Assignments for Prob/Stat/Discrete Math Unit 2: Counting Methods and Probability Theory (CH 11)

(B) Means blue 123 A and B days

(G) means Green 456 A and B days

Day	Date	Assignment (Due the next class meeting)
Friday	9/9-10/20 (B)	2.1 Worksheet
Tuesday	9/11/20 (G)	2.2 Worksheet
Wednesday	9/15-16/20 (B)	2.3 Worksheet
Thursday	9/14/20 (G)	2.4 Worksheet
Friday	9/21/20 (B)	2.5 Worksheet
Monday	9/17/20(G)	
Tuesday	9/22/20 (B)	2.6 Worksheet
Wednesday	9/18/20 (G)	
Thursday	9/25/20 (B)	2.7 Worksheet
Friday	9/23/20 (G)	
Monday	9/28/20 (B)	2.8 Worksheet
Tuesday	9/24/20 (G)	
Wednesday	10/01-2/20 (B)	Unit 2 (CH 11) Presetion Test
Thursday	9/29/20 (G)	Unit 2 (CH 11) Practice Test
Friday	10/1-2/20 (B)	Unit 2 (CH 11) Test
Monday	9/30/20 (G)	

NOTE: You should be prepared for daily quizzes.

HW reminders:

- > If you cannot solve a problem, get help **before** the assignment is due.
- > Help is available before school, during lunch, or during IC, or online at mathguy.us
- For extra practice, visit <u>www.interactmath.com</u>

11.1 and 11.2: The Fundamental Counting Principle and Permutations

Objectives

- 1. Use the Fundamental Counting Principle to determine the number of possible outcomes in a given situation.
- 2. Use the Fundamental Counting Principle to count permutations.
- 3. Evaluate Factorial Expressions
- 4. Find the number of permutations of duplicate items.

The Fundamental Counting Principle

If you can choose ______ item from a group of M items and a ______ item from a group of N items, then the total number of two-item choices is $M \cdot N$.

Example 1: The Greasy Spoon Restaurant offers 6 appetizers and 14 main courses. In how many ways can a person choose a two-course meal?

Example 2: This is the semester that you will take your required psychology and social science courses. Because you decide to register early, there are 15 sections of psychology from which you can choose, and 9 sections of social science. In how many ways can you create a two-course schedule taking both psychology and social science?

The Fundamental Counting Principle

The number of ways in which a series of successive things can occur is found by _____

the number of ways in which each thing can occur.

Example 3: Next semester you are planning to take 3 courses-math, English, and science. There are 8 sections of math, 5 of English, and 4 science. Assuming there are no schedule conflicts how many different 3-course schedules are possible?

Example 4: Car manufacturers are now experimenting with lightweight 3-wheel cars, designed for one person, and considered to be ideal for city driving. Suppose you could order such a car with a choice of 9 colors, with or without A/C, electric or gas powered, and with or without navigation. In how many ways can one of these cars be ordered?

Example 5: You are taking a multiple choice test that has 10 questions. Each of the questions has 4 answer choices with one correct answer per question. If you make a choice for each question, in how many ways can you answer the questions?

Example 6: Telephone numbers in the US begin with a 3-digit area code and are followed by 7-digit local telephone numbers. Area codes and local phone numbers cannot begin with 0 or 1. How many different telephone numbers are possible?

A ______ is an ordered arrangement of items that occurs when:

No item is used more than once

The order of arrangement makes a difference. (order matters)

Example 7: You need to arrange 7 of your favorite books on a shelf. How many different ways can you arrange the books assuming the order of the books matters to you. (For example longest to shortest or newest to oldest)

Factorial Notation

If *n* is a positive integer, the notation *n*! (read "____") is the product of all positive integers from *n* down through 1.

Example 8: Evaluate the following factorial expressions.

a. $\frac{8!}{5!}$ b. $\frac{26!}{21!}$ c. $\frac{500!}{499!}$

Permutations of *n* things taken *r* at a time

The number of possible permutations if r items are taken from n items is:

$$_{n}P_{r}=\frac{n!}{(n-r)!}$$

Example 9: You and 19 of your friends have decided to form an internet marketing consulting firm. The group needs to choose three offices-a CEO, an operating manager, and a treasurer. In how many ways can those offices be filled?

Permutations of duplicate items:

The number of permutations of n items, where p items are identical, q items are identical, r items are identical, and so on is given by: $\frac{n!}{p!q!r!...}$

Example 10: In how many distinct ways can the letters of the word MISSISSIPPI be arranged?

<u>11.3 and 11.4: Combinations and Fundamentals of Probability</u>

Objectives

- 5. Distinguish between permutation and combination problems.
- 6. Solve problems involving combinations using the combinations formula.
- 7. Compute Theoretical Probability.
- 8. Compute Empirical Probability.

A _____ of items occurs when:

The items are selected from the same group.

No item is used more than once.

The order of the items makes no difference. (order doesn't matter)

Example 1: For each of the following problems, determine whether the problem is involving permutations or combinations. (You do not have to solve.)

- A. Six students are running for student government president, vice-president, and treasurer. The student with the highest amount of votes will be president and so on. How many different outcomes are possible to fill the 3 positions?
- B. Six people are on the board of supervisors for your neighborhood park. A 3-person committee is needed to study the possibility of expanding the park. How many different 3-person committees could be formed from the 6 people?
- C. Baskin-Robbins offers 31 different flavors of ice cream. One of their items is a bowl consisting of 3 scoops of ice cream, each a different flavor. How many such bowls are possible?

Combinations of *n* things taken *r* at a time

The number of possible combinations if r items are taken from n items is:

$${}_{n}C_{r}=\frac{n!}{(n-r)!\,r!}$$

Example 2: A 3-person committee is needed to study ways of improving public transportation. How many committees could be formed from the 8 people on the board of supervisors?

Example 3: In poker, a person is dealt 5 cards from a standard 52-card deck. How many different 5-card poker hands are possible?

Example 4: The U.S. Senate of the 104th Congress consisted of 54 Republicans and 46 Democrats. How many committees can be formed if each committee must have 3 Republicans and 2 Democrats?

The ______ probability of an event is the number of ways that the event can occur, divided by the total number of outcomes. It is finding the probability of events that come from a sample space of known equally likely outcomes.

Computing Theoretical Probability

If an event *E* has n(E) equally likely outcomes and its sample space *S* has n(S) equally likely outcomes, the

of an event E, denoted by P(E), is:

 $P(E) = \frac{number of outcomes in event E}{total number of possible outcomes} = \frac{n(E)}{n(S)}$

Example 5: A die is rolled once. Find the probability of rolling:

a. a 3 b. an even number c. a number less than 5

d. a number less than 10

e. a number greater than 6

The sum of the ______ of all possible ______ in the sample space is 1.

Example 6: You are dealt one card from a standard 52-card deck. Find the probability of being dealt:

a. a king

b. a heart

c. the king of hearts

Example 7: Each person carries two genes that are related to the absence or presence of the disease cystic fibrosis. Most Americans have two normal genes for this trait and are unaffected by cystic fibrosis. However, 1 in 25 Americans carries one normal gene and one defective gene. If we use *c* to represent a defective gene and *C* a normal gene, such a carrier can be designated as *Cc*. Thus, *CC* is a person who neither carries nor has cystic fibrosis, *Cc* is a carrier who is not actually sick, and *cc* is a person sick with the disease. The following table shows four equally likely outcomes for a child's genetic inheritance from two parents who are both carrying one cystic fibrosis gene. One copy of each gene is passed on to the child from the parents.

First parent	Second Parent	If eac
	C c	the p
С	CC Cc	
с	Cc cc	

ch parent carries one cystic fibrosis gene, what is probability that their child will have cystic fibrosis?

The ______ probability of an event is an "estimate" that the event will happen based on how often the event occurs after collecting data or running an experiment (in a large number of trials). It is based specifically on direct observations or experiences.

Computing Empirical Probability

The empirical probability of event E is:

 $P(E) = \frac{observed \ number \ of \ times \ E \ occurs}{total \ number \ of \ observed \ occurrences}$

							iny sciected
	Never Married	Married	Widowed	Divorced	Total	from the population de	escribed, find
						the probability, to the i	nearest
Male	28.6	62.1	2.7	9.0	102.4	tenth, that the person	
Female	23.3	62.8	11.3	12.7	110.1	a. is divorced	b. is female
Total	51.9	124.9	14.0	21.7	212.5		

Example 8: The following table shows the distribution, by marital status and gender, of the 212.5 million Americans ages 18 or older. If one person is randomly selected

11.5: Probability with the Fundamental Counting Principle, Permutations, and **Combinations**

Objectives

- 9. Compute Probabilities with permutations.
- 10. Computer Probabilities with combinations.

Probability and permutations

Example 1: There are 5 bands playing at the same show (The Rolling Stones, Beatles, U2, Bruce Springsteen, and Aerosmith), they agree to determine the order of performance based on a random selection. Each band's name is written on one of five cards. The cards are placed in a hat and then five cards are drawn, one at a time. The order in which the cards are drawn determines the order in which the bands perform. What is the probability that the Rolling Stones perform fourth and the Beatles last?

Begin with applying the definition of probability to this situation.

 $P(\text{Rolling Stones fourth and Beatles last}) = \frac{number of permutations with Rolling Stones fou}{Beatles last}$ total number of possible permutations

Use the Fundamental Counting Principle to find the total number of possible permutations. (# of ways you can put together the concert)

Probability and combinations: Winning the lottery

Example 2: Florida's lottery game, LOTTO, is set up so that each player chooses six different numbers from 1 to 53. If the six numbers chosen match the six numbers drawn randomly, the player wins (or shares) the top cash prize. With one LOTTO ticket what is the probability of winning this prize?

Because the order of the six numbers does not matter, this is a situation involving ______ We begin with the formula for probability. $P(winning) = \frac{number \ of \ ways \ of \ winning}{total \ number \ of \ possible \ combination}$

There is only one way of winning, matching all 6 numbers. To find the total number of possible combinations we use ${}_{53}C_6$

If a person buys 5,000 tickets (for \$1 each, and each being different numbers) what is the probability of winning?

Would you expect them to win the main prize?

Probability and combinations

Example 3: A club consists of five men and seven women. Three members are selected at random to attend a conference. Find the probability that the selected group consists of

a. three men

b. one man and two women

Does the order in which the three people are selected matter? Combination or Permutation?

a.
$$P(3 men) = \frac{number of ways of selecting 3 men}{total number of possible combinations}$$

b. $P(1 man, 2 women) = \frac{number of ways of selecting 1 man and 2 women}{total number of possible combinations}$

11.6: Events involving NOT and OR; ODDS

Objectives

- 11. Find the probability that an event will not occur.
- 12. Find the probability of one event or a second event occurring.
- 13. Understand and use odds.

Probability of an event not occurring

If we know P(E), we can determine the probability that the event will not occur, denoted by P(not E). The

event not *E* is the ______ of *E* because it is the set of all outcomes in the sample space

S that are not outcomes in the event E.

Complement Rules of Probability

The Probability that an event E will not occur is equal to 1 minus the probability that it will occur.

P(not E) = 1 - P(E)

The Probability that an event E will occur is equal to 1 minus the probability that it will not occur.

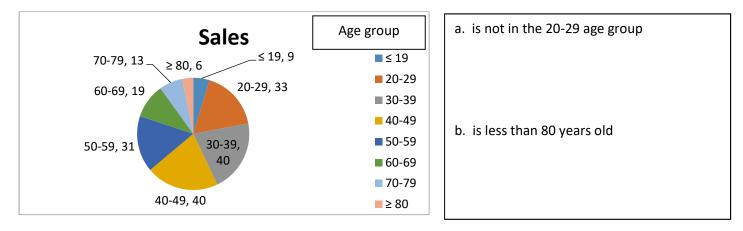
P(E) = 1 - P(not E)

Using set notation, if E' is the complement of E, then P(E') = 1 - P(E) and P(E) = 1 - P(E').

Example 1: If you are dealt one card from a standard 52-card deck, find the probability that you are not dealt a queen.

Example 2: Find the probability that you are not dealt a diamond.

Example 3: The circle graph below shows the distribution, by age group, of the 191 million car drivers in the US, with all numbers being rounded to the nearest million. If one driver is randomly selected from this population, find the probability that the person:



Or Probabilities with Mutually Exclusive Events

Mutually Exclusive Events

If it is impossible for events A and B to occur simultaneously, the events are said to be

In general, if *A* and *B* are mutually exclusive events, the probability that either *A* or *B* will occur is determined by adding their individual probabilities.

Or Probabilities with Mutually Exclusive Events

If A and B are mutually exclusive events, then P(A or B) = P(A) + P(B).

Using set notation, $P(A \cup B) = P(A) + P(B)$.

Example 4: If one card is selected from a deck of cards, what is the probability of selecting a king or a queen?

Or Probabilities with Events That Are Not Mutually Exclusive

If *A* and *B* are not mutually exclusive events, then P(A or B) = P(A) + P(B) - P(A and B). Using set notation, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Example 5: In a group of 25 baboons, 18 enjoy grooming their neighbors, 16 enjoy screeching wildly, while 10 enjoy grooming their neighbors and screeching wildly. If one baboon is selected at random from the group, find the probability that it enjoys grooming its neighbors or screeching wildly.

<u>Odds</u>

If we know the probability of an event E, we can also speak of the odds in favor, or the odds against, the event.

Probability to Odds

If P(E) is the probability of an event E occurring, then:

1. The ______ of *E* are found by taking the probability that *E* will occur

and dividing by the probability that ${\it E}$ will not occur.

Odds in favor of
$$E = \frac{P(E)}{P(not E)}$$

2. The ______ *E* are found by taking the probability that *E* will not occur and dividing by the probability that *E* will occur.

$$Odds \ against \ E = \frac{P(not \ E)}{P(E)}$$

The odds against E can also be found by reversing the ratio representing the odds in favor of E.

Example 6: You roll a single six-sided die.

- a. Find the odds in favor of rolling a 2.
- b. Find the odds against rolling a 2.

Example 7: The winner of a raffle will receive a new sports utility vehicle. If 500 raffle tickets were sold and you purchased 10 tickets, what are the odds against your winning the car?

P(not winning the car) =

P(winning the car) =

Odds against you winning:

Odds to Probability

If the odds in favor of event *E* are *a* to *b*, then the probability of the event is given by $P(E) = \frac{a}{a+b}$.

Example 8: The odds in favor of a particular horse winning a race are 2 to 5. What is the probability that this horse will win the race?

11.7: Events involving AND; conditional probability

Objectives

- 14. Find the probability of one event and a second event occurring.
- 15. Compute Conditional Probabilities

And Probabilities with Independent Events

AND Probabilities with Independent Events

If *A* and *B* are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Example 1: A U.S. Roulette wheel has 38 numbered slots (1-36, 0, 00). Of the 38 compartments, 18 are black and 18 are red, and 2 are green. A play has the dealer spin the wheel and a small ball in opposite directions. As the ball slows to a stop, it can land with equal probability on any one of the 38 numbered slots. Find the probability of red occurring on 2 consecutive plays.

Example 2: If the probability of a child being born a female is $\frac{1}{2}$, find the probability of a family having 9 girls in a row.

Example 3: If the probability that South Florida will be hit by a hurricane is $\frac{5}{19}$.

a. What is the probability that South Florida will be hit by a hurricane in three consecutive years?

b. What is the probability that South Florida will not be hit by a hurricane in the next ten years?

The probability of an event happening at least once

P(event happening at least once) = 1 - P(event does not happen)

c. What is the probability that South Florida will be hit by a hurricane at least once in the next 4 years?

And Probabilities with Dependent Events

Two events are	 if the occurrence of one of them has an effect on
the other.	

And Probabilities with Dependent Events

If *A* and *B* are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \text{ given that } A \text{ has occurred})$.

Example 4: Good news: You won a free trip to Spain and can take two people with you, all expenses paid. Bad News: Ten of your cousins have appeared out of nowhere and are begging you to take them. You write each cousin's name on a card, place the cards in a hat, and select one name. Then you select a second name without replacing the first card. If three of your ten cousins speak Spanish, find the probability of selecting two Spanish-speaking cousins.

Example 5: Three people are randomly selected, one person at a time, from five freshmen, two sophomores, and four juniors. Find the probability that the first two people selected are freshmen and the third is a junior.

The probability of event *B*, assuming that event *A* has already occurred, is called the _____

_____ of *B*, given *A*. This probability is denoted by P(B|A).

It is helpful to think of the conditional probability P(B|A) as the probability that event *B* occurs if the sample space is restricted to the outcomes associated with event *A*.

Example 6: A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes the letter h.

11.8: Expected Value

Objectives

- 16. Compute Expected Value
- 17. Use expected value to solve applied problems.
- 18. Use expected value to determine the average payoff or loss in a game of chance.

An ______ is a mathematical way to use probabilities to determine what to expect in various situations over the long run. The standard way to find expected value is to multiply each possible outcome by its probability, and then add these products.

Example 1: Find the expected value for the outcome of the roll of a fair die.

Example 2: Find the expected value for the number of girls for a family with three children.

Outcome # of girls	Probability
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

Amount of Claim	Probability
(to the nearest \$2,00)	
\$0	0.70
\$2 ,000	0.15
\$2,000	0.15
\$4,000	0.08
\$1,000	0.00
\$6,000	0.05
\$8,000	0.01
\$10,000	0.01

Example 3: An automobile insurance company has determined the probabilities for various claim amounts for drivers ages 16-21.

a. Calculate the expected value and describe what this means in practical terms.

b. How much should the company charge as an average premium so that it does not lose or gain money on its claim costs.

Example 4: You are a realtor considering listing a \$500,000 house. The cost of advertising and providing food for other realtors during open showings is anticipated to cost you \$5,000. The house is quite unusual, and you are given a fourmonth listing. Use the information given in the table to help decide whether or not to list the house if you only decide to list the house if you expect to make at least \$6,000.

My Cost:	\$5,000			
My Possible Income:				
I sell house:	\$30,000			
Another Agent Sells House:	\$15,000			
House does not sell after 4 months:	\$0			
The Probabilities:				
I sell house:	0.3			
Another Agent Sells House:	0.2			
House does not sell after 4 months:	0.5			
My Bottom Line:				
I take the listing only if I anticipate earning at				
least \$6,000.				

Example 5: A game is played using one die. If the die is rolled and shows 1, 2 or 3, the player wins nothing. If the die shows 4 or 5, the player wins \$3. If the die shows 6, the player wins \$9. If there is a charge of \$1 to play the game, what is the game's expected value? Describe what this means in context.

Example 6: One way to bet in roulette is to place \$1 on a single number. If the ball lands on that number, you are awarded \$35 and get to keep the \$1 that you paid to play the game. If the ball lands on any one of the other 37 slots, you are awarded with nothing and you lose the \$1 you paid to play the game. Find the expected value for playing roulette if you bet \$1 on number 20. Describe what this means in context.

Discrete Math Unit 2 Objectives Name_____



Objective #1: Can you use the Fundamental Counting Principle to determine the number of possible outcomes in a given situation?

A) A restaurant offers 8 entrees and 7 desserts. In how many ways can a person order a two-course meal?

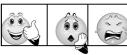
B) A restaurant offers 12 entrees and 6 desserts. In how many ways can a person order a two-course meal?

C) A restaurant offers 4 appetizers, 9 entrees, and 5 desserts. In how many ways can a person order a three-course meal?

A) In how many ways can a girl choose a two-piece outfit from 5 blouses and 8 skirts?

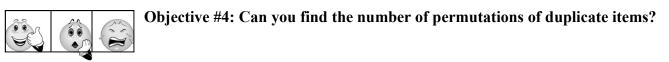
B) In how many ways can a girl choose a two-piece outfit from 4 blouses and 6 skirts?

C) In how many ways can a girl choose a three-piece outfit from 8 blouses and 5 skirts and 6 pairs of shoes?



Objective #3: Can you evaluate factorial expressions?

- A) You need to arrange 5 of your favorite books on a shelf. How many different ways can you arrange the books assuming the order of the books matters to you. (For example longest to shortest or newest to oldest)
- B) You need to arrange 9 of your favorite movies on a shelf. How many different ways can you arrange the books assuming the order of the books matters to you.



- A) In how many ways can the letters of the word **OSMOSIS** be arranged?
- B) In how many ways can the letters of the word **TALLAHASSEE** be arranged?



Objective #5: Can you distinguish between permutation and combination problems?

Does the problem involve permutations or combinations? Explain (you don't have to solve)

- A) A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?
- B) How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?

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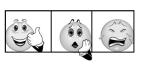
Objective #6: Can you solve problems involving combinations using the combinations formula?

Solve by using the combinations formula?

A)
$${}_{6}C_{5}$$
 B) ${}_{9}C_{5}$

C)
$$_{30}C_3$$
 D) $\frac{_{7}C_3}{_{5}C_4}$

E) An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?



Objective #7: Can you compute theoretical probability?

A die is rolled. The se of equally likely outcomes is $\{1, 2, 3, 4, 5, 6\}$, find the probability of rolling:

A) a 4

B) an odd number

C) a number less than 3

D) a number greater than 7

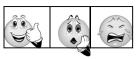


Objective #8: Can you compute empirical probability?

		Gender/Age	15-24	25-34	35-44	45-64	65-74	≥75	Total
	Use the information given in the table	Male	0.7	2.2	2.6	4.3	1.3	1.4	12.5
	to find the following probabilities.	Female	0.8	1.6	1.6	5.0	2.9	4.9	16.8
		Total	1.5	3.8	4.2	9.3	4.2	6.3	29.3

A) in the 25-34 age range

B) a woman in the 15-24 age range



Objective #9: Can you compute probabilities with permutations?

Martha, Lee, Nancy, Paul, and Armando have all been invited to a dinner party. They arrive randomly and each person arrives at a different time.

- A) In how many ways can they arrive?
- B) In how many ways can Martha arrive first and Armando last?
- C) Find the probability that Martha will arrive first and Armando last.

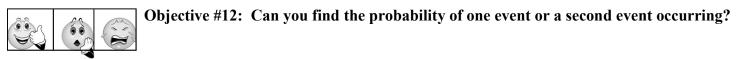
Objective #10: Can you compute probabilities with combinations?

Given a standard 52-card deck, find the following, if a poker hand consists of five cards:

- A) The total number of possible five-card poker hands.
- B) A diamond flush (all 5 cards are diamonds)
- C) The probability of being dealt a diamond flush.

Given a standard 52-card deck of cards, find the probability that you are not dealt:

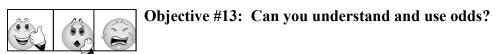
A) An ace B) a heart C) a face card (Jacks, Queens, Kings)



You are dealt one card from a standard 52-card deck of cards, find the probability that you are dealt:

A) A 7 or a red card

B) a heart or a face card



One card is randomly selected from a deck of cards. Find the odds:

A) in favor of drawing a heart B) in favor of drawing a red card

C) against drawing a 9

D) against drawing a face card



Objective #14: Can you find the probability of one event and a second event occurring?

A single die is rolled twice, find the probability of rolling:

A) a 2 and then a 3 B) an even number and then a number greater than 2



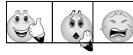
Objective #15: Can you compute conditional probabilities?

The following table gives information on car accidents in Florida.

	Wore Seat Belt	No Seat Belt	Total
Driver Survived	412,368	162,527	574,895
Driver Died	510	1601	2,111
Total	412,878	164,128	577,006

A) Find the probability of surviving a car accident, given that the driver wore a seat belt.

B) Find the probability of wearing a seat belt, given that a driver survived a car accident.



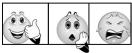
Objective #16: Can you compute expected value?

A game is played using a fair coin that is tossed twice. The sample space is {HH, HT, TH, TT}. If two heads occur, the player receives \$15, and if exactly one head occurs, the player receives \$10. If no heads occur (both tails) then the player receives nothing. There is a 10 dollar charge to play the game. What is the expected value and what does it mean in the context of this problem?



Objective #17: Can you use expected value to solve applied problems?

A construction company is planning to bid on a building contract. The bid costs the company \$3,000. The probability that the bid is accepted is $\frac{1}{4}$. If the bid is accepted the company will make \$30,000 minus the cost of the bid. Find the expected value of the situation.



Objective #18: Can you use expected value to determine the average payoff or loss in a game of chance?

The table below shows the medical insurance claims and their probabilities for an insurance company. For example the company can expect to pay 6% of policyholders a claim amount of \$20,000. The company can also expect that 76% of policyholders will not put in a claim.

Probabilities for Medical Insurance Claims						
Amount of the claim Probability Probabili						
(to the nearest \$10,000)	(as a percent)	(as a decimal)				
\$0	76%	0.76				
\$10,000	15%	0.15				
\$20,000	6%	0.06				
\$30,000	2%	0.02				
\$40,000	1%	0.01				

A) Calculate the expected value.

B) How much should the company charge per policy so that it breaks even on its claim costs?

C) How much should the company charge to make a profit of \$300 per policy?