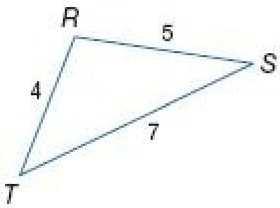
List the angles and sides of each triangle in order from smallest to largest.

15. 

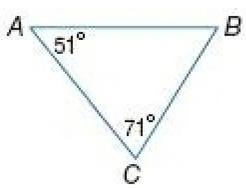
SOLUTION:

Based on the diagram, we see that $RT < RS < TS$. By Theorem 5.9, the measure of the angle opposite the longer side has a greater measure than the angle opposite the shorter side, therefore $m\angle S < m\angle T < m\angle R$.

Angle: $\angle S, \angle T, \angle R$
Side: $\overline{RT}, \overline{RS}, \overline{ST}$

ANSWER:

$\angle S, \angle T, \angle R$; $\overline{RT}, \overline{RS}, \overline{ST}$

18. 

SOLUTION:

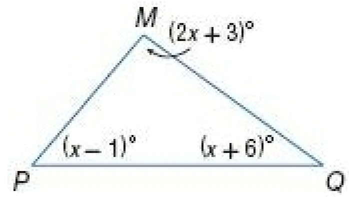
By the Triangle Angle-Sum Theorem, $m\angle B = 180 - (51 + 71) = 58$.
So, $m\angle A < m\angle B < m\angle C$. Therefore, by Theorem 5.10 we know that the side opposite the greater angle is longer than the side opposite a lesser angle and $BC < AC < AB$.

Angle: $\angle A, \angle B, \angle C$
Side: $\overline{BC}, \overline{AC}, \overline{AB}$

ANSWER:

$\angle A, \angle B, \angle C$; $\overline{BC}, \overline{AC}, \overline{AB}$

List the angles and sides of each triangle in order from smallest to largest.

23. 

SOLUTION:

Using the Triangle Angle-Sum Theorem, we can solve for x, as shown below.

$$\begin{aligned}(2x+3)+(x-1)+(x+6)&=180\\4x+8&=180\\4x&=172\\x&=43\end{aligned}$$

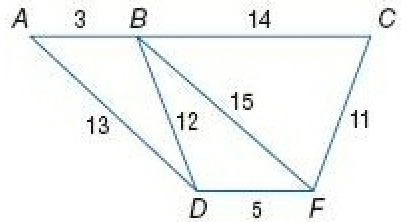
$m\angle M = 2(43) + 3 = 89$ degrees, $m\angle P = (43) - 1 = 42$ degrees and the $m\angle Q = 43 + 6 = 49$ degrees. Therefore, $m\angle P < m\angle Q < m\angle M$. By Theorem 5.10, we know that the lengths of sides across from larger angles are longer than those across from shorter angles so $MQ < PM < PQ$.

Angle: $\angle P, \angle Q, \angle M$
Side: $\overline{MQ}, \overline{PM}, \overline{PQ}$

ANSWER:

$\angle P, \angle Q, \angle M$; $\overline{MQ}, \overline{PM}, \overline{PQ}$

SENSE-MAKING Use the figure to determine the relationship between the measures of the given angles.



32. $\angle BFD$, $\angle BDF$

SOLUTION:

The side opposite $\angle BFD$ is $\overline{BD}$, which is of length 12.
The side opposite $\angle BDF$ is $\overline{BF}$, which is of length 15.
Since $\overline{BD} < \overline{BF}$ in ΔBDF, $m\angle BFD < m\angle BDF$ by Theorem 5.9.

ANSWER:

$m\angle BFD < m\angle BDF$

33. $\angle DBF$, $\angle BFD$

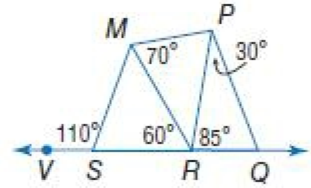
SOLUTION:

The side opposite $\angle DBF$ is $\overline{DF}$, which is of length 5.
The side opposite $\angle BFD$ is $\overline{BD}$, which is of length 12.
Since $\overline{DF} < \overline{BD}$ in ΔBDF, $m\angle DBF < m\angle BFD$ by Theorem 5.9.

ANSWER:

$m\angle DBF < m\angle BFD$

Use the figure to determine the relationship between the lengths of the given sides.



34. *SM*, *MR*

SOLUTION:

Since $\angle MSV$ and $\angle MSR$ are a linear pair, $m\angle MSR = 180 - 110 = 70$.
The side opposite $\angle SRM$ is $\overline{SM}$. The side opposite $\angle MSR$ is $\overline{MR}$. In ΔSRM, $m\angle SRM < m\angle MSR$, since 60 < 70. Therefore, by Theorem 5.10, $SM < MR$.

ANSWER:

$SM < MR$

35. *RP*, *MP*

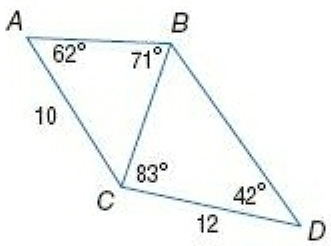
SOLUTION:

Since $\angle SRM$, $\angle MRP$, and $\angle PRQ$ form a straight angle, $m\angle MRP = 180 - (60 + 85)$ or 35.
The side opposite $\angle PMR$ is $\overline{RP}$. The side opposite $\angle MRP$ is $\overline{MP}$. In ΔPRM, $m\angle PMR > m\angle MRP$, since 70 < 35. Therefore, by Theorem 5.10, $RP > MP$.

ANSWER:

$RP > MP$

41. List the side lengths of the triangles in the figure from shortest to longest. Explain your reasoning.



SOLUTION:

AB, *BC*, *AC*, *CD*, *BD*;
In $\triangle ABC$, *AB* < *BC* < *AC* and in $\triangle BCD$, *BC* < *CD* < *BD*.
By the figure *AC* < *CD*, so *BC* < *AC* < *CD*.

ANSWER:

AB, *BC*, *AC*, *CD*, *BD*; In $\triangle ABC$, *AB* < *BC* < *AC* and in $\triangle BCD$, *BC* < *CD* < *BD*. By the figure *AC* < *CD*, so *BC* < *AC* < *CD*.

State the assumption you would make to start an indirect proof of each statement.

13. If two lines have the same slope, the lines are parallel.

ANSWER:

The lines are not parallel.

15. If a triangle is not equilateral, the triangle is not equiangular.

ANSWER:

The triangle is equiangular.

47. Colin is given a figure of triangle *PQR* that has some of the angle measures given. He is asked to write an indirect proof that $m\angle P < 45$. What assumption should Colin make to start his indirect proof?

A Assume that $m\angle P < 45$.
B Assume that $m\angle P \geq 45$.
C Assume that $m\angle P = 45$.
D Assume that $m\angle P > 45$.

ANSWER:

B

Is it possible to form a triangle with the given side lengths? If not, explain why not.

6. 4 ft, 9 ft, 15 ft

ANSWER:

No; $4 + 9 \ngtr 15$

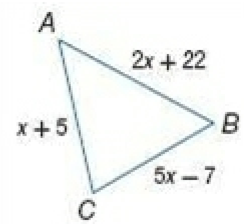
Find the range for the measure of the third side of a triangle given the measures of two sides.

14. 2.7 cm, 4.2 cm

ANSWER:

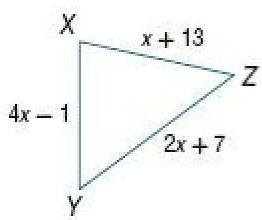
1.5 cm < n < 6.9 cm

SENSE-MAKING Determine the possible values of *x*.

20. 

ANSWER:

$6 < x < 17$

21. 

ANSWER:

$\frac{7}{5} < x < 21$

Find the range of possible measures of *x* if each set of expressions represents measures of the sides of a triangle.

26. 8, x, 12

ANSWER:

$4 < x < 20$

28. $x - 2$, 10, 12

ANSWER:

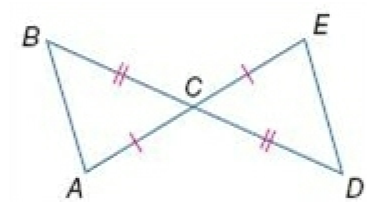
$4 < x < 24$

30. x, $2x + 1$, $x + 4$

ANSWER:

$x > \frac{3}{2}$

43. **REASONING** What is the range of possible perimeters for figure *ABCDE* if $AC = 7$ and $DC = 9$? Explain your reasoning.



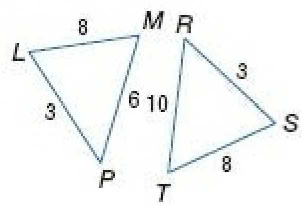
ANSWER:

The perimeter is greater than 36 and less than 64. Sample answer: From the diagram we know that $\overline{AC} \cong \overline{EC}$ and $\overline{DC} \cong \overline{BC}$, and $\angle ACB \cong \angle ECD$ because vertical angles are congruent, so $\Delta ACB \cong \Delta ECD$. Using the Triangle Inequality Theorem, the minimum value of *AB* and *ED* is 2 and the maximum value is 16. Therefore, the minimum value of the perimeter is greater than 2(2 + 7 + 9) or 36, and the maximum value of the perimeter is less than 2(16 + 7 + 9) or 64.

5-6 Inequalities in Two Triangles

Compare the given measures.

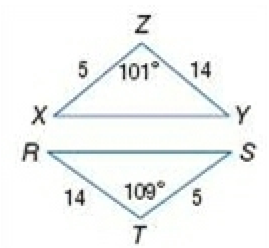
11. $m\angle MLP$ and $m\angle TSR$



SOLUTION:

In ΔMLP and ΔRST, $LP \cong RS, LM \cong ST$, and $MP < RT$. By the converse of the Hinge Theorem, $m\angle MLP < m\angle TSR$.

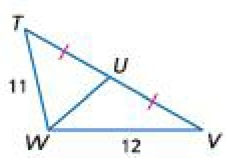
12. SR and XY



SOLUTION:

In ΔXYZ and ΔTSR, $ST \cong XZ, RT \cong ZY$, and $m\angle RTS > m\angle XZY$. By the Hinge Theorem, $SR > XY$.

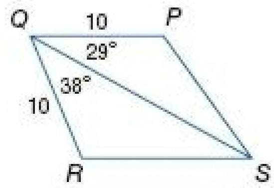
13. $m\angle TUW$ and $m\angle VUW$



SOLUTION:

In ΔTUW and ΔVUW, $TU \cong UV, UW \cong UW$, and $TW > WV$. By the converse of the Hinge Theorem, $m\angle TUW < m\angle VUW$.

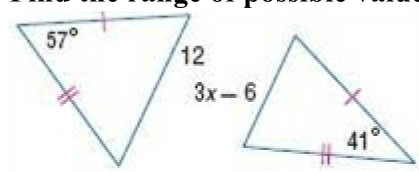
14. PS and SR



SOLUTION:

In ΔQPS and ΔQRS, $QP \cong QR, QS \cong QS$, and $m\angle RQS > m\angle SQP$. By the Hinge Theorem, $PS < SR$.

Find the range of possible values for *x*.

17. 

SOLUTION:

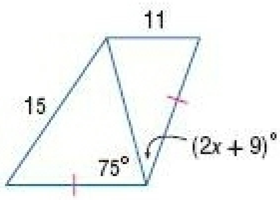
Since we have two pairs of congruent sides, by the Hinge Theorem, we can state that the side opposite the 57° angle is greater than the 41° angle. Therefore, we can write the inequality $12 > 3x - 6$.

$$12 + 6 > 3x - 6 + 6$$
$$18 > 3x$$
$$6 > x$$

Using the fact that the measure of any side is greater than 0, we can write a second inequality.

$$3x - 6 > 0$$
$$3x - 6 + 6 > 0 + 6$$
$$3x > 6$$
$$x > 2$$

Write $x > 2$ and $x < 6$ as the compound inequality $2 < x < 6$.

18. 

SOLUTION:

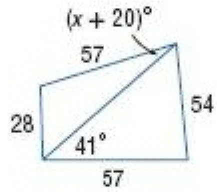
Since we have two pairs of congruent sides, by the Converse of the Hinge Theorem, we can state that the angle opposite the side with a length of 15 is greater than the angle opposite the side with a length of 11. Therefore, we can write the inequality $75 > 2x + 9$.

$$\begin{aligned} 75 &> 2x + 9 \\ 75 - 9 &> 2x + 9 - 9 \\ 66 &> 2x \\ 33 &> x \end{aligned}$$

Using the fact that the measure of any angle in a polygon is greater than 0, we can write a second inequality:

$$\begin{aligned} 2x + 9 &> 0 \\ 2x + 9 - 9 &> 0 - 9 \\ x &> -\frac{9}{2} \\ x &> -4.5 \end{aligned}$$

Write $x > -4.5$ and $x < 33$ as the compound inequality $-4.5 < x < 33$.

19. 

SOLUTION:

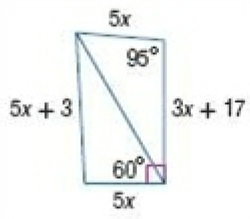
Since we have two pairs of congruent sides, by the Converse of the Hinge Theorem, we can state that the angle opposite the side with a length of 57 is greater than the angle opposite the side with a length of 54. Therefore, we can write and solve the inequality $41 > x + 20$.

$$\begin{aligned} 41 &> x + 20 \\ 41 - 20 &> x + 20 - 20 \\ 21 &> x \end{aligned}$$

Using the fact that the measure of any angle is greater than 0, we can write a second inequality:

$$\begin{aligned} x + 20 &> 0 \\ x + 20 - 20 &> 0 - 20 \\ x &> -20 \end{aligned}$$

Write $x > -20$ and $x < 21$ as the compound inequality $-20 < x < 21$.

20. 

SOLUTION:

First, find the missing angle measures in the diagram. Notice that the 60° angle is part of a right angle, which makes the angle adjacent to the 60° angle equal 30° Since you already know that another angle in this triangle is 95°, you can find the missing angle, across from the $3x + 17$ side, measures 55°, using the Triangle Sum Theorem.

Since we have two pairs of congruent sides, by the Hinge Theorem, we can state that the side opposite the 60-degree angle is greater than the 55-degree angle. Therefore, we can write the inequality $5x + 3 > 3x + 17$.

$$\begin{aligned} 5x+3 &> 3x+17 \\ 5x+3-3x &> 3x+17-3x \\ 2x+3 &> 17 \\ 2x &> 14 \\ x &> 7 \end{aligned}$$

Using the fact that any value of x greater than 7 will result in side lengths that are greater than zero, we can conclude the answer is $x < 7$.