equation obtained from the words.

Algebraic Translations of English Phrases

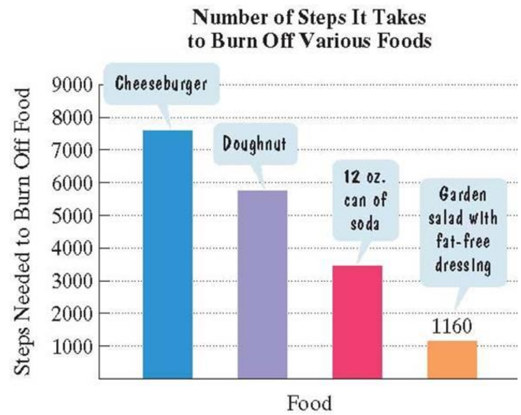
Addition	**Subtraction**	**Multiplication**	**Division**
sum	minus	times	divided by
more than	decreased by	product of	quotient
increased by	subtracted from	percent of a number	reciprocal
	difference between	multiplied by	
	less than	twice	
	fewer than		

Example 1: Solving a Word Problem

Nine subtracted from eight times a number is 39. Find the number.

Example 2: Walk it Off

This graph shows the number of steps needed to burn off various foods.



The number of steps needed to burn off a cheeseburger exceeds the number needed to burn off a 12-ounce soda by 4140. The number needed to burn off a doughnut exceeds the number needed to burn off a 12-ounce soda by 2300.

- Determine the number of steps it takes to burn off each of a cheeseburger, doughnut and 12-ounce soda if a 16,790 step walk is needed to burn off all three combined.

Solution:

Let x = number of steps needed to burn off a 12-ounce soda.

Let __________ = number of steps needed to burn off a cheeseburger.

Let _________ = number of steps needed to burn off a doughnut.

Write an equation that models the conditions:

Example 3: You are choosing between two long-distance telephone plans. Plan A has a monthly fee of \$20 with a charge of \$0.05 per minute for all long-distance calls. Plan B has a monthly fee of \$5 with a charge of \$0.10 per minute for all long-distance calls. For how many minutes of long-distance calls will the costs for the two-plans be the same?

Section 10.2: Ratio, Proportion, and Variation

Objectives

1. Solve proportions.
2. Solve problems using proportions.
3. Solve direct variation problems.
4. Solve inverse variation problems.

Proportions:

- ______________ compares quantities by division.

Example 1: a group contains 60 women and 30 men. The ratio of women to men is:

- **A** __________________ is a statement that says two ratios are equal: $\frac{a}{b} = \frac{c}{d}$

Example 1
The Cross-Products Principle for Proportions

If $\frac{a}{b} = \frac{c}{d}$ then *ad* = *bc*. (b ≠ 0 and d ≠ 0)

The cross products *ad* and *bc* are equal.

Solve this proportion for x: Check the solution:

$$\frac{63}{x} = \frac{7}{5}$$

Applications of Proportions Solving Applied Problems Using Proportions

1. Read the problem and represent the unknown ___________________ by x (or any letter).
2. Set up a proportion by listing the given ______________ on one side and the _____________ with the unknown quantity on the other side. Each respective quantity should occupy the same corresponding position on each side of the proportion.
3. Drop units and apply the _______________-____________________ principle.
4. Solve for x and answer the question.

Example 2
Calculating Taxes

- The property tax on a house whose assessed value is

$65,000 is $825. Determine the property tax on a house with

an assessed value of $180,000, assuming the same tax rate.

- Solution:

Step 1. Let x = tax on a $180,000 house.

Step 2. Set up the proportion:

Step 3 Drop the units and apply the cross products principle

Step 4 Solve for x and answer the question.

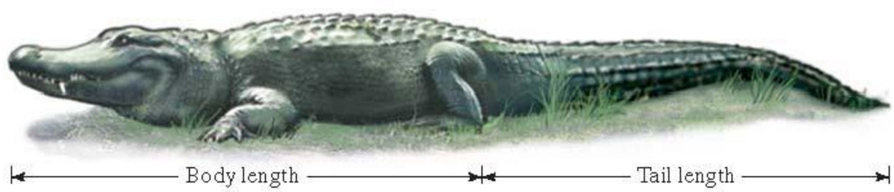
The property tax on the $180,000 house is approximately $______________

Example 3
Direct Variation

As one quantity increases, the other quantity increases and vice versa.

- An alligator's tail length varies directly as its body length.

  An alligator with a body length of 4 feet has a tail length of 3.6 feet. What is the tail length of an alligator whose body length is 6 ft?



- Solution

  Step 1. Let x = tail length of an alligator whose body length is 6 feet.

  Step 2. Set up the proportion:

  Step 3. Apply the cross-products principle, solve and answer the question.

An alligator whose body length is 6 feet has a tail length measuring ________ feet.

Inverse Variation

- As one quantity increases, the other quantity decreases and vice versa.
- Setting up a Proportion when y varies inversely as x

$$\frac{The\ first\ value\ for\ y}{The\ value\ for\ x\ corresponding\ to\ the\ second\ value\ for\ y} = \frac{The\ second\ value\ for\ y}{The\ value\ for\ x\ corresponding\ to\ the\ first\ value\ for\ y}$$

Example 4
Inverse Variation

A bicyclist tips the cycle when making a turn. The angle B, formed by the vertical direction and the bicycle is called the **banking angle**. The banking angle varies inversely as the cycle's turning radius. When the turning radius is 4 feet the banking angle is 28°.

- What is the banking angle when the turning radius is 3.5 feet?

- Solution

Step 1. Represent the unknown x

x = banking angle when

turning radius is 3.5 feet.

Step 2. Set up the proportion

Step 3 and 4. Apply the cross products principle, solve, and answer the question.

When the turning radius is 3.5 feet, the banking angle is ____°.

Section 10.3: Linear Inequalities in One Variable

Objectives

1. Graph subsets of real numbers on a number line.
2. Solve linear inequalities.
3. Solve applied problems using linear inequalities.

- Linear Inequalities

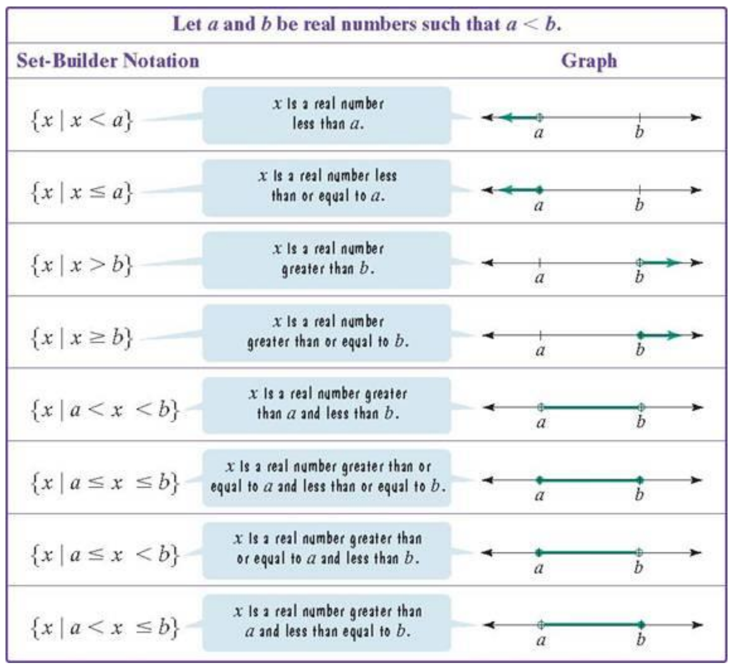
A **linear inequality :** ax + b [inequality symbol] c, where the inequality symbol can be ____________

- **Solving an inequality** is the process of finding the set of numbers that make an inequality a ________statement.
- **A solution set** is the set of all numbers that __________ the inequality.

Graphing Subsets of Real Numbers on a Number Line

Open dots – indicate a number is not included in a set.

Closed dots- indicate a number is included in a set.



Let a and b be real numbers such that $a < b$.		
Set-Builder Notation		**Graph**
$\{x \mid x < a\}$	x is a real number less than a.	
$\{x \mid x \leq a\}$	x is a real number less than or equal to a.	
$\{x \mid x > b\}$	x is a real number greater than b.	
$\{x \mid x \geq b\}$	x is a real number greater than or equal to b.	
$\{x \mid a < x < b\}$	x is a real number greater than a and less than b.	
$\{x \mid a \leq x \leq b\}$	x is a real number greater than or equal to a and less than or equal to b.	
$\{x \mid a \leq x < b\}$	x is a real number greater than or equal to a and less than b.	
$\{x \mid a < x \leq b\}$	x is a real number greater than a and less than equal to b.	

Example 1: Graphing Subsets of Real Numbers

a) $\{x : x < 3\}$

b) $\{x : x \geq -1\}$

c) $\{x : -1 < x \leq 3\}$

Solving Linear Inequalities in One Variable

- The procedure for solving linear inequalities is the same as the procedure for solving linear equations, with one important exception:

  Remember!! **When multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality symbol, changing the sense of the inequality.

Example 2: Solving a Linear Inequality

Solve and graph the solution set: $6x - 12 > 8x + 2$

Example 3: Solving a Linear Inequality

Solve the three part inequality: $-3 < 2x + 1 \leq 3.$

$2x + 1$ Is greater than -3 and less than or equal to 3.

Example 4: To earn an A in a course, you must have a final average of at least 90%. On the first four examinations, you have grades of 86%, 88%, 92%, and 84%. If the final examination counts as two grades, what must you get on the final to earn an A in the course?

Section 10.4: Systems of Linear Equations in Two Variables

Objectives

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve linear systems by graphing.
3. Solve linear systems by substitution.
4. Solve linear systems by addition.
5. Identify systems that do not have exactly one ordered-pair solution.
6. Solve problems using systems of linear equations.

Systems of Linear Equations & Their Solutions

- Two linear equations are called a ***system of linear equations*** or a ***linear system***.
- A ***solution to a system of linear equations in two variables*** is an ____________________ that satisfies both equations in the system.

Example 1: Determine whether (1,2) is a solution of the system:

$2x - 3y = -4$
$2x + \ \ y = \ 4$

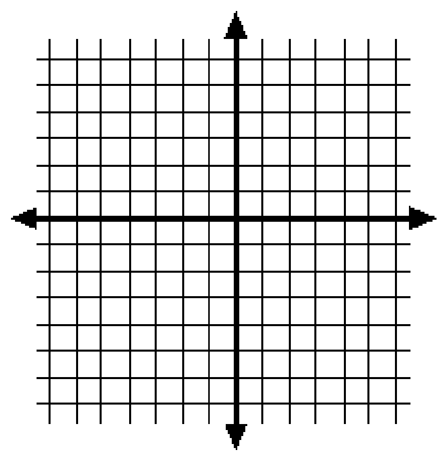
Solving Linear Systems by **Graphing**

- For a system with one solution, *the pair of* ***coordinates of the point of intersection of the lines is the system's solution***.

Example 2: Solve by graphing:

$x + 2y = 2$

$x - 2y = 6.$



Solving Linear Systems by the **Substitution** Method

- This method involves converting the system to one equation in one variable by an appropriate substitution.

Example 3: Solve by the substitution method:

$y = -x - 1$

$4x - 3y = 24$

Solving Linear Systems by the Addition Method ____________________

Example 4: Solve by the addition (elimination) method:

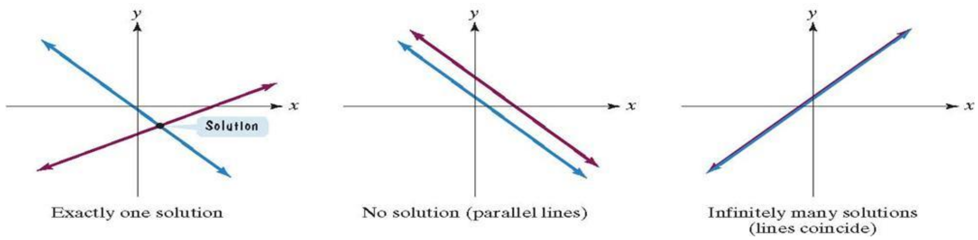
$3x + 2y = 48$

$9x - 8y = -24.$

Linear Systems Having No Solution or Infinitely Many Solutions

The number of solutions to a system of two linear equations in two variables is given by one of the following:

Number of Solutions	What This Means Graphically
Exactly one ordered-pair solution	The two lines intersect at one point.
No Solution	The two lines are parallel, i.e., have the same slope.
Infinitely many solutions	The two lines are identical.



Example 5: Solve the system:

$4x + 6y = 12$

$6x + 9y = 12.$

Example 6: Solve the system:

$y = 3x - 2$

$15x - 5y = 10.$

Modeling with Systems of Equations: Making Money

Revenue and Cost Functions

A company produces and sells *x* units of a product.

- Revenue Function: $R(x)$ = (price per unit sold)x
- Cost Function: $C(x)$ = fixed cost + (cost per unit produced)x

The point of intersection of the graphs of the revenue and cost functions is called the ___________________

Modeling with Systems of Equations: Making Money

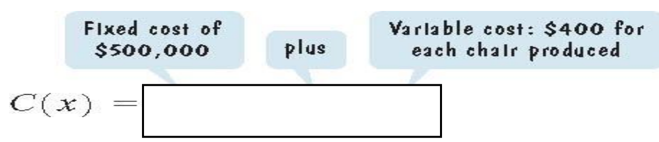
Finding a Break-even Point

Example 7: A company is planning to manufacture radically different wheelchairs. Fixed cost will be \$500,000 and it will cost \$400 to produce each wheelchair. Each wheelchair will be sold for \$600.

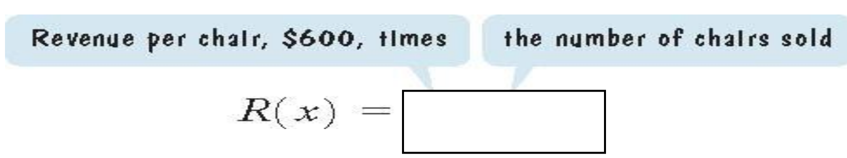
a. Write the cost function, C, of producing x wheelchairs.
b. Write the revenue function, R, from the sale of x wheelchairs.
c. Determine the break-even point. Describe what this means.

Solution:

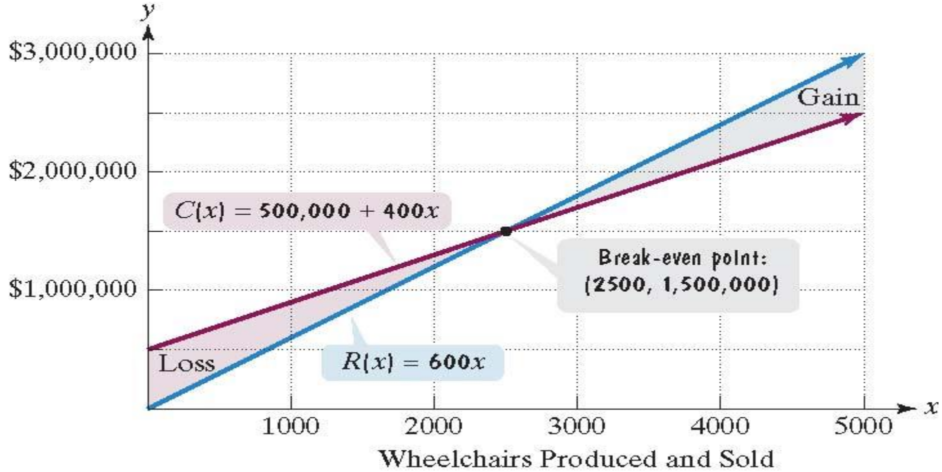
- The cost function is the sum of the fixed cost and the variable cost.



- The revenue function is the money generated from the sale of x wheelchairs.



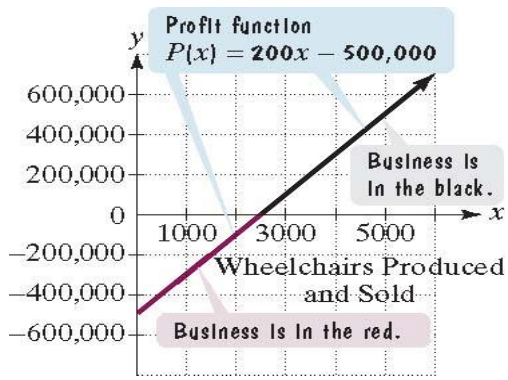
The break-even point is (____________________). This means that the company will break even if it produces and sells __________ wheelchairs for $__________________________.



- The profit, $P(x)$, generated after producing and selling x units of a product is given by the **profit function**

$$P(x) = R(x) - C(x),$$

where R and C are the revenue and cost, respectively.



Example: The profit function, $P(x)$, for the previous example is

$P(x) = R(x) - C(x)$

=

Section 10.5: Linear Inequalities in Two Variables

Objectives

1. Graph a linear inequality in two variables.
2. Use mathematical models involving linear inequalities.
3. Graph a system of linear inequalities.

Linear Inequalities in Two Variables and Their Solutions

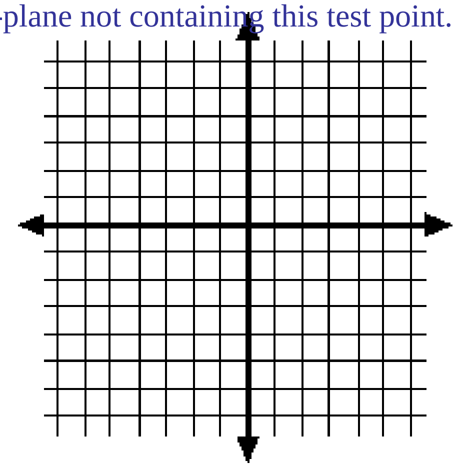
- The ***graph of an inequality in two variables*** is the set of all _________whose coordinates satisfy the inequality.

Graphing a linear inequality in two variables:

1. Replace the inequality with an equal sign and graph the linear equation.
   - Draw a _______line if the original inequality has a $\leq$ or $\geq$.
   - Draw a _________line if the original inequality has a $<$ or $>$.
2. Choose a test point in one of the half-planes that is not on the line. Substitute the coordinates of the test point into the inequality.
3. If a true statement results, shade the half-plane containing this test point.

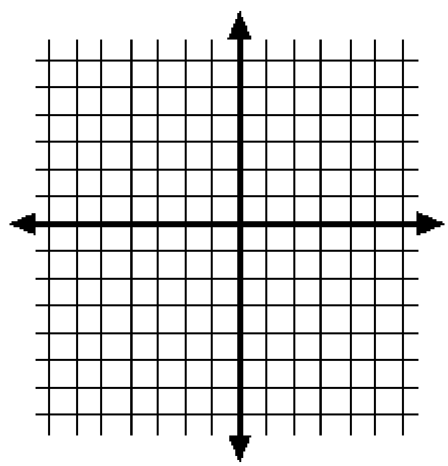
   If a false statement results, shade the half-plane not containing this test point.

Example 1: Graph: $3x - 5y \geq 15$.



The Graph of a Linear Inequality in Two Variables

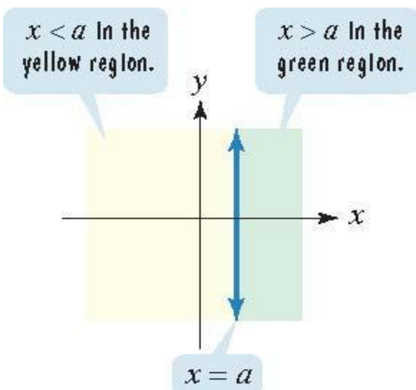
Example 2: Graph: $y > -\frac{2}{3}x$.

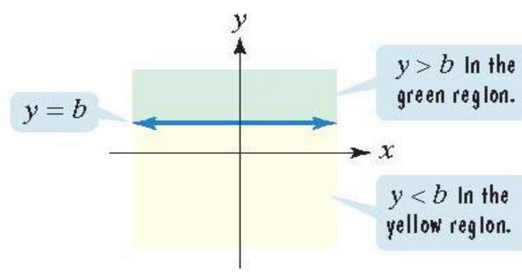


Graphing Linear Inequalities without Using Test Points

For the vertical line $x = a$:
- If $x > a$, shade the half-plane to the right of $x = a$.
- If $x < a$, shade the half-plane to the left of $x = a$.

For the horizontal line $y = b$:
- If $y > b$, shade the half-plane above $y = b$.
- If $y < b$, shade the half-plane below $y = b$.



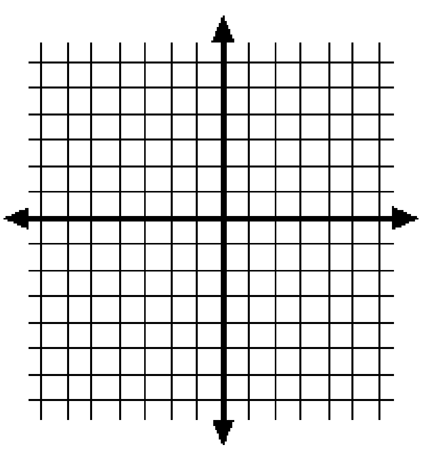


Graphing Linear Inequalities without Using Test Points

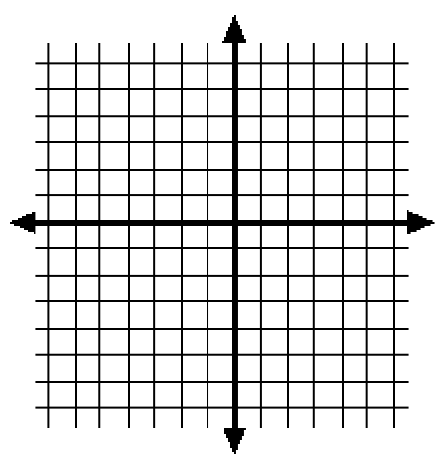
Example 3:

Graph each inequality in a rectangular coordinate system:

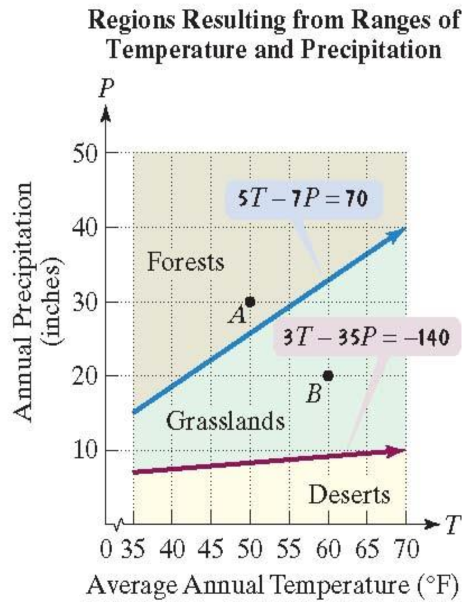
a. $y \leq -3$



b. $x > 2$



Modeling with Systems of Linear Inequalities



Example 4: The graph displays three kinds of regions- deserts, grasslands, and forests- temperatures, $T \geq 35$, and precipitation, P. Show that point A is a solution of the system of inequalities that describes where forests occur.

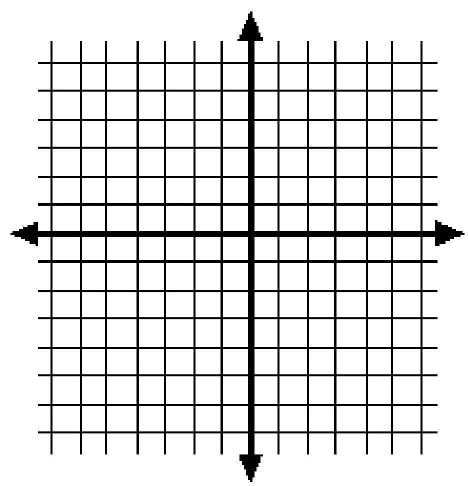
A forest occurs if: $T \geq 35$ and $5T - 7P < 70$.

- Graphing Systems of Linear Inequalities: The **solution set of a system of linear inequalities in two variables** is the set of all __________________ that satisfy ___________ inequalities in the system.

Example 5: Graph the solution set of the system:

$x - y < 1$

$2x + 3y \geq 12$.



Section 10.6: Linear Programming

Objectives

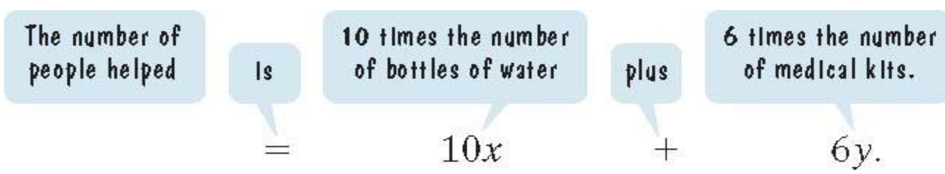
1. Write an objective function describing a quantity that must be maximized or minimized.
2. Use inequalities to describe limitations in a situation.
3. Use linear programming to solve problems.

Objective Functions in Linear Programming

- A method for solving problems in which a particular quantity that must be _____________ or __________________is limited by other factors is called ***linear programming***.
- An ________________________ is an algebraic expression in two or more variables describing a quantity that must be maximized or minimized.

Example 1: Bottled water and medical supplies are to be shipped to victims of an earthquake by plane. Each container of bottled water will serve 10 people and each medical kit will aid 6 people. Let x represent the number of bottles of water to be shipped and y the number of medical kits. Write the objective function that describes the number of people that can be helped.

Solution: Because each water serves 10 people and each medical kit aids 6 people, we have



Using z to represent the number of people helped, the objective function is

$$\boxed{z = 10x + 6y.}$$

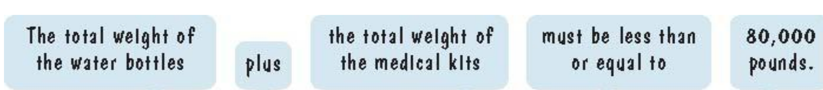
For a value of x and a value for y, there is only one value of z. Thus, z is a function of x and y.

Constraints on Linear Programming

- A constraint is expressed as an _________________________.
- The list of constraints forms a system of linear inequalities.

Example 2: Each plane can carry no more than 80,000 pounds. The bottled water weighs 20 pounds per container and each medical kit weighs 10 pounds. Let x represent the number of bottles of water to be shipped and y the number of medical kits. Write an inequality that describes this constraint.

Solution: Because each plane carries no more than 80,000 pounds, we have

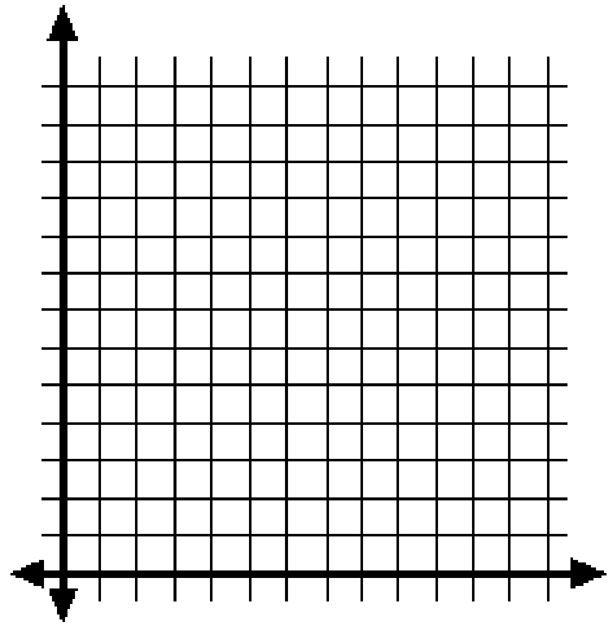


Solving Problems with Linear Programming

Let $z = ax + by$ be an objective function that depends on x and y. Furthermore, z is subject to a number of constraints on x and y. If a maximum or minimum value exists, it can be determined as follows:

1. Graph the system of inequalities representing the constraints.
2. Find the value of the objective function at each corner, or ***vertex***, of the graphed region. The maximum and minimum of the objective function occur at one or more of the corner points.

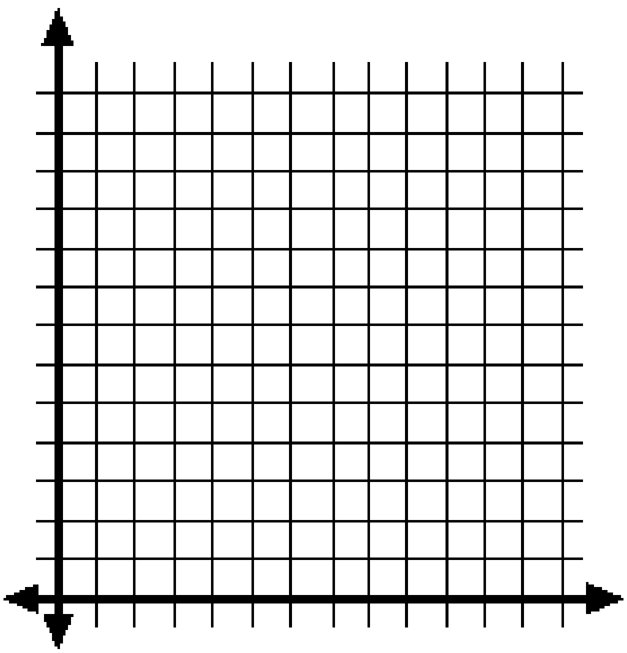
The Japanese Club at Damonte Ranch specializes in creating origami (paper making) for their fundraiser each school year. They produce paper cranes and paper flowers. Each paper crane sold yields 50 cents and each paper flower sold yields 75 cents. They can produce up to 80 pieces of origami per week. Based on the popularity of paper cranes, they must produce at least 20 per week, but no more than 50 pieces. They know that the minimum number of paper flowers sold per week is 10 pieces. How many of each type of origami should they produce to maximize their profits?



Let $z = ax + by$ be an objective function that depends on x and y. Furthermore, z is subject to a number of constraints on x and y. If a maximum or minimum value exists, it can be determined as follows:

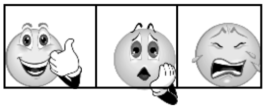
1. Graph the system of inequalities representing the constraints.
2. Find the value of the objective function at each corner, or ***vertex***, of the graphed region. The maximum and minimum of the objective function occur at one or more of the corner points.

At an electronics booth, Allan plans to sell televisions at $225 each and Streaming Devices at $75 each. Due to space limitations, he can only store at most 300 items for the day, However, because more people already own televisions, Allan knows that the number of Streaming Devices sales will at least double the number of television sales. How many of each item should Bruce bring to the flea market to maximize his sales?



Prob/Stat/Discrete **Name____**

Unit 10 Objectives

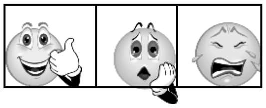


Objective #1: Can you use linear equations to solve problems?

a) A car rental agency charges \$150 per week plus \$0.20 per mile to rent a car. How many miles can you travel in one week for \$190?

b) Basketball, bicycle riding, and football are the three sports and recreational activities in the United States with the greatest number of medically treated injuries. In 2004, the number of injuries from basketball exceeded those from football by 0.6 million. The number of injuries from bicycling exceeded those from football by 0.3 million. Combined, basketball, bicycling, and football accounted for 3.9 million injuries. Determine the number of medically treated injuries from each of these recreational activities in 2004.

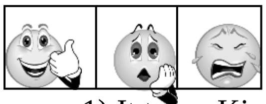
c) In a film, the actor Charles Coburn plays an elderly "uncle" character criticized for marrying a woman when he is 3 times her age. He wittily replies, "Ah, but in 20 years time I shall only be twice her age." How old are the "uncle" and the woman?



Objective #2: Can you solve proportions?

1) $\frac{10}{15} = \frac{x}{39}$

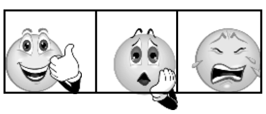
2) $\frac{12}{x} = \frac{17}{34}$



Objective #3: Solve problems using proportions?

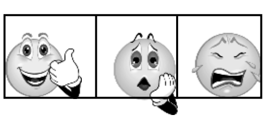
1) It takes Kim 26 minutes to type and spell check 14 pages of a manuscript. Find how long it takes her to type and spell check 63 pages. Round answers to the nearest whole number if necessary

.



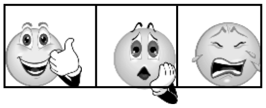
Objective #4: Can you solve direct variation problems?

The perimeter of a square varies directly as the length of one of its sides. If the perimeter is 132 feet for a square with a 33-foot side, what is the perimeter for a square with a 97-foot side?



Objective #5: Can you solve inverse variation problems?

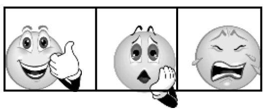
When the temperature stays the same, the volume of a gas varies inversely as the pressure of the gas. If a balloon is filled with 102 cubic inches of a gas at a pressure of 14 pounds per square inch, find the new pressure of the gas if the volume is decreased to 51 cubic inches.



Objective #6: Can you graph subsets of real numbers on a number line?

a) $\{x : x \le 4\}$

b) $\{x : -5 \le x < 2\}$



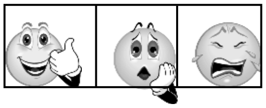
Objective #7: Can you solve and graph linear inequalities?

a) $\frac{x}{-6} \le 5$

b) 7x-3 > 13x +33

c) $2(\mathrm{x}-3)+5\mathrm{x} \le 8(x-1)$

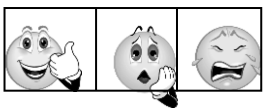
d) $-11 < 2x - 1 \le -5$



Objective #8: Solve applied problems using linear inequalities?

a) Mary has been put in charge of buying soft drinks and chips for a party. If the soft drinks total \$25 and chips are \$3.29 per bag, how may bags of chips can Mary buy is she wants to spend at most \$100?

b) A company manufactures and sells personalized stationery. The weekly fixed cost is \$3000 and it cost \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to generate a profit?

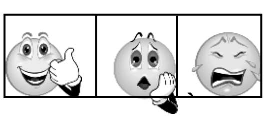


Objective #9: Can you decide whether an ordered pair is a solution of a linear system?

For Exercises 1 and 2, determine whether the given ordered pair is a solution to the system.

1. (6,2)
2x + y = 14
3x + 2y = 22

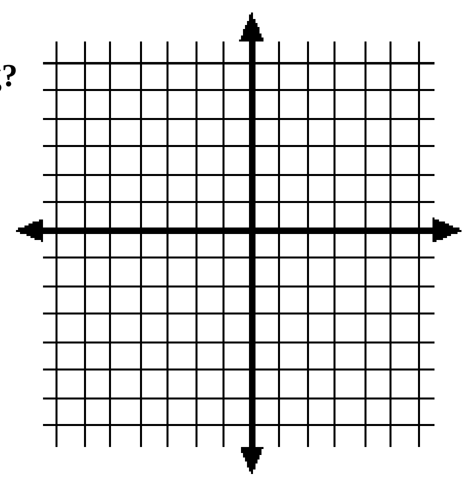
2.(3,3)
2x + y = 3
4x + 2y =6

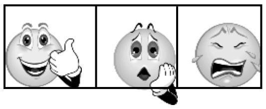


Objective #10: Can you solve linear systems by graphing?

Solve the system by graphing. Check the coordinates of the intersection point in both equations.

y = x +1
y= 5x - 3

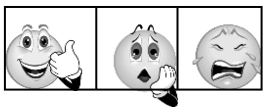




Objective #11: Can you solve linear systems by substitution?

4. Solve the system by the substitution method.

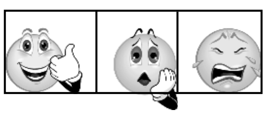
$5x + 6y = 15$
$y = -4x - 7$



Objective #12: Can you solve linear systems by addition (elimination)?

5. Solve the system by the addition method.

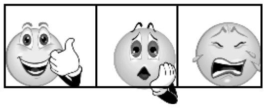
$x + y = 11$
$2x – y = 13$



Objective #13: Can you identify systems that do not have exactly one ordered-pair solution?

8. Solve by the method of your choice.
$4x + 2y = 3$
$-2x – y = 1$

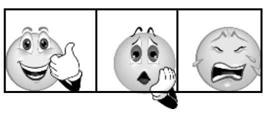
9. Solve by the meathod of your choice.
$2x + y = 7$
$8x + 4y = 28$



Objective #14: Can you solve problems using systems of linear equations?

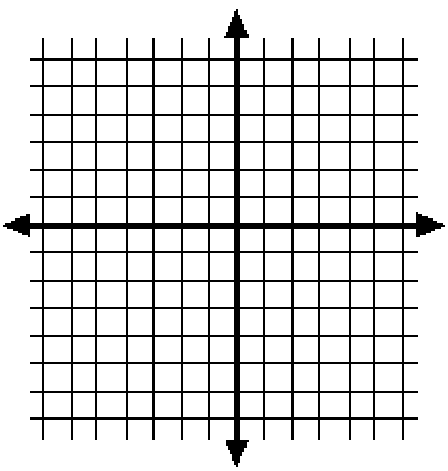
For the following, write a) the cost function, C; b) the revenue function, R; and c) determine the break-even point. Describe what this means.

A company that manufactures small canoes has a fixed cost of $18,000. It costs $20 to produce each canoe. The selling price is $80 per canoe. (In solving this exercise, let *x* represent the number of canoes produced and sold.

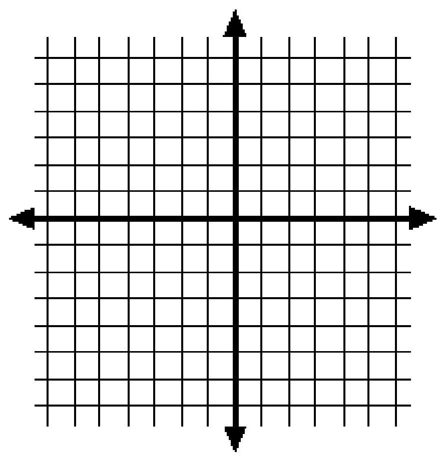


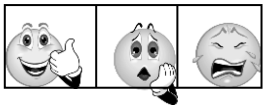
Objective #15: Can you graph a linear inequality in two variables?

1. $x + y \le 7$



3. $x > 3$

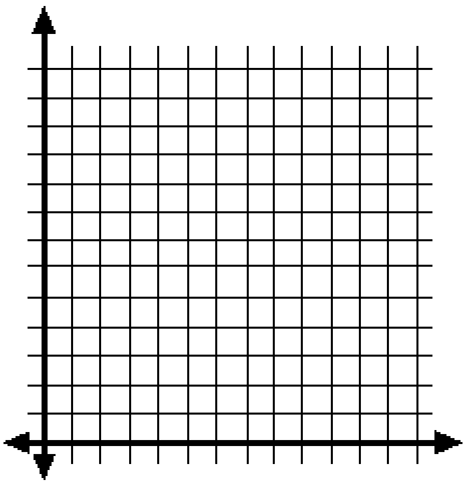




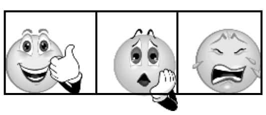
Objective #16: Can you use mathematical models involving linear inequalities?

a) Many elevators have a capacity of 2000 pounds. If a child average 50 pounds and adult 150 pounds, write an inequality that describes when *x* children and *y* adults will cause the elevator to be overloaded.

b) Graph the inequality. Because *x* and *y* must be positive, limit the graph to quadrant I only.

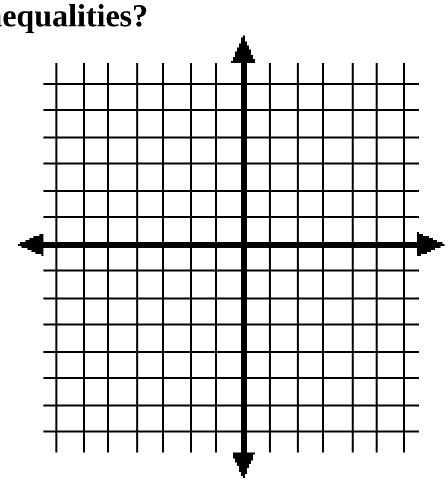


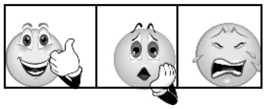
c) Select an ordered pair satisfying the inequality. What are its coordinates and what do they represent in this situation?



Objective #17: Can you graph a system of linear inequalities?

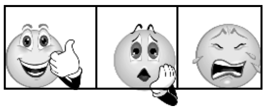
$x + 3y > -9$
$y \le 4$





Objective #18: Can you use inequalities to describe limitations in a situation?

Why are a majority of the linear programming problems limited to the first quadrant?



Objective #19: Can you linear programming to solve problems?

4. At a flea market, Bruce plans to sell televisions at $125 each and DVD players at $100 each. Due to space limitations, he can only store at most 150 items for the day, However, because more people already own television, Bruce knows that the number of DVD sales much at least match the number of television sales. How many of each item should Bruce bring to the flea market to maximize his sales?