# Unit 4 Notes

Tuesday, January 16, 2024 8:49 AM



## 4.1 Solving Systems of Linear Equations by Graphing

**EXAMPLE 1** Determining Whether an Ordered Pair Is a Solution

Yes/No ?

Decide whether the ordered pair (4, -3) is a solution of each equation. **What conclusion can we make?**

x y

**(a)** $x + 4y = -8$

$4 + 4(-3) = -8$

$4 + {}^-12 = -8$

$-8 = -8$ Yes!

**(b)** $2x + 5y = -7$

$2(4) + 5(-3) = -7$

$8 + {}^-15 = -7$

$-7 = -7$ Yes!

Since (4, -3) makes both true it is a SOLUTION to the SYSTEM

$\begin{cases} x + 4y = -8 \\ 2x + 5y = -7 \end{cases}$

**EXAMPLE 2** Solving a System by Graphing

Solve the system of equations by graphing both equations on the same axes.

2a) $y = -\frac{2}{3}x + 5$

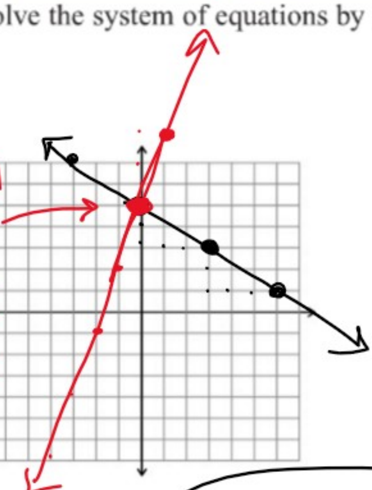
slope $\frac{-2}{3}$ $\frac{\text{rise}}{\text{run}}$

y-int. (0, 5)

$y = 3x + 5$

y-int (0, 5)

slope $= \frac{3}{1}$ $\frac{\text{rise}}{\text{run}}$



Solution (0, 5)

2b)

$y = \frac{1}{2}x - 3$

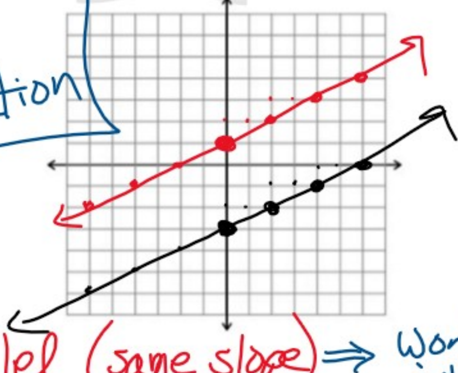
y-int (0, -3)

slope $\frac{1}{2}$ $\frac{\text{rise}}{\text{run}}$

$y = \frac{1}{2}x + 1$

y-int (0, 1)

No solution



parallel (same slope) ⟹ won't intersect!

2c)

$y = 3x - 3$

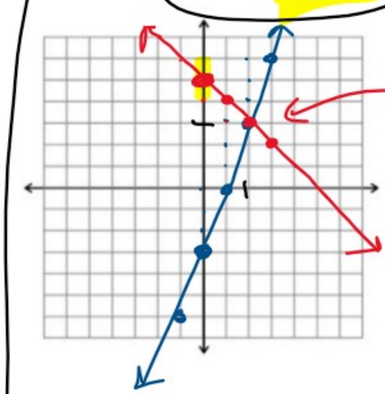
y-int (0, -3)

slope: $\frac{3}{1}$ $\frac{\text{rise}}{\text{run}}$

$y = -1x + 5$

y-int (0, 5)

slope: $\frac{-1}{1}$ $\frac{\text{rise}}{\text{run}}$



(2, 3)
x y

parallel (same slope) ⇒ won't intersect!



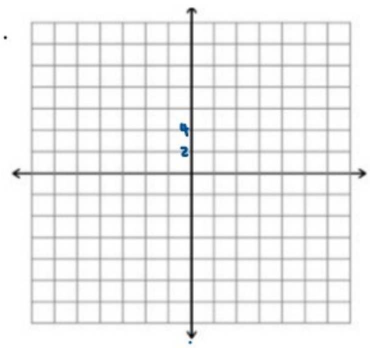
## EXAMPLE 3 Solving Special Systems by Graphing

Solve each system by graphing.

**(a)** $y = -2x + 2$

$y = -2x + 8$

HOLD



**(b)** $y = \boxed{\frac{-2}{5}}x + 2$

slope

y-int

$\boxed{6x + 15y = 30}$

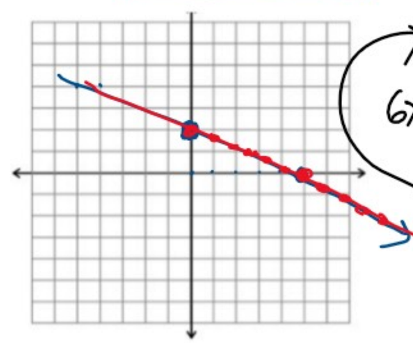
(0, 2)

(5, 0)

$6x + 15\cdot 0 = 30$

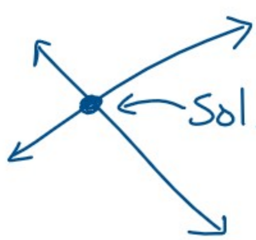
$\frac{6x}{6} = \frac{30}{6}$

$x = 5$

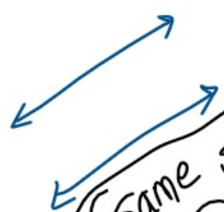


*Based on the slopes and y-intercepts of the lines, linear systems can have:* IMS.

| One solution | No solution | Infinitely Many Solutions |
| --- | --- | --- |
| | **"Inconsistent System"** | **"Dependent Equations"** |



Sol.



parallel

same slope different y-int

same line

· same slope

· same y-int.

## EXAMPLE 4 Identifying the Three Cases by Using Slopes

Describe each system without graphing. State the number of solutions.

**(a)** $3x + 2y = 6$

$-3x \qquad -3x$

$-2y = 3x - 5$

$\frac{-2y}{-2} = \frac{3x}{-2} - \frac{5}{-2}$

$\boxed{y = -\frac{3}{2}x + \frac{5}{2}}$

$\frac{2y}{2} = \frac{-3x}{2} + \frac{6}{2}$

$\boxed{y = -\frac{3}{2}x + 3}$

Same slope! ⇒ parallel → Incons. (No sol.)

**(b)** $2x - y = 4$

$x = \frac{y}{2} + 2$

**(c)** $x - 3y = 5$

$2x + y = 8$

$\boxed{y = \frac{\;}{2}x + 3}$ → ...allel
Inconsistent
(No sol.)



## 4.2 Solving Systems of Linear Equations by Substitution

**Steps:**

1) ✓ Solve one of the equations for a variable. (Get $x$ or $y$ by itself.)
2) ✓ **Substitute** whatever that variable equals into the *other* equation.
3) ✓ Solve that other equation. Now you have an answer for either $x$ or $y$.
4) Plug your answer back in to find the missing variable.
5) Write your answer as an ordered pair $(x, y)$.

**EXAMPLE 1** Using the Substitution Method

Solve the system by the substitution method.

$3x + 5y = 26$ → $3x+5(2x)=26$

y is by itself → $y = \boxed{2x}$

$3x + 5\cdot 2\cdot x = 26$

like terms: $3x + 10x = 26$

$\frac{13x}{13} = \frac{26}{13}$

$\boxed{x=2}$

$y = 2(2)$

$\boxed{y=4}$ → $(2, 4)$
x, y

**EXAMPLE 2** Using the Substitution Method

Solve the system by the substitution method.

$2x + 5y = 7$ → $2(-1-y)+5y=7$

$x = \boxed{-1 - y}$

$-2-2y+5y=7$

$-2+3y=7$
$+2 \qquad +2$

$\frac{3y}{3}=\frac{9}{3}$

$\boxed{y=3}$

$x=-1-(3)$

$x=-1-3$

$\boxed{x=-4}$

Sol: $(-4, 3)$
x, y

UNDO
P
E
MD
AS

**EXAMPLE 3** Using the Substitution Method

Use substitution to solve the system.

$y = \boxed{4 - 2x}$
$5x + 3y = 10$ → $5x+3(4-2x)=10$

$5x+12-6x=10$

$-1x+12=10$
$-12 \qquad -12$

$\frac{-1x}{-1} = \frac{-2}{-1}$

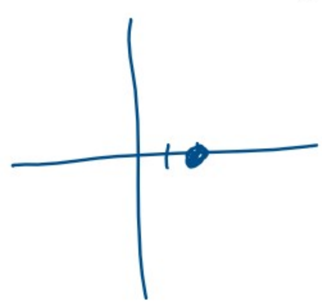
$\boxed{x=2}$

$y=4-2\cdot 2$

$y=4-4$

$\boxed{y=0}$

Sol. $(2, 0)$
x, y



**EXAMPLE 4** Solving an Inconsistent System by Substitution

parallel lines

Use substitution to solve the system.

$x = \boxed{5 - 2y}$

$2x + 4y = 6$ → $2(5-2y)+4y=6$

$10-\cancel{4y}+\cancel{4y}=6$

$10=6$

False → "No Solution"

**EXAMPLE 5** Solving a System with Dependent Equations by Substitution

same line

Solve the system by the substitution method.

Almost by itself

$-y = 4 - 3x$ → $-1y=4-3x$

$-9x + 3y = -12$

$\frac{-1y}{-1}=\frac{4}{-1}\frac{-3x}{-1}$

$y=\boxed{-4+3x}$

$-9x+3(-4+3x)=-12$

$\cancel{-9x}-12+\cancel{9x}=-12$

$-12=-12$

TRUE → "Infinitely Many Solutions"

$\{(x,y) \mid y=-4+3x\}$

**EXAMPLE 6** Using the Substitution Method

Solve the system by the substitution method.

Almost!

$12x + y = 8$ → $12x+y=8$

$2x + 3y = -10$

$-12x \quad -12x$

$y=\boxed{8-12x}$

$2x+3(8-12x)=-10$

$2x+24-36x=-10$

$-34x+24=-10$

$-24 \quad -24$

$-34x=-34$

$\frac{-34x}{-34}=\frac{-34}{-34}$

$\boxed{x=1}$

$y=8-12\cdot1$

$y=8-12$

$\boxed{y=-4}$

~~y=-12x+8~~

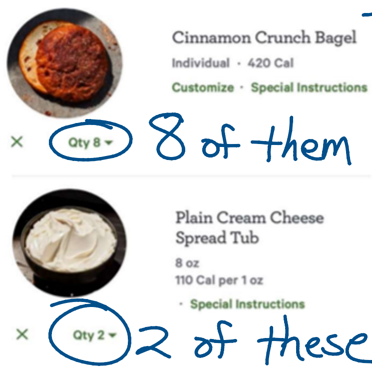
$\boxed{\text{Sol}: (1,-4)}$

x y

## 4.3 Solving Systems of Linear Equations by Elimination

Panera Bread is known for their delicious bagels, a popular choice for breakfast or a mid-afternoon snack. Peyton, Kelly, and Carter work together and take turns bringing food from Panera for their department each Friday.

1. Peyton's Panera Bread order is shown below. What is a ***possible*** cost for a single bagel and what is a ***possible*** cost for one tub of cream cheese? Explain your thinking. Answers will vary as there are many possibilities.




8 of them

2 of these

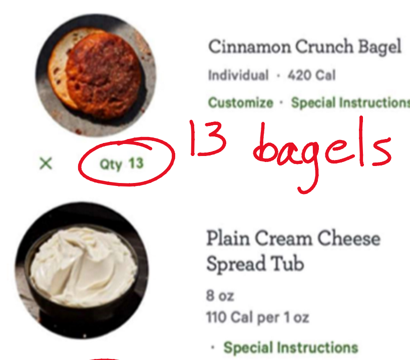
Subtotal: $19.50

2. Kelly bought 4 Cinnamon Crunch bagels and 1 tub of plain cream cheese. How much should Kelly's order cost? How do you know?

half! $19.50 \div 2 = 9.75$

3. Find the cost of a single bagel and a single tub of plain cream cheese **or** explain why this is not possible.

4. When it was Carter's turn to bring food, he ordered more bagels than Peyton and double the amount of cream cheese. His order is shown.




13 bagels

4 cheese

a. Should Carter's subtotal be exactly double Peyton's subtotal? Explain.

No – didn't double bagels

Peyton's ×2 39.00

b. Carter's subtotal $\boxed{\text{was } \$34.53}$. Can you figure out the cost of a single bagel?

Peyton ×2 − Carter's = 4.47 more

\$39 − \$34.53 = 4.47

bagels: 16 − 13 bagels = 3 bagels

$\$4.47 / 3 = \$1.49$

c. What is the cost of a single tub of cream cheese?

Carter: 13 bagels × 1.49 = 19.37

34.53 − 19.37 = \$15.16

\$15.16 ÷ 4 cheese = $\boxed{3.79}$

**EXAMPLE 1** Using the Elimination Method

Use the elimination method to solve the system.

$1x + \cancel{1y} = 5$
$+ 1x - \cancel{1y} = 3$

$\frac{2x}{2} = \frac{8}{2}$

$\boxed{x = 4}$

$(4, 1)$ x y

You are allowed to ADD equations!
It eliminates a var. if something cancels

$x + y = 5$
$4 + y = 5$
$-4 \quad -4$
$y = 1$

**EXAMPLE 2** Using the Elimination Method

Solve the system.

$x + 4y = 5$
$2x - y = 1$

Target to cancel

Need $4y$ & $-4y$

$1x + \cancel{4y} = 5$
$8x - \cancel{4y} = 4$

Now add!

$\frac{9x}{9} = \frac{9}{9}$

$\boxed{x = 1}$

Sol $(1, 1)$ x

$x + 4y = 5$
$1 + 4y = 5$
$-1 \quad -1$
$\frac{4y}{4} = \frac{4}{4}$
$y = 1$

**EXAMPLE 3** Using the Elimination Method

Solve the system.

$2x + 3y = -15 \xrightarrow{2} 4x + 6y = -30$
$5x + 2y = 1 \xrightarrow{x(-3)} -15x - 6y = -3$

Goal: ✓ $6y$ ✓ & $-6y$

$\frac{-11x}{-11} = \frac{-33}{-11}$

$\boxed{x = 3}$

$(3, -7)$ x y

$5x + 2y = 1$
$5(3) + 2y = 1$



STEPS
1. Get the equations so that something cancels
2. ADD
3. Solve little equation
4. Subst. back in to find the other

④ Subst. back in to find the other



3b) $2x + y = 8$ $\xrightarrow{\times 2}$ $4x + 2\cancel{y} = 16$

$5x - 2y = -16$ $\longrightarrow$ $5x - 2\cancel{y} = -16$

$9x = 0$

$\frac{9x}{9} = \frac{0}{9}$

$\boxed{x = 0}$

$2x + y = 8$

$2(0) + y = 8$

$y = 8$

Sol: $(0, 8)$ — x, y

3c) $2(6x - 2y) = -22 \cdot 2 \rightarrow 12x - \cancel{4y} = -44$

$-3x + 4y = 17 \rightarrow -3x + \cancel{4y} = 17$

$\frac{9x}{9} = \frac{-27}{9}$

$\boxed{x = -3}$

Sol. $(-3, 2)$

**EXAMPLE 5** Solving Special Systems Using the Elimination Method

Solve each system by the elimination method.

a. $2x + 4y = 5$ $\xrightarrow{\times(-2)}$ $-\cancel{4x} - \cancel{8y} = -10$

$4x + 8y = -9$ $\longrightarrow$ $\cancel{4x} + \cancel{8y} = -9$

$0 = -19$

Get $-4x$ & $4x$

No Sol — False $\Leftarrow$ Inconsistent System

b. $3(3x - y) = 4 \cdot 3$ $\xrightarrow{\times 3}$ $\cancel{9x} - \cancel{3y} = 12$

$-9x + 3y = -12$ $\rightarrow$ $-\cancel{9x} + \cancel{3y} = -12$

$0 = 0$

Get $-3y$, $3y$

Infinitely Many Sol. — True $\Leftarrow$ Dependent Equations

## 4.4 Applications of Linear Systems

**Example 1:** A coffee shop sells teas for \$4 each and coffees for \$5 each. If the coffee shop sold 9 drinks for a total of \$40, how many of each type of drink were sold?

Let $x = $ # of tea

$y = $ # of coffee

a) Write two equations to model this situation.

$x + y = 9$ drinks

$\frac{4x}{4} + 5y = \frac{\$40}{4}$

No tea $(0, 8)$

$(10, 0)$ No coffee

Solve for y

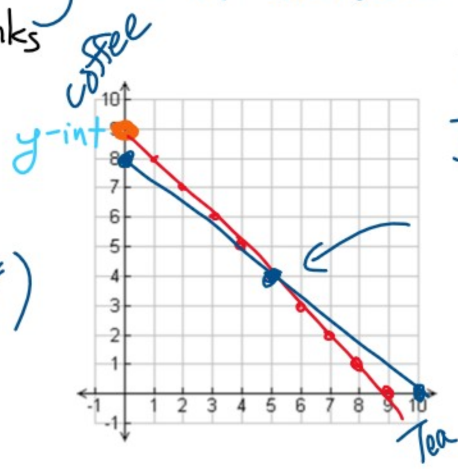
$-x + x + y = 9 - x$

$y = \boxed{9 - 1x}$

Subst. in for y

$y = -1x + 9$

$y = mx + b$




b) Solve the system.

$(5, 4)$

Teas = 5

Coffees = 4

**EXAMPLE 2** Solving a Problem about Quantities and Costs

For a production of the musical *Wicked* at the Ford Center in Chicago, main floor tickets cost $148, while the best balcony tickets cost $65. Suppose that the members of a club spent a total of $2614 for 30 tickets to *Wicked*. How many tickets of each kind did they buy? (*Source:* www.ticketmaster.com)

Let m = main floor
b = balcony tickets

$148m + 65b = \$2614$

$m + b = 30$
$-m \quad\quad -m$

$b = \boxed{30 - m}$

$148m + 65(30 - m) = 2614$
$148m + 1950 - 65m = 2614$
$83m + 1950 = 2614$
$83m = 664$

$8 + b = 30$
$-8 \quad\quad -8$
$\boxed{b = 22}$

**Example 3:**
Jonathan, a second grader, counted the money in his piggy bank. He had only quarters and dimes. When he added up his money, he had 39 coins worth a total of $7.50. How many coins of each kind did he have?

$q + d = 39 \rightarrow q + d = 39$

$.25q + .10d = \$7.50 \xrightarrow{\times 100} 25q + 10d = 750$

**Example 4:** y = cost x = how many customers
Lindsey and Gretchen work at two different hair salons and pay different amounts for their station. Lindsey pays $140 for rent, and $10 per customer that she works on that month. Gretchen only pays $100 for rent, but has to pay $18 per customer. How many customers would it take for them to pay the same amount?

L: $190 = \boxed{140 + 10 \cdot 5}$ $\rightarrow$ $y = 140 + 10 \cdot 5$

G: $190 = 100 + 18 \cdot 5$

$y = 140 + 50$

$\boxed{y = 190}$

$\boxed{(5, 190)}$
x y

Subst:
$140 + 10x = 100 + 18x$
$-10x \quad\quad -10x$
$140 = 100 + 8x$

## 4.5 Solving Systems of Linear Inequalities Date:

| Inequality symbols | Shading rules |
|---|---|
| For $<$ and $>$, draw a dotted boundary line | For $y <$ (Less), shade below |
| For $\leq$ and $\geq$, draw a solid boundary | For $y >$ (Greater), shade above |

Find overlap

**EXAMPLE 1** Solving a System of Linear Inequalities

Graph the solution set of the system.

$$\begin{cases} y > -\frac{1}{2}x + 1 \\ y \leq x + 4 \end{cases}$$

dotted (shade above)

solid (shade below)

y-int

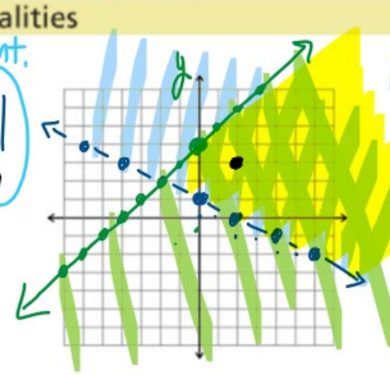
$m = \frac{1}{1}\ \frac{\text{rise}}{\text{run}}$

boundary $y = -\frac{1}{2}x + 1$

y-int.

$y = mx + b$

$m = -\frac{1}{2}\ \frac{\text{rise (down)}}{\text{run (rt.)}}$



Is $(2, 3)$ a solution? YES

$3 > -\frac{1}{2} \cdot \frac{2}{1} + 1$ ?

$3 > -1 + 1$ Yes :)

$3 \leq 2 + 4$

$3 \leq 6$ Yes :)

**EXAMPLE 2** Solving a System of Linear Inequalities

Graph the solution set of the system.

$$\begin{cases} y > x + 3 \\ -x + y \leq -5 \end{cases}$$

Dotted (shade above)

Get y alone

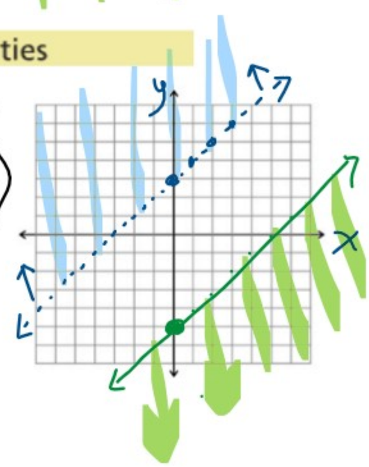
$+x \quad +x$

$y \leq x - 5$

y-int. $(0, -5)$

Solid (shade below)

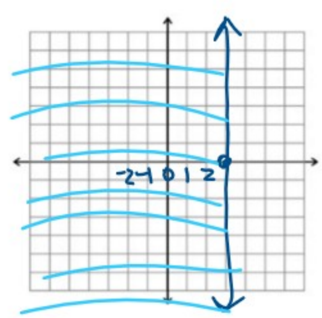
One big rule: If mult/divide BY a neg. turn the inequality!



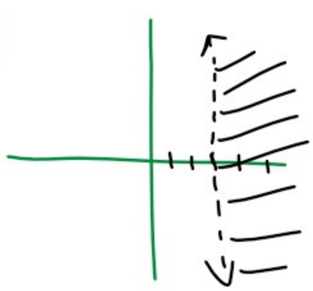
No overlap: No sol.

| More Shading rules |
|---|
| For $x <$ , shade left |
| For $x >$ , shade right |

$x \leq 3$




$x > 3$



**EXAMPLE 3** Solving a System of Three Linear Inequalities

Graph the solution set of the system.

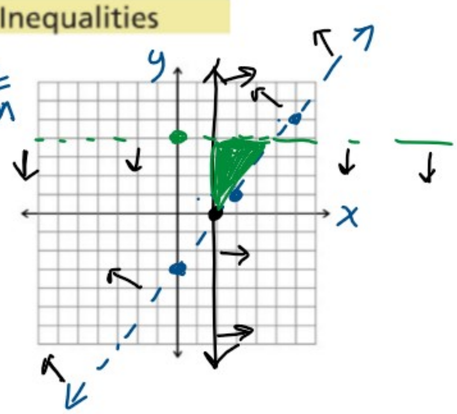
$$\begin{cases} y > \frac{4}{3}x - 3 \\ x \geq 2 \\ \boxed{y < 4} \end{cases}$$

dotted (shade up)

vertical

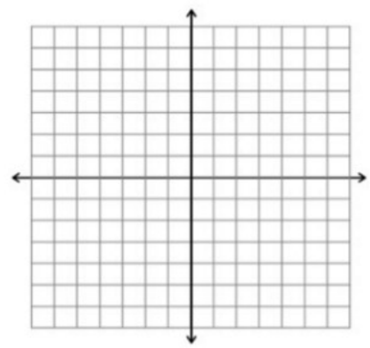
horiz.

$m = \frac{4}{3} \frac{\text{rise}}{\text{run}}$



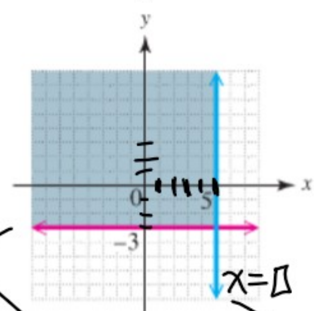

**Example 4:**

$$\begin{cases} x + y < 2 \\ x \geq -2 \\ y \leq 4 \end{cases}$$

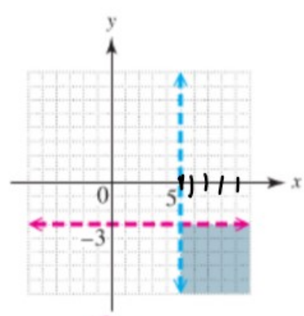


**Example 5**

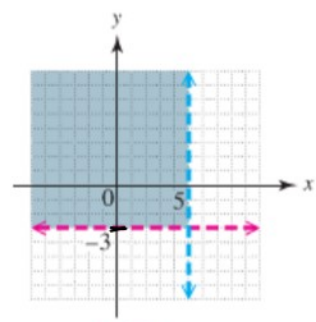
Write the equations for each system graphed below:




$$\begin{cases} y \geq -3 \\ x \leq 5 \end{cases}$$




$$\begin{cases} y < -3 \\ x > 5 \end{cases}$$




$$\begin{cases} y > -3 \\ x < 5 \end{cases}$$